



Corso Luigi Einaudi, 55 - Torino

Appunti universitari

Tesi di laurea

Cartoleria e cancelleria

Stampa file e fotocopie

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Rilegature

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DATA: 17/07/2013

A P P U N T I

STUDENTE: Fracchia

MATERIA: Aerodinamica Applicata

Prof. Di Cicca

Il presente lavoro nasce dall'impegno dell'autore ed è distribuito in accordo con il Centro Appunti.

Tutti i diritti sono riservati. È vietata qualsiasi riproduzione, copia totale o parziale, dei contenuti inseriti nel presente volume, ivi inclusa la memorizzazione, rielaborazione, diffusione o distribuzione dei contenuti stessi mediante qualunque supporto magnetico o cartaceo, piattaforma tecnologica o rete telematica, senza previa autorizzazione scritta dell'autore.

**ATTENZIONE: QUESTI APPUNTI SONO FATTI DA STUDENTIE NON SONO STATI VISIONATI DAL DOCENTE.
IL NOME DEL PROFESSORE, SERVE SOLO PER IDENTIFICARE IL CORSO.**

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Libri di testo:

Aerodinamica, 2^a edizione } ~ 36 €
F. Queri
L&B

Gasdinamica, Problemi risolti e richiami di teoria } ~ 29 €
F. Queri, G. Iuso
L&B

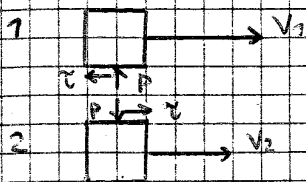
Aircraft Design. A Conceptual Approach } ~ 100 €
D.P. Raymer
AIAA Educational Series

Aerodynamics, Aeronautics and Flight Mechanics
Bernard W. McCormick
John Wiley & Sons

Modalità d'esame: il primo appello è scritto e gli altri orali.
Lo scritto è formato da domande a risposta multiple ed esercizi.

A.A. 2011/2012

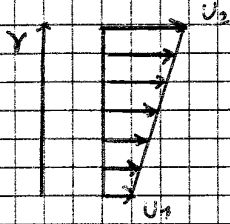
Esame da 5 crediti.



$v_1 > v_2$ (Gradiente di velocità)

Se non c'è nessun gradiente di velocità, cioè tutte le particelle si muovono con la stessa velocità, non ci sono sforzi di taglio.

Nel fluido possono essere presenti gradienti di temperatura. In quel caso si hanno dei flussi termici nel fluido (\vec{q}).



$$\tau \propto \frac{du}{dy} \rightarrow \tau = \mu \frac{du}{dy} \rightarrow \text{Legge di Newton}$$

$$[\mu] = \frac{\text{kg}}{\text{m} \cdot \text{s}} = \text{Pa} \cdot \text{s}$$

μ = Coefficiente di viscosità dinamica

ν = Coefficiente di viscosità cinematica

$$[\nu] = \frac{\text{m}^2}{\text{s}}$$

$$\nu = \frac{\mu}{\rho}$$

Nell'aria standard:

$$T = 15^\circ\text{C} = 288 \text{ K}$$

$$p = 101325 \text{ Pa}$$

$$\rho = 1,225 \text{ kg/m}^3$$

$$\nu = 1,454 \cdot 10^{-5} \text{ m}^2/\text{s}$$

$$\mu = 1,787 \cdot 10^{-5} \text{ kg/(m} \cdot \text{s)}$$

$$\vec{q} = -K \cdot \text{grad } T \rightarrow \text{Legge di Fourier}$$

K = Coefficiente di conducibilità termica

$$[K] = \frac{\text{W}}{\text{m} \cdot \text{K}}$$

$$\begin{cases} q_x = -K \frac{dT}{dx} \\ q_y = -K \frac{dT}{dy} \\ q_z = -K \frac{dT}{dz} \end{cases}$$

$M < 1 \rightarrow$ Subsonico

$M = 1 \rightarrow$ Sonico

$M > 1 \rightarrow$ Supersonico

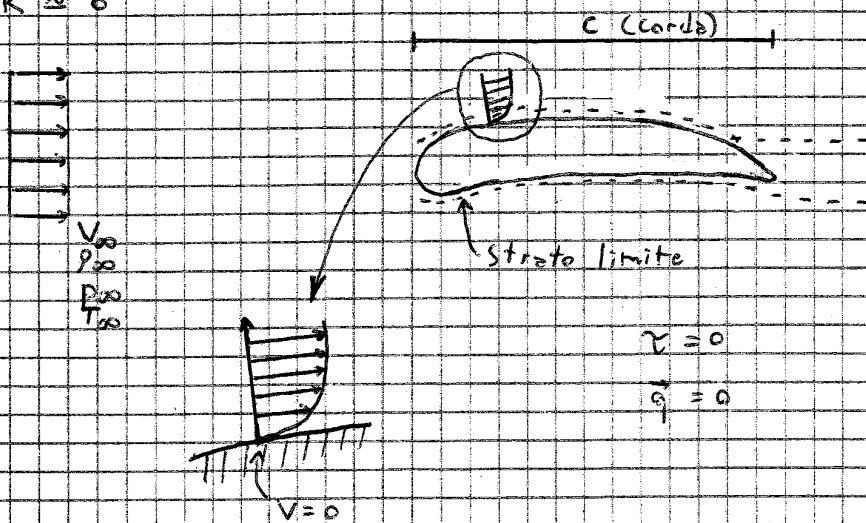
Nell'aria standard: $c_s = \sqrt{\gamma R^* T} = \sqrt{1,4 \cdot 287 \cdot 288} \approx 340 \text{ m/s}$

Nell'acqua: $c_s \approx 1000 \text{ m/s} = 1 \text{ km/s} \rightarrow$ Quindi nell'acqua si raggiungono solo Mach bassi.

Approssimazione di fluido ideale

$M \approx 0$

$K \approx 0$



$$Re = \frac{\rho V L}{\mu} = \frac{\rho V L}{\nu} \rightarrow \text{Numero di Reynolds}$$

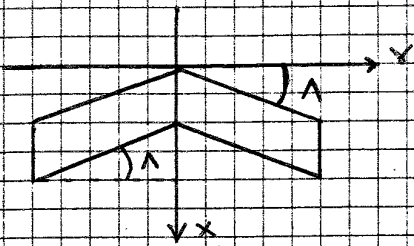
Se Re è elevato, gli effetti viscosi sono trascurabili rispetto a quelli inerziali.

Nell'esempio di prisma:

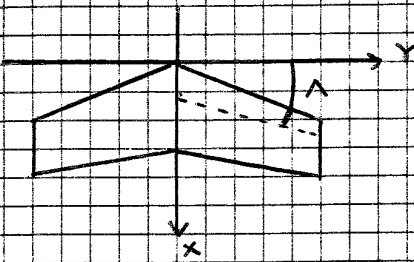
$$Re = \frac{\rho_\infty V_\infty C}{\mu_\infty}$$

dove C è la corda dell'ala.

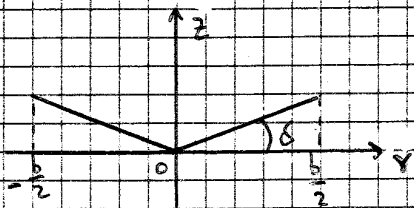
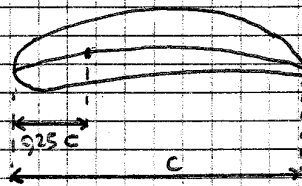
Ala a freccia



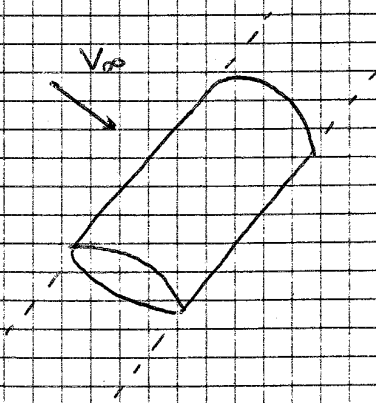
$\Delta = \text{Angolo di freccia}$



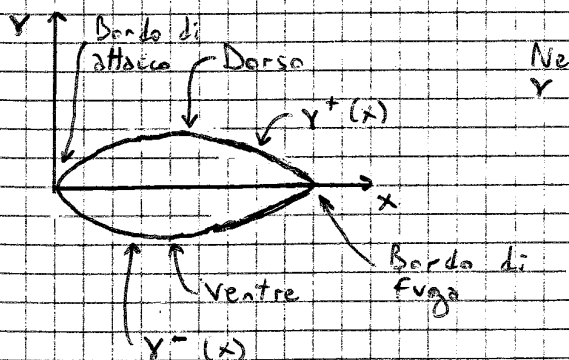
In questo caso l'angolo di freccia è l'angolo formato con la linea dei fuochi (la linea passante per i quarti anteriori delle corde).



$\delta = \text{Angolo diedro}$



(2D)



Nel sistema 2D si usano solo x e y. Y prende il posto di z (verticale).

$$R_x = \int_0^c dF_x^+ + \int_0^c dF_x^-$$

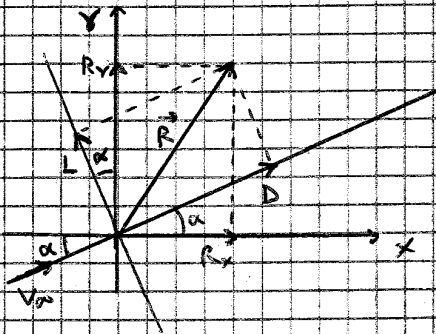
$$R_x = \int_0^c dF_x^+ + \int_0^c dF_x^- = - \int_0^c (p^+ - p_\infty) ds^+ \cos \beta + \int_0^c (p^- - p_\infty) ds^- \cos \beta$$

$$R_x = \int_0^c [(p^+ - p_\infty) - (p^- - p_\infty)] dx$$

$\hookrightarrow > 0$ $\hookrightarrow < 0$

R_x elevata \rightarrow Importante nel progetto

$$|R_x| \ll |R_y|$$



$D =$ Resistenza (drag)

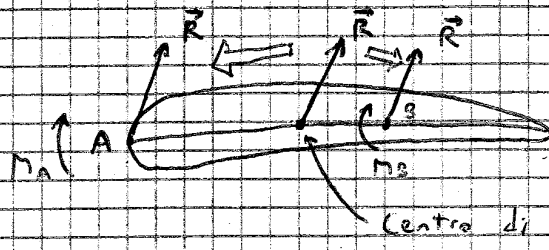
$L =$ Portanza (lift)

$$\begin{cases} L = R_y \cos \alpha - R_x \sin \alpha \\ D = R_y \sin \alpha + R_x \cos \alpha \end{cases}$$

\rightarrow Trascurabile

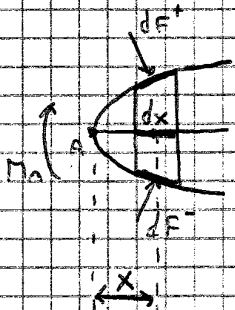
α piccolo $\rightarrow \sin \alpha \approx \alpha, \cos \alpha \approx 1$

$$L \approx R_y = \int_0^c [(p^- - p_\infty) - (p^+ - p_\infty)] dx$$



Centro di pressione (C.p.)

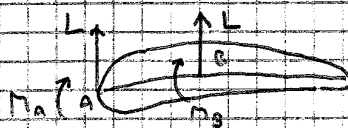
Traslando la risultante delle forze R dal centro di pressione al bordo d'attacco A si crea un momento M_A .
Lo stesso vale per un qualunque altro punto B.



$$dM_A = -dF_x^+ x - dF_x^- x = [(p^+ - p_\infty) - (p^- - p_\infty)] x dx$$

$$M_A = \int_0^c [(p^+ - p_\infty) - (p^- - p_\infty)] x dx$$

$$M_B = M_A + Lx$$



Si vuole trovare quel particolare punto B per cui il momento è nullo (B è un punto generico).

$$M_B = 0$$

$$M_A + Lx_B = 0$$

A/D

$$C_L = \frac{L}{\frac{1}{2} \rho_{\infty} V_{\infty}^2 S} = \text{Coefficiente di portanza}$$

$$C_D = \frac{D}{\frac{1}{2} \rho_{\infty} V_{\infty}^2 S} = \text{Coefficiente di resistenza}$$

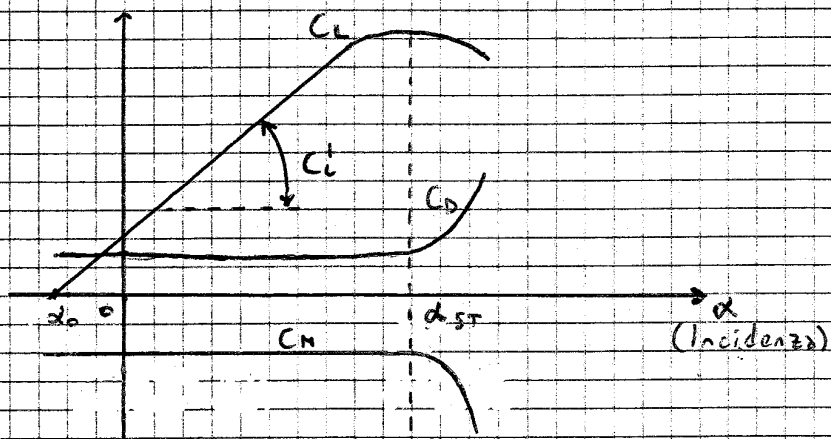
$$C_M = \frac{M}{\frac{1}{2} \rho_{\infty} V_{\infty}^2 S c} = \text{Coefficiente di momento}$$

Profilo

$$C_L = \frac{L}{\frac{1}{2} \rho_{\infty} V_{\infty}^2 c}$$

$$C_D = \frac{D}{\frac{1}{2} \rho_{\infty} V_{\infty}^2 c}$$

$$C_M = \frac{M}{\frac{1}{2} \rho_{\infty} V_{\infty}^2 c^2}$$



$\alpha_0 =$ Incidenza di portanza nulla

$\alpha_{ST} =$ Incidenza di stallo

$\alpha_2 = \alpha - \alpha_0 =$ Incidenza zero dinamica

$$\frac{dC_L}{d\alpha} = C_L' = \text{Coefficiente angolare di portanza}$$

$$C_L = C_L' (\alpha - \alpha_0) = 2\pi (\alpha - \alpha_0)$$

Coefficiente di pressione

$$C_p = \frac{p - p_{\infty}}{\frac{1}{2} \rho_{\infty} V_{\infty}^2}$$

$$L = \int_0^c [(p^- - p_{\infty}) - (p^+ - p_{\infty})] dx$$

$$C_L = \frac{L}{\frac{1}{2} \rho_{\infty} V_{\infty}^2 c} = \int_0^c \frac{(p^- - p_{\infty}) - (p^+ - p_{\infty})}{\frac{1}{2} \rho_{\infty} V_{\infty}^2 c} dx = \int_0^c (C_p^- - C_p^+) \frac{1}{c} dx = \int_0^1 (C_p^- - C_p^+) d\left(\frac{x}{c}\right)$$

$$\Delta C_p = C_p^- - C_p^+$$

$$C_L = \int_0^1 \Delta C_p d\left(\frac{x}{c}\right)$$

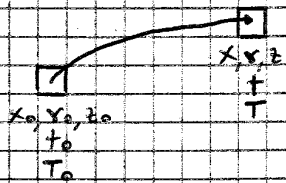
Linee di corrente e traiettorie

Le linee di corrente sono delle linee sempre tangenti al vettore velocità

La traiettoria è l'insieme dei punti che la particella occupa nel tempo.

Se il flusso è stazionario le linee di corrente coincidono con le traiettorie.

Metodo lagrangiano → Studia una singola particella



$$\begin{aligned} x &= x(x_0, y_0, z_0, t) & x_0 &= x(x_0, y_0, z_0, t) \\ y &= y(x_0, y_0, z_0, t) & y_0 &= y(x_0, y_0, z_0, t) \\ z &= z(x_0, y_0, z_0, t) & z_0 &= z(x_0, y_0, z_0, t) \\ T &= T(x_0, y_0, z_0, t) & T_0 &= T(x_0, y_0, z_0, t) \end{aligned}$$

$$U = \left(\frac{dx}{dt} \right)_{x_0, y_0, z_0}$$

$$\partial_x = \left(\frac{d^2x}{dt^2} \right)_{x_0, y_0, z_0}$$

$$V = \left(\frac{dy}{dt} \right)_{x_0, y_0, z_0}$$

$$\partial_y = \left(\frac{d^2y}{dt^2} \right)_{x_0, y_0, z_0}$$

$$W = \left(\frac{dz}{dt} \right)_{x_0, y_0, z_0}$$

$$\partial_z = \left(\frac{d^2z}{dt^2} \right)_{x_0, y_0, z_0}$$

Metodo euleriano → Studia un punto fisso

Funzioni di campo:

$$u(x, y, z, t)$$

$$v(x, y, z, t)$$

$$w(x, y, z, t)$$

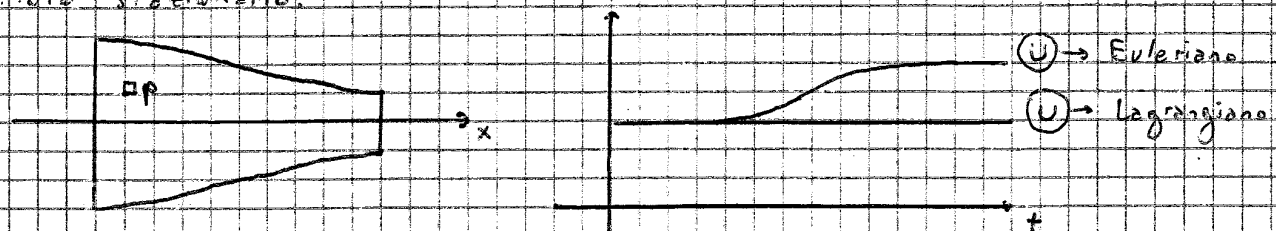
$$T(x, y, z, t)$$

Per rappresentare le velocità:

$$\frac{\partial u}{\partial t} \quad \frac{\partial v}{\partial t} \quad \frac{\partial w}{\partial t}$$

Esempio

Moto stazionario.



F scalare.

$$df = \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz$$

$$U = \frac{dx}{dt} \rightarrow \begin{cases} dx = u dt \\ dy = v dt \\ dz = w dt \end{cases}$$

Equazioni di bilancio in forma differenziale

Teorema di Gauss o della divergenza

$$\int_{\delta} \vec{F}_q \cdot \vec{n} d\delta = \int_{\tau} \nabla \cdot \vec{F}_q d\tau$$

$$\int_{\delta} \vec{F}_q \cdot \vec{n} d\delta = \int_{\tau} \nabla \cdot \vec{F}_q d\tau$$

Teorema del gradiente

$$\int_{\delta} F \vec{n} d\delta = \int_{\tau} \nabla F d\tau$$

Equazione di continuità

$$\int_{\tau} \frac{\partial \rho}{\partial t} d\tau = - \int_{\tau} \nabla \cdot (\rho \vec{v}) d\tau$$

$$\int_{\tau} \left[\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) \right] d\tau = 0$$

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0}$$

Equazione della quantità di moto

$$\int_{\tau} \frac{\partial (\rho \vec{v})}{\partial t} d\tau = - \int_{\tau} \nabla \cdot (\rho \vec{v} \vec{v}) d\tau - \int_{\tau} \nabla p d\tau + \int_{\tau} \rho \vec{F} d\tau$$

$$\int_{\tau} \left[\frac{\partial (\rho \vec{v})}{\partial t} + \nabla \cdot (\rho \vec{v} \vec{v}) \right] d\tau = - \int_{\tau} \nabla p d\tau + \int_{\tau} \rho \vec{F} d\tau$$

$$\frac{\partial (\rho \vec{v})}{\partial t} + \nabla \cdot (\rho \vec{v} \vec{v}) = - \nabla p + \rho \vec{F}$$

$$\rho \frac{\partial \vec{v}}{\partial t} + \vec{v} \frac{\partial \rho}{\partial t} + \vec{v} \nabla \cdot (\rho \vec{v}) + \rho \vec{v} \cdot \nabla \vec{v} = - \nabla p + \rho \vec{F}$$

$$\rho \left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \nabla \cdot \vec{v} \right) + \vec{v} \left[\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) \right] = - \nabla p + \rho \vec{F}$$

= 0 perché è l'equazione di continuità

$$\boxed{\frac{\partial \vec{v}}{\partial t} + \vec{v} \nabla \cdot \vec{v} = - \frac{1}{\rho} \nabla p + \vec{F}}$$

Equazione dell'energia

$$\int_{\tau} \frac{\partial (\rho E)}{\partial t} d\tau = - \int_{\tau} \nabla \cdot (\rho E \vec{v}) d\tau - \int_{\tau} \nabla \cdot (\rho \vec{v}) d\tau + \int_{\tau} \rho \vec{F} \cdot \vec{v} d\tau$$

$$\frac{\partial (\rho E)}{\partial t} + \nabla \cdot (\rho E \vec{v}) = - \nabla \cdot (\rho \vec{v}) + \rho \vec{F} \cdot \vec{v}$$

$$\rho \frac{\partial E}{\partial t} + E \frac{\partial \rho}{\partial t} + E \nabla \cdot (\rho \vec{v}) + \rho \vec{v} \cdot \nabla E = - \nabla \cdot (\rho \vec{v}) + \rho \vec{F} \cdot \vec{v}$$

$$\rho \left(\frac{\partial E}{\partial t} + \vec{v} \cdot \nabla E \right) + E \left[\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) \right] = - \nabla \cdot (\rho \vec{v}) + \rho \vec{F} \cdot \vec{v}$$

= 0 perché è l'eq. di continuità

$$\boxed{\frac{\partial E}{\partial t} + \vec{v} \cdot \nabla E = - \frac{1}{\rho} \nabla \cdot (\rho \vec{v}) + \vec{F} \cdot \vec{v}}$$

$$\frac{D}{Dt} \left(\frac{v^2}{2} \right) = -\frac{1}{\rho} \vec{v} \cdot \nabla p + \vec{F} \cdot \vec{v}$$

$$\frac{De}{Dt} = -\frac{1}{\rho} p \nabla \cdot \vec{v} + \frac{D}{Dt} \left(\frac{v^2}{2} \right)$$

$$\frac{D}{Dt} \left(e - \frac{v^2}{2} \right) = -\frac{1}{\rho} p \nabla \cdot \vec{v}$$

$$\boxed{\frac{De}{Dt} = -\frac{p}{\rho} \nabla \cdot \vec{v}}$$

$$\frac{Dp}{Dt} = -\rho \nabla \cdot \vec{v}$$

$$\frac{D}{Dt} \left(\frac{1}{\rho} \right) = -\frac{1}{\rho^2} \frac{Dp}{Dt}$$

$$\frac{Dp}{Dt} = -\rho^2 \frac{D}{Dt} \left(\frac{1}{\rho} \right) = -\rho \nabla \cdot \vec{v}$$

$$\nabla \cdot \vec{v} = \rho \frac{D}{Dt} \left(\frac{1}{\rho} \right)$$

$$\frac{De}{Dt} = -\frac{p}{\rho} \rho \frac{D}{Dt} \left(\frac{1}{\rho} \right)$$

$$\boxed{\frac{De}{Dt} + p \frac{D}{Dt} \left(\frac{1}{\rho} \right) = 0} \rightarrow \text{Aerodinamica}$$

$$de + p d\left(\frac{1}{\rho}\right) = 0 \rightarrow \text{Termodinamica}$$

$$de + p d\left(\frac{1}{\rho}\right) = T ds$$

$$\frac{De}{Dt} + p \frac{D}{Dt} \left(\frac{1}{\rho} \right) = T \frac{Ds}{Dt}$$

$\frac{Ds}{Dt} = 0 \rightarrow$ Isentropica (l'entropia di una particella è costante lungo la sua traiettoria).
 $\nabla s = 0 \rightarrow$ Omoentropica (entropia costante in tutto il campo).

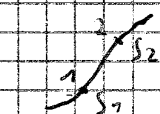
$$T ds = de + p d\left(\frac{1}{\rho}\right)$$

$$ds = \frac{de}{T} + \frac{p}{T} d\left(\frac{1}{\rho}\right)$$

Poiché $\begin{cases} p = \rho R T \\ \frac{p}{\rho} = R^* T \end{cases}$

$$ds = c_v \frac{dT}{T} + p R^* d\left(\frac{1}{\rho}\right)$$

$$ds = c_v \frac{dT}{T} + R^* \frac{d\left(\frac{1}{\rho}\right)}{\left(\frac{1}{\rho}\right)}$$



$$\int_1^2 ds = c_v \int_1^2 \frac{dT}{T} + R^* \int_1^2 \frac{d\left(\frac{1}{\rho}\right)}{\left(\frac{1}{\rho}\right)}$$

$$s_2 - s_1 = c_v \ln \frac{T_2}{T_1} + R^* \ln \frac{\rho_1}{\rho_2}$$

$\nabla s = 0 =$

→ Campo stazionario

→ $h + \frac{v^2}{2} = \text{cost.} = H$

→ $\vec{v} \times \vec{\omega} = 0 \rightarrow \vec{v} = 0$ oppure $\vec{\omega} = 0$ (In quest'ultimo caso si ha che il campo è irrotazionale).
 Se queste 3 condizioni sono soddisfatte, il campo è ortocentrico.

Ora si considera di avere un fluido stazionario e incompressibile.

Stazionaria $\rightarrow \frac{\partial}{\partial t} = 0$

Incompressibile $\rightarrow \rho = \text{cost.}$

$\frac{\partial \vec{v}}{\partial t} = (\vec{v} \times \vec{\omega}) = -\frac{1}{\rho} \nabla p - \nabla \left(\frac{v^2}{2} \right)$

$\vec{v} = v \vec{e}$

$-(\vec{v} \times \vec{\omega}) \cdot \vec{e} = -\frac{1}{\rho} \nabla p \cdot \vec{e} - \nabla \left(\frac{v^2}{2} \right) \cdot \vec{e}$

$= 0$ perché è perpendicolare sia a \vec{v} che a $\vec{\omega}$.

$+\frac{1}{\rho} \frac{dp}{ds} + \frac{d}{ds} \left(\frac{v^2}{2} \right) = 0$

$\frac{d}{ds} \left(p + \frac{1}{2} \rho v^2 \right) = 0$

$p + \frac{1}{2} \rho v^2 = \text{cost. (lungo la linea di corrente)}$

In un fluido stazionario e incompressibile

Pressione statica
 Pressione dinamica

Ora si considera di avere un fluido incompressibile e irrotazionale.

Incompressibile $\rightarrow \rho = \text{cost.} \rightarrow \nabla \cdot \vec{v} = 0$

Irrotazionale $\rightarrow \nabla \times \vec{v} = 0 \rightarrow \vec{\omega} = 0$

Se un campo è irrotazionale, ammette un potenziale $\phi \rightarrow \vec{v} = \nabla \phi$

$\nabla \cdot \vec{v} = \nabla \cdot (\nabla \phi) = 0$

$\text{div}(\text{grad } \phi) = 0$

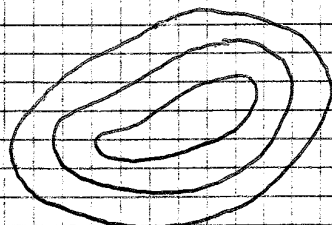
$\text{div}(\text{grad}) = \nabla^2$

$\nabla^2 \phi = 0 \rightarrow$ Equazione di Laplace

Operatore Laplaciano

L'equazione di Laplace è lineare.

$$\begin{cases} u = \frac{d\phi}{dx} \\ v = \frac{d\phi}{dy} \\ w = \frac{d\phi}{dz} \end{cases}$$



Linee ϕ costante (su ogni linea $\phi = \text{cost.}$)

$\frac{\partial \vec{v}}{\partial t} = (\vec{v} \times \vec{\omega}) = -\frac{1}{\rho} \nabla p - \nabla \left(\frac{v^2}{2} \right) + \vec{g}$

$= 0$ perché $\vec{\omega} = 0$.

$\frac{dv}{dt} = \frac{\partial}{\partial t} (\nabla \phi) = \nabla \left(\frac{d\phi}{dt} \right)$

$\frac{\partial u}{\partial t} = \frac{\partial}{\partial t} \left(\frac{d\phi}{dx} \right)$
 $\frac{\partial v}{\partial t} = \frac{\partial}{\partial t} \left(\frac{d\phi}{dy} \right)$
 $\frac{\partial w}{\partial t} = \frac{\partial}{\partial t} \left(\frac{d\phi}{dz} \right)$

Equazione della vorticità

$$\frac{\partial \vec{v}}{\partial t} - (\vec{v} \times \vec{\omega}) = -\frac{1}{\rho} \nabla p - \nabla \left(\frac{v^2}{2} \right) + \vec{F}$$

Bisogna calcolare il rotore di questa equazione

$$\nabla \times \frac{\partial \vec{v}}{\partial t} - \nabla \times (\vec{v} \times \vec{\omega}) = -\nabla \times \left(\frac{1}{\rho} \nabla p \right) - \nabla \times \nabla \left(\frac{v^2}{2} \right) + \nabla \times \vec{F}$$

$$1) \nabla \times \frac{\partial \vec{v}}{\partial t} = \frac{\partial}{\partial t} (\nabla \times \vec{v}) = \frac{\partial \vec{\omega}}{\partial t}$$

$$2) -\nabla \times (\vec{v} \times \vec{\omega}) = -\vec{v} (\nabla \cdot \vec{\omega}) + \vec{v} \cdot \nabla \vec{\omega} - \vec{\omega} \cdot \nabla \vec{v} + \vec{\omega} (\nabla \cdot \vec{v})$$

$$3) -\nabla \times \left(\frac{1}{\rho} \nabla p \right) = -\frac{1}{\rho} (\nabla \times \nabla p) - \nabla \left(\frac{1}{\rho} \right) \times \nabla p$$

$$\frac{\partial \vec{\omega}}{\partial t} + (\vec{v} \cdot \nabla) \vec{\omega} = (\vec{\omega} \cdot \nabla) \vec{v} - \vec{\omega} (\nabla \cdot \vec{v}) - \nabla \left(\frac{1}{\rho} \right) \times \nabla p + \nabla \times \vec{F}$$

Questi termini servono a distribuire la vorticità nel campo di moto. | Questi termini servono a generare la vorticità.

Ora si analizza il caso incomprimibile ($\rho = \text{cost.}$):

$$\nabla \cdot \vec{v} = 0$$

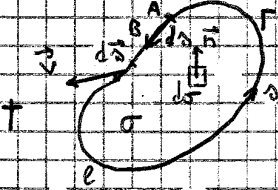
$$\frac{D\vec{\omega}}{Dt} = \vec{\omega} \cdot \nabla \vec{v} + \nabla \times \vec{F}$$

Si suppone ora che:

- a) $\nabla \times \vec{F} \equiv 0$
- b) il campo è bidimensionale $\rightarrow \vec{\omega} \perp \vec{v}$

Allora $\frac{D\vec{\omega}}{Dt} = 0 \rightarrow \vec{\omega}$ è costante lungo tutta la traiettoria della particella. Se il campo inizialmente è irrotazionale, rimane irrotazionale.

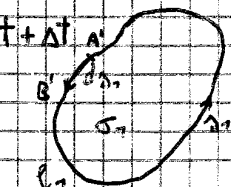
Prendiamo una superficie σ contornata da una linea Γ :



$$\Gamma = \oint_{\Gamma} \vec{v} \cdot d\vec{s} = \int_{\sigma} (\nabla \times \vec{v}) \cdot \vec{n} \, d\sigma$$

Circuitazione \rightarrow per il teorema di Gauss

$$d\vec{s} = d\sigma \cdot \vec{n}$$



$$\frac{D\Gamma}{Dt} = \lim_{\Delta t \rightarrow 0} \frac{\Gamma_2 - \Gamma_1}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} \left[\oint_{\Gamma_2} \vec{v}_2 \cdot d\vec{s}_2 - \oint_{\Gamma_1} \vec{v}_1 \cdot d\vec{s}_1 \right] =$$

$$\frac{D\Gamma}{Dt} = \oint_{\Gamma} \frac{D}{Dt} (\vec{v} \cdot d\vec{s}) = \oint_{\Gamma} \vec{v} \cdot \frac{D}{Dt} (d\vec{s}) + \oint_{\Gamma} \frac{D\vec{v}}{Dt} \cdot d\vec{s} = *$$

$$\frac{D}{Dt} (d\vec{s}) = \lim_{\Delta t \rightarrow 0} \frac{d\vec{s}_2 - d\vec{s}_1}{\Delta t}$$

$$\vec{A}A_2 + d\vec{s}_2 = \vec{B}B_2 + d\vec{s}_1 \rightarrow d\vec{s}_2 - d\vec{s}_1 = \vec{B}B_2 - \vec{A}A_2 = (\vec{v} + \vec{v}') \Delta t = -\vec{v} \Delta t = d\vec{v} \Delta t$$

$$* = \oint_{\Gamma} \vec{v} \cdot d\vec{v} + \oint_{\Gamma} \frac{D\vec{v}}{Dt} \cdot d\vec{s} = \oint_{\Gamma} d\left(\frac{v^2}{2} \right) = 0$$

Teorema di Bernoulli
 È valido per una corrente stazionaria, incomprimibile e irrotazionale.
 Si consideri un fluido stazionario e incomprimibile. $\nabla \times \vec{V} = 0$
 $\frac{\partial \rho}{\partial t} = 0$ $\rho = \text{cost.}$

$$\frac{DE}{Dt} = - \frac{1}{\rho} \nabla \cdot (\rho \vec{V}) = *$$

$$\nabla \cdot (\rho \vec{V}) = \rho \nabla \cdot \vec{V} + \vec{V} \cdot \nabla \rho$$

$$* = - \frac{1}{\rho} \rho (\nabla \cdot \vec{V}) - \frac{1}{\rho} \vec{V} \cdot \nabla \rho$$

$$\frac{D\rho}{Dt} = - \rho (\nabla \cdot \vec{V})$$

$$\frac{D}{Dt} \left(\frac{1}{\rho} \right) = - \frac{1}{\rho^2} \frac{D\rho}{Dt}$$

$$\frac{D\rho}{Dt} = - \rho^2 \frac{D}{Dt} \left(\frac{1}{\rho} \right)$$

$$\nabla \cdot \vec{V} = - \frac{1}{\rho} \frac{D\rho}{Dt} = \rho \frac{D}{Dt} \left(\frac{1}{\rho} \right)$$

$$\frac{DE}{Dt} = - \frac{1}{\rho} \rho \rho \frac{D}{Dt} \left(\frac{1}{\rho} \right) - \frac{1}{\rho} \vec{V} \cdot \nabla \rho$$

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + \vec{V} \cdot \nabla \rho$$

$$\vec{V} \cdot \nabla \rho = \frac{D\rho}{Dt} - \frac{\partial \rho}{\partial t}$$

$$\frac{DE}{Dt} = - \rho \frac{D}{Dt} \left(\frac{1}{\rho} \right) - \frac{1}{\rho} \frac{D\rho}{Dt} + \frac{1}{\rho} \frac{\partial \rho}{\partial t}$$

$$- \frac{D}{Dt} \left(\frac{\rho}{\rho} \right)$$

$$\frac{DE}{Dt} + \frac{D}{Dt} \left(\frac{\rho}{\rho} \right) = \frac{1}{\rho} \frac{\partial \rho}{\partial t}$$

$$E = e + \frac{v^2}{2}$$

$$h = \text{Entalpia} = e + \frac{p}{\rho}$$

$$\frac{D}{Dt} \left(e + \frac{v^2}{2} \right) + \frac{D}{Dt} \left(\frac{p}{\rho} \right) = \frac{1}{\rho} \frac{\partial p}{\partial t}$$

$$\frac{D}{Dt} \left(h + \frac{v^2}{2} \right) = \frac{1}{\rho} \frac{\partial p}{\partial t} \rightarrow \text{Altra forma dell'equazione dell'energia}$$

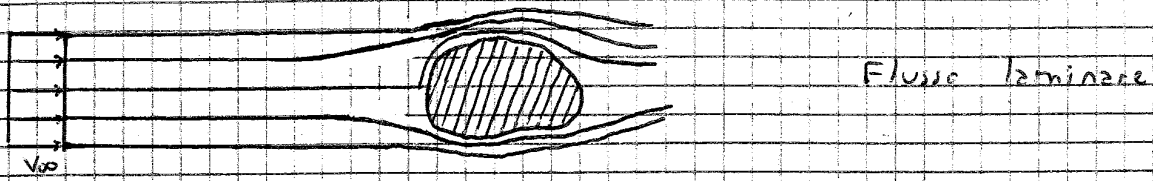
~~$$\frac{\partial}{\partial t} \left(h + \frac{v^2}{2} \right) + \vec{V} \cdot \nabla \left(h + \frac{v^2}{2} \right) = \frac{1}{\rho} \frac{\partial p}{\partial t}$$~~

$$\vec{V} \cdot \nabla \left(h + \frac{v^2}{2} \right) = 0$$

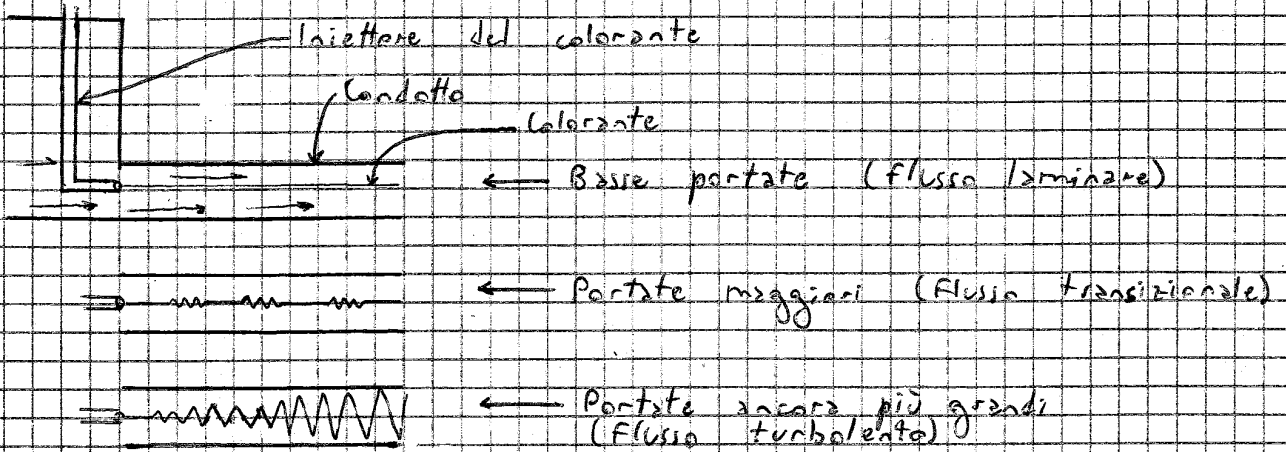
$$\frac{d}{ds} \left(h + \frac{v^2}{2} \right) = 0$$

$$h + \frac{v^2}{2} = \text{cost. su una linea di corrente}$$

Turbolenza



Esperimento di Reynolds



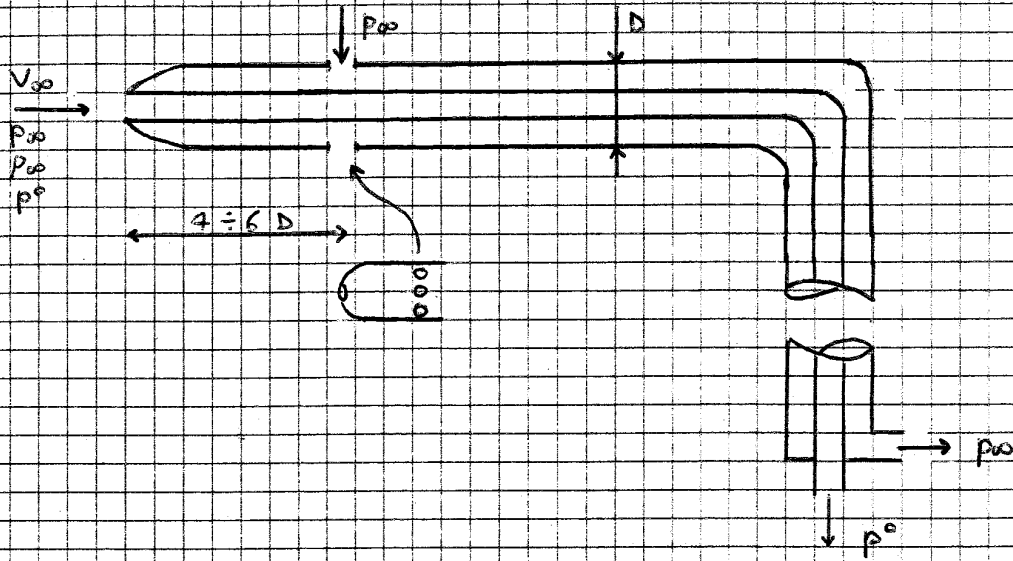
Reynolds ha notato che la transizione tra il flusso laminare e quello turbolento cominciava quando

$$\frac{UD}{\nu} \cong 2300$$

da cui $Re = \frac{UD}{\nu} = \text{Numero di Reynolds}$

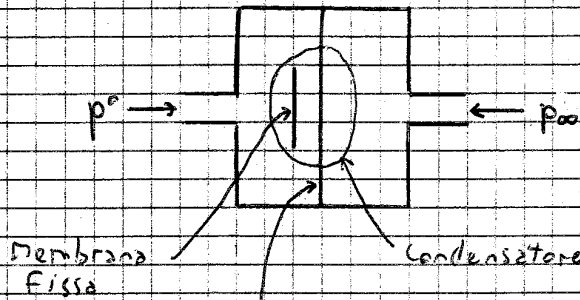
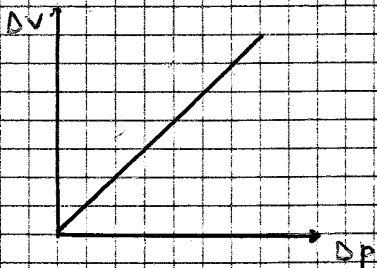
Esercitazione

Il tubo di Pitot



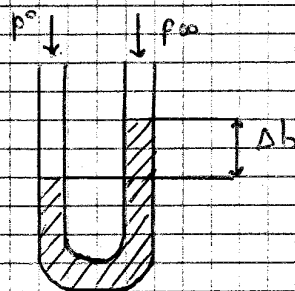
Quando il sistema è a regime, non c'è movimento di fluido nel tubo di Pitot.

Trasduttore di pressione:



Questa membrana viene deformata dalla variazione di pressione avvicinandosi o allontanandosi dalla membrana fissa e modificando le proprietà del condensatore.

Tubo di pressione:



$$\Delta p = p^0 - p_{00} = \frac{\rho}{2} V_{\infty}^2$$

$$V_{\infty} = \sqrt{\frac{2 \Delta p}{\rho}}$$

$$\Delta p = \rho \cdot g \cdot \Delta h$$

↓ Densità del liquido

Esercizio

$(\Delta h)_1 = 100 \text{ mm H}_2\text{O}$

$V_{\infty 1} = ?$

$(\Delta h)_2 = 200 \text{ mm H}_2\text{O}$

$V_{\infty 2} = ?$

$T_{amb} = 16 \text{ }^\circ\text{C}$

$p_{amb} = 760 \text{ mm Hg}$

$\gamma_{Hg} = 13,59 \text{ kgf/dm}^3$

$$\rho_{H_2O} = 1 \text{ kg/dm}^3 = 9810 \text{ N/m}^3$$

$$\Delta p = \rho_{H_2O} \cdot g \cdot Dh = \rho_{H_2O} \cdot Dh = 9810 \cdot 0,03 = 294,3 \text{ Pa}$$

$$v_c = \sqrt{\frac{2 \cdot 294,3}{1,225 \cdot (4^2 - 1)}} = 5,66 \text{ m/s}$$

Equazioni di Navier - Stokes mediate alla Reynolds

$$\left\{ \begin{aligned} \frac{Dp}{Dt} &= -\rho \nabla \cdot \vec{v} \end{aligned} \right.$$

$$\left\{ \begin{aligned} \frac{D\vec{v}}{Dt} &= -\frac{1}{\rho} \nabla p + \vec{g} + \nu \nabla^2 \vec{v} + \frac{\nu}{3} \nabla (\nabla \cdot \vec{v}) \end{aligned} \right.$$

$$\left\{ \begin{aligned} \rho \frac{De}{Dt} &= K \nabla^2 T - p (\nabla \cdot \vec{v}) + D \end{aligned} \right.$$

↳ Equazioni di Navier - Stokes originali

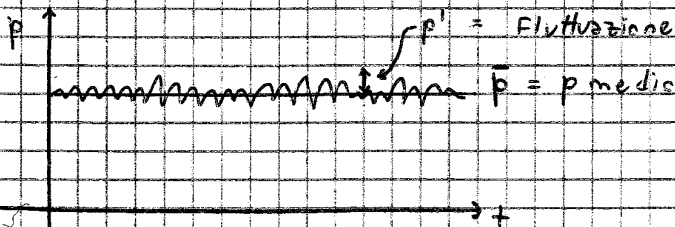
$$e = c_v T$$

$$\frac{p}{\rho} = R^* T$$

$$\frac{D\vec{v}}{Dt} = \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v}$$

Si mediano solo le prime 2 equazioni, tralasciando quelle dell'energia.

$$\left\{ \begin{aligned} \nabla \cdot \vec{v} &= 0 \\ \frac{D\vec{v}}{Dt} &= -\nabla p + \nu \nabla^2 \vec{v} \end{aligned} \right.$$



$$\vec{v} = \vec{\bar{v}} + \vec{v}'$$

$$p = \bar{p} + p'$$

$$\rho \left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = -\nabla p + \mu \nabla^2 \vec{v}$$

$$\nabla \cdot (\vec{\bar{v}} + \vec{v}') = 0$$

La linea sopra indica che ne facciamo la media.

$$\rho \left[\frac{\partial (\vec{\bar{v}} + \vec{v}')}{\partial t} + (\vec{\bar{v}} + \vec{v}') \cdot \nabla (\vec{\bar{v}} + \vec{v}') \right] = -\nabla (\bar{p} + p') + \mu \nabla^2 (\vec{\bar{v}} + \vec{v}')$$

$$\nabla \cdot \vec{\bar{v}} = 0$$

$$\rho \left[\frac{\partial \vec{\bar{v}}}{\partial t} + \overline{(\vec{\bar{v}} + \vec{v}') \cdot \nabla (\vec{\bar{v}} + \vec{v}')} \right] = -\nabla \bar{p} + \mu \nabla^2 \vec{\bar{v}}$$

$$\nabla \cdot (\vec{\bar{v}} \vec{\bar{v}}) = \vec{\bar{v}} \cdot \nabla \vec{\bar{v}} + \vec{\bar{v}} \cdot \nabla (\vec{\bar{v}} \cdot \vec{\bar{v}})$$

$$\overline{(\vec{\bar{v}} + \vec{v}') \cdot \nabla (\vec{\bar{v}} + \vec{v}')} = \nabla \cdot [(\vec{\bar{v}} + \vec{v}')(\vec{\bar{v}} + \vec{v}')]$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{u_e}{L} \frac{v}{\delta} = 0 \rightarrow \boxed{v \propto \frac{u_e \delta}{L}}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

$$u_e \frac{u_e}{L} \frac{u_e^2 \delta}{L \delta} = \frac{1}{\rho} \left(\frac{\rho u_e^3}{L} \right) \left(\frac{u_e}{L^2} \frac{u_e}{\delta^2} \right)$$

$$u_e \frac{\partial u_e}{\partial x} + v \frac{\partial u_e}{\partial y} = -\frac{1}{\rho} \frac{\partial p_e}{\partial x}$$

$$u_e \frac{u_e}{L} = \frac{1}{\rho} \frac{p_e}{L}$$

$$\frac{p_e}{L} \propto \frac{\rho u_e^2}{L} \Rightarrow \frac{u_e^2}{L} \propto \frac{u_e^2}{L} \nu \left(\frac{u_e}{L^2} \frac{u_e}{\delta^2} \right)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2}$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)$$

$$u_e \frac{u_e \delta}{L^2} \frac{u_e^2 \delta^2}{L^2 \delta} \frac{1}{\rho} = \frac{1}{\rho} \frac{\rho u_e^2}{\delta} \nu \left(\frac{u_e \delta}{L^3} \frac{u_e \delta}{L \delta^2} \right)$$

$$\frac{u_e^2 \delta}{L^2} \frac{u_e^2 \delta}{L^2} = \frac{u_e^2}{\delta} \nu \left(\frac{u_e \delta}{L^3} \frac{u_e}{L \delta} \right)$$

$$\frac{u_e^2}{L} \propto \nu \frac{u_e}{\delta^2}$$

$$\boxed{\nu \propto \frac{u_e \delta^2}{L}}$$

$$\frac{u_e^2 \delta}{L^2} \frac{u_e^2 \delta}{L^2} = \frac{u_e^2}{\delta} \frac{u_e \delta^2}{L} \frac{u_e}{L \delta}$$

$$\frac{u_e^2 \delta}{L^2} \frac{u_e^2 \delta}{L^2} = \frac{u_e^2}{\delta} \frac{u_e \delta^2}{L} \frac{u_e}{L \delta}$$

$$\frac{\partial p}{\partial y} = 0$$

$$p = \text{cost. lungo } y$$

$$\begin{cases} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \\ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} \\ \frac{\partial p}{\partial y} = 0 \end{cases} \rightarrow \text{Equazioni di Prandtl per lo strato limite}$$

$$u_e \frac{\partial u_e}{\partial x} + v \frac{\partial u_e}{\partial y} = -\frac{1}{\rho} \frac{\partial p_e}{\partial x}$$

$$\boxed{u_e \frac{d u_e}{d x} = -\frac{1}{\rho} \frac{d p_e}{d x}}$$

$$C_f(x) = \frac{\tau_p(x)}{\frac{1}{2} \rho U_0^2} = \text{Coefficiente di attrito locale}$$

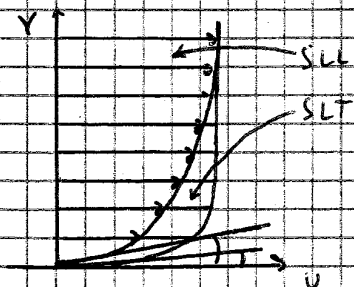
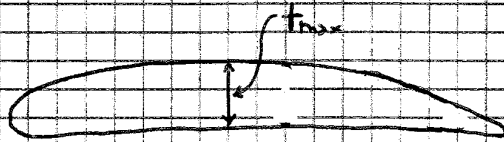
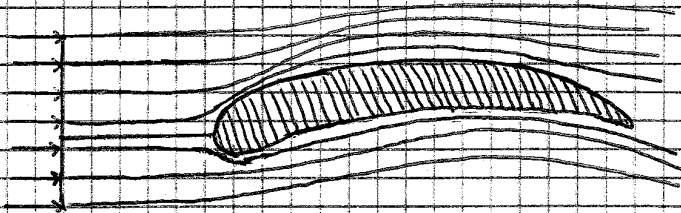
$$C_D = \frac{1}{L} \int_0^L C_f(x) dx$$

$$D = \frac{1}{2} \rho \sqrt{U_0^2} C_D S = \tau_p S$$

$$Re_x = \frac{U_0 x}{\nu}$$

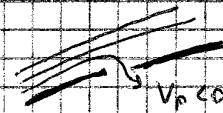

$$Re_L = \frac{U_0 L}{\nu}$$

$$Re_\theta = \frac{U_0 \theta}{\nu}$$



$$\tau_p = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}$$

Re ← Controllo dello strato limite

- o) Gradienti di pressione →
- o) Aspirazione o soffiatura dello strato limite → 
- o) Rugosità della parete →  Superficie rugosa
- o) Temperatura della parete
- o) Turbolenza della corrente a monte

$$\rho \left[\frac{d\bar{u}^2}{dx} + \frac{d\bar{v}^2}{dy} + \frac{d}{dy} (\bar{u}\bar{v}) + \frac{d}{dx} (\bar{u}'\bar{v}') \right] = - \frac{d\bar{p}}{dx} + \mu \frac{d^2\bar{u}}{dy^2}$$

$$\rho \left[\cancel{0 \frac{d\bar{u}}{dx}} + \cancel{\bar{u} \frac{d\bar{u}}{dx}} + \cancel{\bar{v} \frac{d\bar{v}}{dy}} + \bar{v} \frac{d\bar{u}}{dy} \right] = - \frac{d\bar{p}}{dx} + \mu \frac{d^2\bar{u}}{dy^2} - \rho \left[\cancel{\frac{d(\bar{u}^2)}{dx}} + \frac{d}{dy} (\bar{u}'\bar{v}') \right]$$

Trascurabile

$$\rho \left(\bar{v} \frac{d\bar{u}}{dy} + \bar{v} \frac{d\bar{u}}{dy} \right) = - \frac{d\bar{p}}{dx} + \frac{d}{dy} \left(\bar{v} \frac{d\bar{u}}{dy} \right) + \frac{d}{dy} \left(- \rho \bar{u}'\bar{v}' \right)$$

$$\left\{ \begin{aligned} \frac{d\bar{u}}{dx} + \frac{d\bar{v}}{dy} &= 0 \\ \frac{d\bar{u}}{dx} + \frac{d\bar{v}}{dy} &= 0 \end{aligned} \right.$$

τ

Sforzi di Reynolds

Modello di Boussinesq

$$-\rho \bar{u}'\bar{v}' = \mu_T \frac{d\bar{u}}{dy}$$

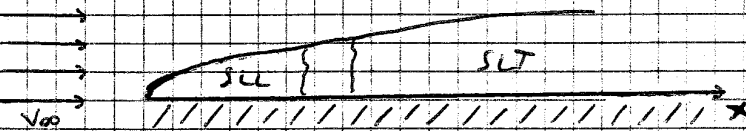
$\mu_T =$ Viscosità turbolenta

$$\tau = (\mu + \mu_T) \frac{d\bar{u}}{dy}$$

$$\left\{ \begin{aligned} \frac{d\bar{u}}{dx} + \frac{d\bar{v}}{dy} &= 0 \\ \rho \left(\bar{v} \frac{d\bar{u}}{dx} + \bar{v} \frac{d\bar{u}}{dy} \right) &= - \frac{d\bar{p}}{dx} + \frac{d\tau}{dy} \end{aligned} \right.$$

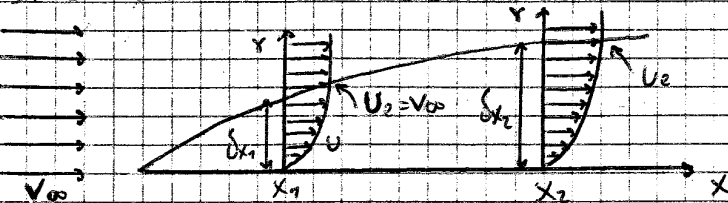
Strato limite che investe una placca piana a incidenza nulla \rightarrow La pressione rimane costante ovunque

$$\left\{ \begin{aligned} \frac{du}{dx} + \frac{dv}{dy} &= 0 \\ u \frac{du}{dx} + v \frac{du}{dy} &= - \frac{1}{\rho} \frac{dp}{dx} + \nu \frac{d^2u}{dy^2} \end{aligned} \right. \rightarrow \text{Bilancio tra forze inerziali e forze visose}$$



$$Re_x = \frac{V_\infty x}{\nu} \rightarrow \text{Reynolds critico} = 500000 \rightarrow \text{Transizione da laminare a turbolenta}$$

Strato limite laminare



A y uguale, u è più basso nella posizione x_2 . Sono simili ma non uguali.

$$\eta = \frac{y}{\delta(x)}$$

$$\eta = \frac{y}{\delta(x)}$$

$$\varphi = \varphi(\eta)$$

$$Re_L = 500.000 = \frac{U_e L}{\nu}$$

$$U_e = \frac{\nu Re_L}{L} = \frac{1,454 \cdot 10^{-5} \cdot 500.000}{2} = 3,635 \text{ m/s}$$

$$\delta(x_1) = 5 \text{ mm}$$

$$\delta(x_2) = 8,0622 \text{ mm}$$

$$\delta(x_3) = 12,46 \text{ mm}$$

$$\delta^*(x) \approx 1,73 \sqrt{\frac{\nu x}{U}}$$

$$\theta(x) = 0,664 \sqrt{\frac{\nu x}{U}}$$

$$H(x) = \frac{\delta^*(x)}{\theta(x)}$$

$$\tau_p(x) = 0,322 \sqrt{\frac{\rho \mu U^3}{x}}$$

$$C_f(x) = 0,644 \sqrt{\frac{\nu}{U x}}$$

$$\nu = \frac{\mu}{\rho}$$

$$\mu = \nu \rho = 1,787 \cdot 10^{-5}$$

Fare calcoli, tabelle e grafici con Excel

Risultati

$$\delta^*(x_1) = 1,73 \text{ mm}$$

$$\delta^*(x_2) = 2,78 \text{ mm}$$

$$\delta^*(x_3) = 4,37 \text{ mm}$$

$$\theta(x_1) = 0,644 \cdot 10^{-3}$$

$$\theta(x_2) = 1,038 \cdot 10^{-3}$$

$$\theta(x_3) = 1,653 \cdot 10^{-3}$$

$$H(x_1) = 2,686$$

$$H(x_2) = 2,688$$

$$H(x_3) = 2,607$$

$$\tau(x_1) = 0,0208$$

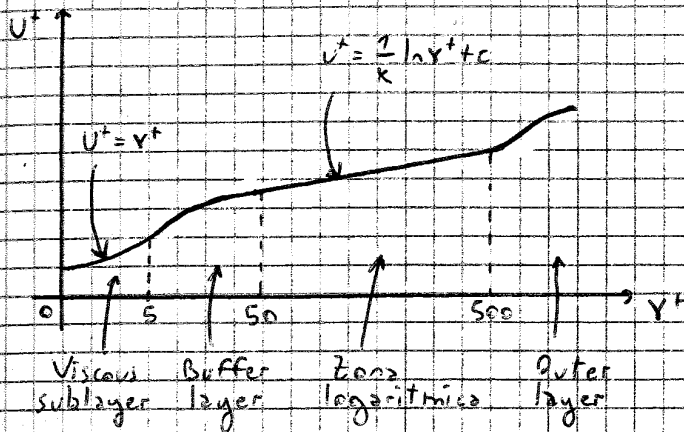
$$\tau(x_2) = 0,0129$$

$$\tau(x_3) = 0,0072 \cdot 10^{-3}$$

$$C_f(x_1) = 2,57 \cdot 10^{-3}$$

$$C_f(x_2) = 1,55 \cdot 10^{-3}$$

$$C_f(x_3) = 0,0077$$



Strato limite laminare su piaça piana senza incidenza

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}$$

$$\delta(x) = 5 \sqrt{\frac{\nu x}{U_0}} = \frac{5x}{\sqrt{Re_x}}$$

$$\delta^*(x) = 1,73 \sqrt{\frac{\nu x}{U_0}}$$

$$\theta(x) = 0,664 \sqrt{\frac{\nu x}{U_0}}$$

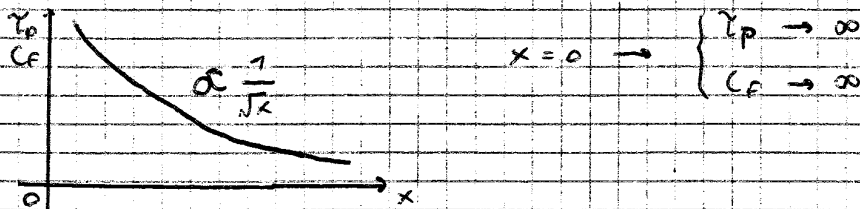
$$H(x) = \frac{\delta^*(x)}{\theta(x)} = 2,605$$

$$\tau_p(x) = 0,332 \sqrt{\frac{\rho \mu U_0^3}{x}} \rightarrow \tau_p \propto \frac{1}{\sqrt{x}}$$

$$C_f(x) = \frac{\tau_p}{\frac{1}{2} \rho U_0^2} = 0,664 \sqrt{\frac{\nu}{U_0 x}} = \frac{0,664}{\sqrt{Re_x}} \rightarrow C_f \propto \frac{1}{\sqrt{x}}$$

$$\bar{\tau}_p = 0,664 \sqrt{\frac{\rho \mu U_0^3}{L}}$$

$$C_D = \frac{\bar{\tau}_p}{\frac{1}{2} \rho U_0^2} = 1,328 \sqrt{\frac{\nu}{U_0 L}} = \frac{1,328}{\sqrt{Re_L}}$$



Strato limite turbolenta su piaça piana senza incidenza

$$5 \cdot 10^5 < Re_x < 10^7$$

$$\delta(x) = \frac{0,37x}{(Re_x)^{1/5}}$$

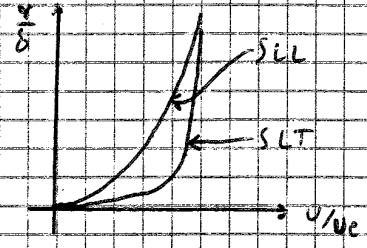
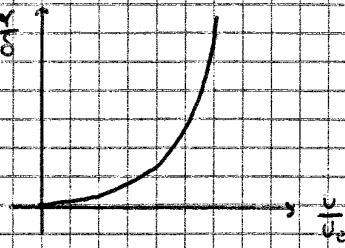
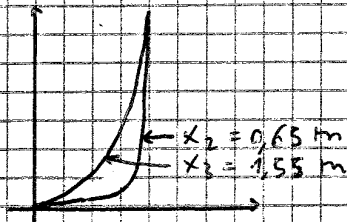
$$\delta^*(x) = \frac{\delta}{8}$$

$$\theta(x) = \frac{7\delta}{72}$$

$$H = \frac{\delta^*}{\theta} = 1,286$$

Esercizio

$U_e = 13,385419 \text{ m/s}$

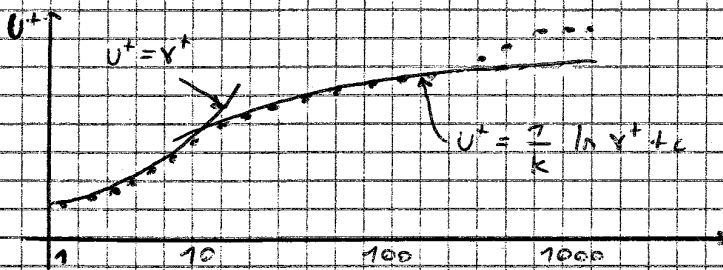


$\delta \rightarrow \gamma \rightarrow \frac{u}{U_e} = 0,9906$

$x_2) C_f = 0,00355 \rightarrow \tau_p \rightarrow U_e$

$x_3) C_f = 0,00305 \rightarrow \tau_p \rightarrow U_e$

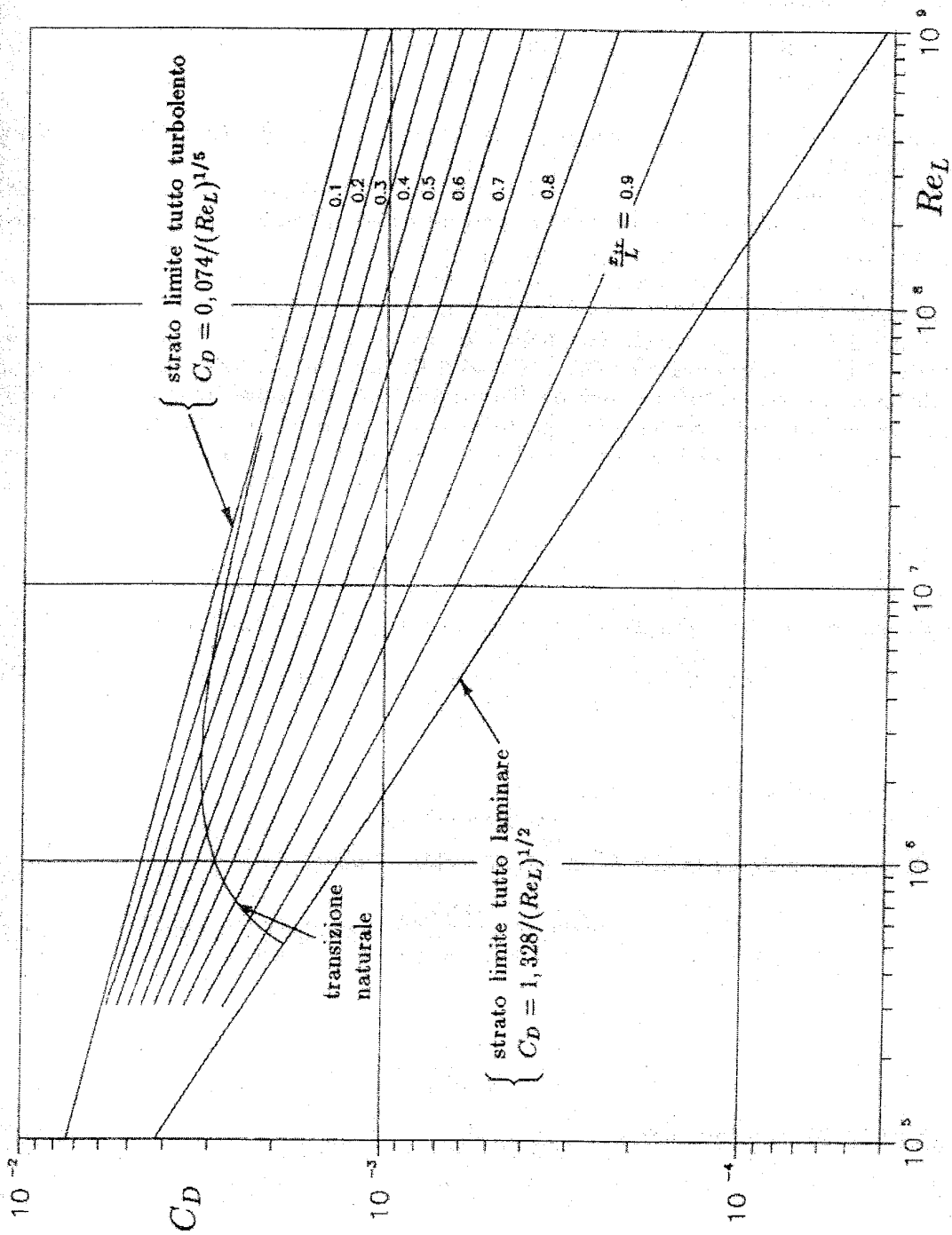
Aria in condizioni standard $\rightarrow \nu = 1,454 \cdot 10^{-5} \text{ m}^2/\text{s}$



$Re_{x_2} = \frac{U_e x_2}{\nu} = \frac{13,38 \cdot 0,65}{1,454 \cdot 10^{-5}} = 6 \cdot 10^5$

$Re_{x_3} = \frac{U_e x_3}{\nu} = \frac{13,38 \cdot 1,55}{1,454 \cdot 10^{-5}} = 1,4 \cdot 10^6$

x (m)	δ (mm)	δ^* (mm)	θ (mm)	H	τ_p (N/m ²)	τ (m/s)
0,65	13,87	2,97	2,19	135	0,385	0,5633
1,55	38,25	5,53	4,087	135	0,335	0,5267



$$T^0 = T \left(1 + \frac{\gamma-1}{2} M^2 \right) = \text{cost.}$$

$$Pr_{aria} = 0,71$$

$$Pr = 1:$$

$$h_{rec} = \text{Entalpia di recupero} = h + R \frac{v^2}{2} = \text{cost.}$$

$$T_{rec} = T \left(1 + R \frac{\gamma-1}{2} M^2 \right)$$

R = Fattore di recupero

$$R = Pr^{1/2} \text{ per S.L.L.}$$

$$R = Pr^{1/3} \text{ per S.L.T.}$$

$$\mathcal{R}_e = \left(\frac{T_p + T_a}{2T_a} + R \frac{\gamma-1}{2} \frac{M_{a0}^2}{4} \right)^{-0,725} \text{ per S.L.L.}$$

$$\mathcal{R}_e = \left(\frac{T_p + T_a}{2T_a} + R \frac{\gamma-1}{2} \frac{M_{a0}^2}{4} \right)^{-0,65} \text{ per S.L.T.}$$

Se la parete è adiabatica $T_p = T_{rec}$

Equazione integrale della strato limite

$$\begin{cases} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \\ U \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U_e \frac{du_e}{dx} + v \frac{\partial^2 u}{\partial y^2} \end{cases}$$

$$\int_0^{\infty} \left(U \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - U_e \frac{du_e}{dx} \right) dy = U_e \int_0^{\infty} \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial y} \right) dy$$

$$v = \frac{m}{\rho}$$

$$\dot{m}_p = \rho \left(\frac{\partial u}{\partial y} \right)_p$$

$$v \int_0^{\infty} \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial y} \right) dy = v \left[\frac{\partial u}{\partial y} \right]_p^{y \rightarrow \infty} = -v \left(\frac{\partial u}{\partial y} \right)_p = -\frac{m}{\rho} \left(\frac{\partial u}{\partial y} \right)_p = -\frac{\dot{m}_p}{\rho}$$

$$\int_0^{\infty} \left(U \frac{du}{dx} + v \frac{du}{dy} - U_e \frac{du_e}{dx} \right) dy = -\frac{\dot{m}_p}{\rho}$$

$$dv = -\frac{\partial u}{\partial x} dy$$

$$\int_0^y dv = -\int_0^y \frac{\partial u}{\partial x} dy$$

$$v = -\int_0^y \frac{\partial u}{\partial x} dy$$

$$\int_0^{\infty} \left(U \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} \int_0^y \frac{\partial u}{\partial x} dy - U_e \frac{du_e}{dx} \right) dy = -\frac{\dot{m}_p}{\rho}$$

$$\int_0^{\infty} \frac{\partial u}{\partial y} \int_0^y \frac{\partial u}{\partial x} dy = U_e \int_0^{\infty} \frac{\partial u}{\partial x} dy - \int_0^{\infty} \left(U \frac{\partial u}{\partial x} \right) dy$$

$$\int_0^{\infty} \left(U \frac{\partial u}{\partial x} - U_e \frac{\partial u}{\partial x} + U \frac{\partial u}{\partial x} - U_e \frac{du_e}{dx} \right) dy = -\frac{\dot{m}_p}{\rho}$$

$$\int_0^{\infty} \left(U \frac{\partial u}{\partial x} - U \frac{du_e}{dx} - U_e \frac{\partial u}{\partial x} + U \frac{\partial u}{\partial x} + U \frac{du_e}{dx} - U_e \frac{du_e}{dx} \right) dy = -\frac{\dot{m}_p}{\rho}$$

$$\frac{U_e}{V} \frac{d\theta^2}{dx} = 0,45 - 6 \frac{\theta^2}{V} \frac{dU_e}{dx}$$

$$\frac{U_e^6}{V} \frac{d\theta^2}{dx} = 0,45 U_e^5 \left(-6 U_e^5 \frac{\theta^2}{V} \frac{dU_e}{dx} \right) \rightarrow = - \frac{\theta^2}{V} \frac{dU_e^6}{dx}$$

$$\frac{U_e^6}{V} \frac{d\theta^2}{dx} + \frac{\theta^2}{V} \frac{dU_e^6}{dx} = 0,45 U_e^5$$

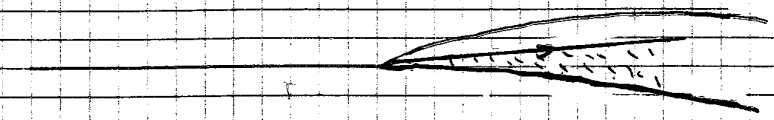
$$\frac{d(U_e^6 \theta^2)}{dx} = 0,45 V U_e^5$$

$$\int_0^x d(U_e^6 \theta^2) = 0,45 V \int_0^x U_e^5(x') dx'$$

$$U_e^6(x) \theta^2(x) - \underbrace{U_e^6(0) \theta^2(0)}_{=0} = 0,45 V \int_0^x U_e^5(x') dx'$$



Per bordo d'attacco arrotondato $U_e(0) = 0$.



Per bordo d'attacco appuntito $\theta(0) = 0$.

$$\theta^*(x) = \frac{0,45 V}{U_e^6(x)} \int_0^x U_e^5(x') dx'$$

Relazioni di Cebeci - Bradshaw

Flussi accelerati

$$\frac{dU_e}{dx} > 0 \leftrightarrow \frac{dp_e}{dx} < 0$$

$$\begin{cases} f(\lambda) = 0,22 + 1,52\lambda - 1,8\lambda^2 \\ H(\lambda) = 2,67 - 3,75\lambda + 5,24\lambda^2 \end{cases}$$

Flussi decelerati

$$\frac{dU_e}{dx} < 0 \leftrightarrow \frac{dp_e}{dx} > 0$$

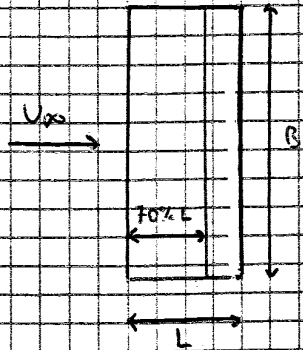
$$\begin{cases} f(\lambda) = 0,22 + 1,402\lambda + \frac{0,078\lambda}{0,707 + \lambda} \\ H(\lambda) = 2,088 + \frac{0,0237}{0,707 + \lambda} \end{cases}$$

se nota $\rightarrow \left\{ \begin{matrix} \theta(x) \\ \frac{dU_e}{dx} \end{matrix} \right\} \rightarrow \lambda \rightarrow \left\{ \begin{matrix} f \rightarrow C_f \\ H \rightarrow \delta^* \end{matrix} \right.$

$$H_{\text{max}} = \frac{\delta^*}{\theta} = 1,287$$

Esercizio n. 7.5 pag. 97

Lamina piana



$$L = 0,6 \text{ m}$$

$$B = 1,5 \text{ m}$$

$$U_{\infty} = 60 \text{ m/s}$$

Aria standard

$$\rho = 1,225 \text{ kg/m}^3$$

$$\mu = 1,787 \cdot 10^{-5} \text{ kg/(m s)}$$

$$\nu = 1,454 \cdot 10^{-5} \text{ m}^2/\text{s}$$

D_{TR} ? → Transizione naturale

D_{TRN} ? → Transizione ritardata fino al 70%

$$Re_L = \frac{U_{\infty} L}{\nu} = 2,476 \cdot 10^6$$

$$C_{DTR} = \frac{0,074}{(Re_L)^{1/5}} - \frac{1742}{Re_L} = 3,151 \cdot 10^{-3}$$

$$x_{TR} = 0,7 \cdot 0,6 = 0,42 \text{ m}$$

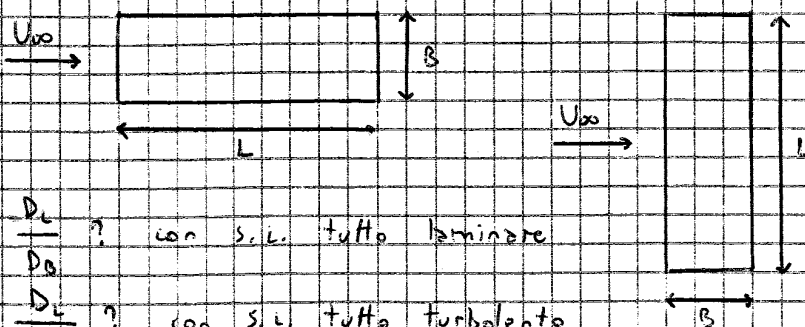
$$\frac{x_{TR}}{L} = 0,7$$

$$D_{TR} = \frac{1}{2} \rho U_{\infty}^2 C_{DTR} L B = 12,7 \text{ N}$$

$$C_{DTRN} = \frac{0,074}{(Re_L)^{1/5}} - \frac{x_{TR}}{L} \left[\frac{0,074}{(\frac{x_{TR}}{L} Re_L)^{1/5}} - \frac{1,328}{(\frac{x_{TR}}{L} Re_L)^{1/2}} \right] = 0,00167$$

$$D_{DTRN} = 6,67 \text{ N}$$

Esercizio



$\frac{D_L}{D_B}$? con s.l. tutto laminare

$\frac{D_L}{D_B}$? con s.l. tutto turbolento

$$D_L = \frac{1}{2} \rho U_{\infty}^2 C_{DL} 2LB$$

$$D_B = \frac{1}{2} \rho U_{\infty}^2 C_{DB} 2LB$$

$$\frac{D_L}{D_B} = \frac{C_{DL}}{C_{DB}}$$

Head ha anche scoperto che:

- se $H \leq 1,6 \rightarrow H_1 = 3,3 + 0,8234 (H - 1,1)^{-1,277}$
- se $H > 1,6 \rightarrow H_1 = 3,3 + 1,5501 (H - 0,6778)^{-3,064}$
- se $H_1 > 5,3 \rightarrow H = 1,1 + 0,8602 (H_1 - 3,3)^{-0,777}$
- se $H_1 \leq 5,3 \rightarrow H = 0,6778 + 1,1536 (H_1 - 3,3)^{0,3264}$

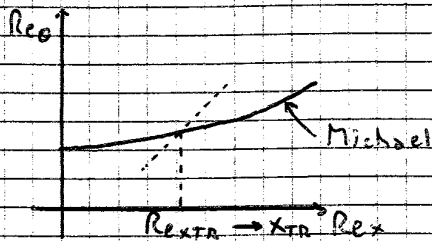
Legge di Ludwig e Tillmann

$$K_E = 0,246 \cdot 10^{0,672H} \quad Re_0 = 0,268$$

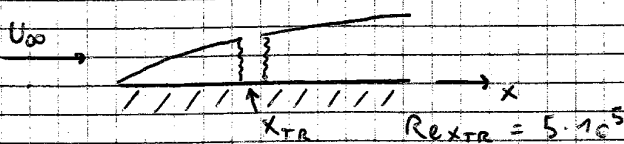
dove $Re_0 = \frac{U_e \cdot D}{\nu}$

Criterio di transizione per i profili alari (criterio di Michel)

$$Re_{0,TR} = 1,174 \left(1 + \frac{22400}{Re_{x,TR}} \right) Re_{x,TR}^{0,996}$$



Piastra piana senza incidenza



$$\theta(x) = 0,664 \sqrt{\frac{\nu x}{U_e}}$$

$$Re_0 = 0,664 \sqrt{\frac{U_x}{\nu}} = 0,664 \sqrt{Re_x}$$

$$0,664 \sqrt{Re_x} = 1,174 \left(1 + \frac{22400}{Re_x} \right) Re_x^{0,996}$$

$$Re_{x,TR} = 2 \cdot 10^6 \Rightarrow 5 \cdot 10^5$$

Matr. bidimensionali

$$\left. \begin{aligned} \frac{\partial p}{\partial t} + \nabla \cdot (p \vec{v}) &= 0 \\ \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} &= -\frac{1}{\rho} \nabla p \\ \frac{\partial E}{\partial t} + \vec{v} \cdot \nabla E &= \frac{1}{\rho} \nabla \cdot (p \vec{v}) \\ \frac{p}{\rho} &= R^* T \end{aligned} \right\} \text{Equazioni di Eulero}$$

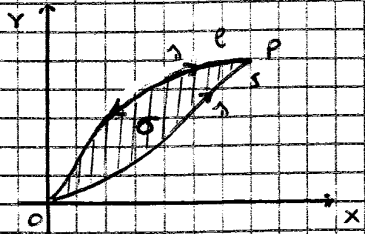
Condizione di tangenza: $\vec{v} \cdot \vec{n} = 0$
 Fluido stazionario e incompressibile

$$\frac{\partial}{\partial t} = 0 \quad \rho = \text{cost.}$$

$$\int_{a,s}^P d\phi = \int_C \vec{v} \cdot \vec{f} \cdot d\vec{s}$$

$$\phi_P - \phi_a = \int_C \vec{v} \cdot \vec{f} \cdot d\vec{s}$$

$$\phi_P = \int_0^P \vec{v} \cdot \vec{f} \cdot d\vec{s}$$



$$\phi_P = \int_{a,s}^P \vec{v} \cdot \vec{f} \cdot d\vec{s}$$

$$\phi_P = \int_{a,e}^P \vec{v} \cdot \vec{f} \cdot d\vec{s}$$

$$\int_{a,s}^P \vec{v} \cdot \vec{f} \cdot d\vec{s} - \int_{a,e}^P \vec{v} \cdot \vec{f} \cdot d\vec{s} = 0$$

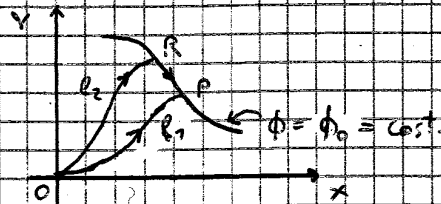
Cambia il verso alla freccia su e.

$$\int_{a,s}^P \vec{v} \cdot \vec{f} \cdot d\vec{s} + \int_{a,e}^P \vec{v} \cdot \vec{f} \cdot d\vec{s} = 0$$

$$\oint_{s-e} \vec{v} \cdot \vec{f} \cdot d\vec{s} = 0$$

$$\Gamma = \oint_{s-e} \vec{v} \cdot \vec{f} \cdot d\vec{s} = \int_{\sigma} (\underbrace{\nabla \times \vec{v}}_{=0}) \cdot \vec{f} \cdot d\sigma = 0$$

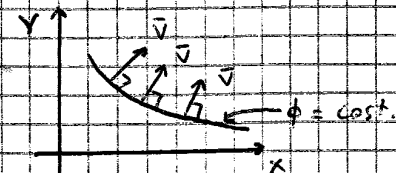
Se $\phi = \text{cost.}$ che succede?



$$\phi_0 = \int_{O,l_1}^P \vec{v} \cdot \vec{f} \cdot d\vec{s}$$

$$\phi_0 = \underbrace{\int_{O,l_2}^R \vec{v} \cdot \vec{f} \cdot d\vec{s}}_{\phi_0} + \int_R^P \vec{v} \cdot \vec{f} \cdot d\vec{s}$$

$$\int_R^P \vec{v} \cdot \vec{f} \cdot d\vec{s} = 0 \rightarrow \vec{v} \cdot \vec{f} = 0 \rightarrow \vec{v} \perp \vec{f} \rightarrow \vec{v} \text{ è sempre } \perp \text{ alle linee } \phi = \text{costante (linee equipotenziali)}$$

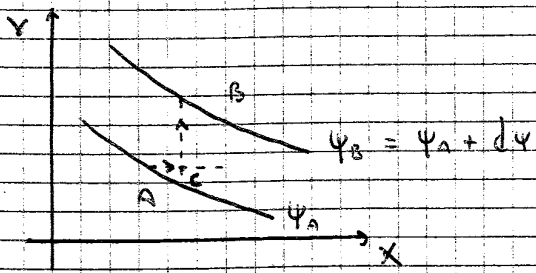


Caso 2D, stazionario, incompressibile, irrotazionale

$$\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$p + \frac{1}{2} \rho v^2 = p_{\infty} + \frac{1}{2} \rho v_{\infty}^2 = \text{cost.}$$

$$\frac{\partial v}{\partial y} - \frac{\partial v}{\partial x} = 0$$



$$d\psi = \vec{v} \cdot d\vec{s} = -v dx + u dy$$

$$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy$$

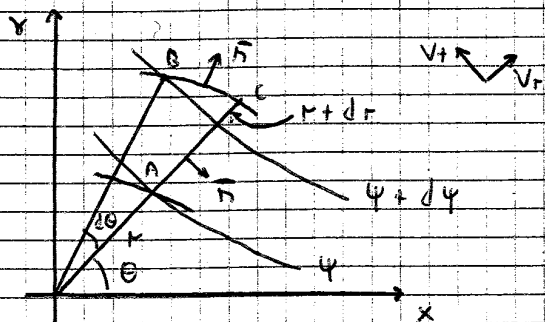
$$u = \frac{\partial \psi}{\partial y}$$

$$v = -\frac{\partial \psi}{\partial x}$$

$$\nabla \cdot \vec{v} = 0 \rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \rightarrow \frac{\partial}{\partial x} \left(\frac{\partial \psi}{\partial y} \right) - \frac{\partial}{\partial y} \left(\frac{\partial \psi}{\partial x} \right) = 0$$

$$\nabla \times \vec{v} = 0 \rightarrow \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0 \rightarrow \frac{\partial}{\partial y} \left(\frac{\partial \psi}{\partial y} \right) + \frac{\partial}{\partial x} \left(\frac{\partial \psi}{\partial x} \right) =$$

$$= \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \boxed{\nabla^2 \psi = 0} \rightarrow \text{Laplace}$$



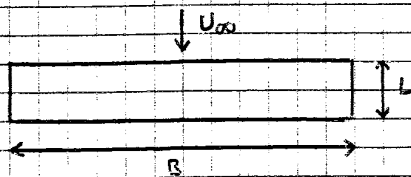
$$d\psi = -v_\theta dr + v_r r d\theta + \cancel{dr d\theta} \rightarrow \text{Trascurabile}$$

$$d\psi = \frac{\partial \psi}{\partial r} dr + \frac{\partial \psi}{\partial \theta} d\theta$$

$$v_r = \frac{1}{r} \frac{\partial \psi}{\partial \theta}$$

$$v_\theta = -\frac{\partial \psi}{\partial r}$$

Esercizio n. 11.16 pag. 217



Condizioni adiabatiche → Non c'è scambio di calore

$$B = 10 \text{ m}$$

$$L = 2 \text{ m}$$

$$p^\circ = 341 \text{ kPa}$$

$$z = 11,000 \text{ m}$$

Atmosfera standard

Dato = ?

$$T_{\infty} = 216,6 \text{ K}$$

$$p_{\infty} = 22,676 \text{ Pa}$$

$$v_{\infty} = 3,886 \cdot 10^{-5} \text{ m}^2/\text{s}$$

$$p^\circ = p_{\infty} \left(1 + \frac{\gamma-1}{2} M_{\infty}^2 \right)^{\frac{\gamma}{\gamma-1}}$$

$$\gamma = 1,4$$

$$\frac{\gamma}{\gamma-1} = 3,5$$

$$3,5 \sqrt{\frac{p^\circ}{p_{\infty}}} = 1 + \frac{\gamma-1}{2} M_{\infty}^2$$

$$\frac{\gamma-1}{2} = 0,2$$

$$\left(\frac{p^\circ}{p_{\infty}} \right)^{1/7} - 1 = 0,2 M_{\infty}^2$$

$$R_{lam} = \left(1 + R \frac{\gamma-1}{2} \frac{3}{4} M_\infty^2 \right)^{-0,725}$$

$$R = P_r^{1/2} = \sqrt{0,71} = 0,843$$

$$R_{lam} = 0,807$$

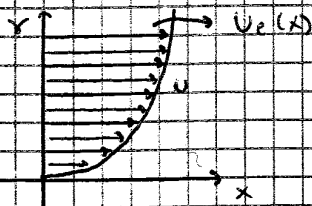
$$R_{turb} = \left(1 + R \frac{\gamma-1}{2} \frac{3}{4} M_\infty^2 \right)^{-0,65}$$

$$R = P_r^{1/3} = 0,852$$

$$R_{turb} = 0,378$$

$$\frac{Co_{laminare}}{Co_{turbolento}} = 0,694 < 1$$

Co laminare



Θ, δ^*, H, CF

$$\Theta^2(x) = \frac{0,45\nu}{U_e^3(x)} \int_0^x U_e^3(x') dx'$$

Nota $U_e(x) \rightarrow \Theta(x)$

$$\lambda = \frac{\Theta^2}{\nu} \frac{dU_e}{dx}$$

$$C_f = \frac{1}{2} C_F Re_\Theta$$

Dove $Re_\Theta = \frac{U_e(x) \Theta(x)}{\nu}$

$\left\{ \begin{array}{l} \Theta(x) = \\ H(x) = \end{array} \right.$ Relazioni differenti per i flussi accelerati e per quelli decelerati



$\Theta(x), H, \delta^*, C_F, \bar{T}_p$

$$U_e = U_0 = cost.$$

$$\Theta^2 = \frac{0,45\nu}{U_0^3} \int_0^x U_0^3 dx' = 0,45 \frac{\nu x}{U_0^2}$$

$$\Theta(x) = \sqrt{0,45} \sqrt{\frac{\nu x}{U_0^2}} = 0,671 \sqrt{\frac{\nu x}{U_0^2}} \rightarrow \text{Thwaites}$$

$$\Theta_B(x) = 0,664 \sqrt{\frac{\nu x}{U_0^2}} \rightarrow \text{Blasius}$$

$$\lambda = \frac{\Theta^2}{\nu} \frac{dU_e}{dx} = 0$$

$$H(\lambda=0) = 2,61$$

$$H_B = 2,605$$

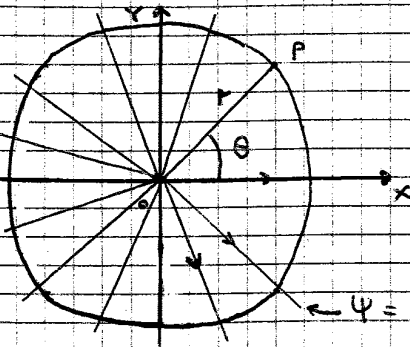
$$H = \frac{\delta^*}{\Theta} \rightarrow \delta^* = H\Theta = 2,61 \cdot 0,671 \sqrt{\frac{\nu x}{U_0^2}} = 1,75 \sqrt{\frac{\nu x}{U_0^2}}$$

$$\delta_B^* = 1,73 \sqrt{\frac{\nu x}{U_0^2}}$$

Sorgente e pozzo

+Q

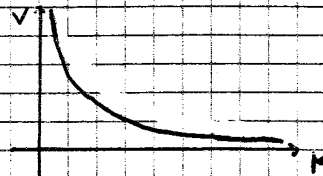
-Q



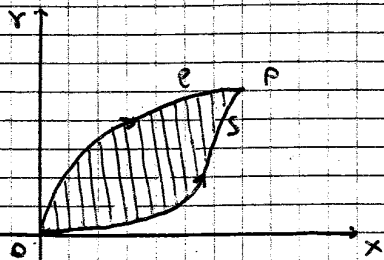
$$\int_{\text{Circonf.}} \vec{v} \cdot \vec{n} d\sigma = Q = \int_{\sigma} \nabla \cdot \vec{v} d\sigma$$

$$2\pi r v = Q$$

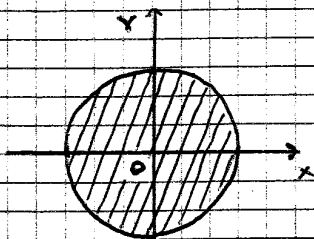
$$v = \frac{Q}{2\pi r}$$



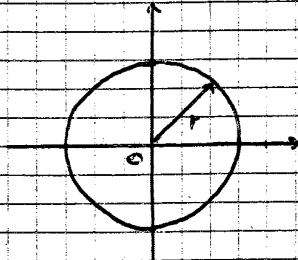
$r \rightarrow 0 \Rightarrow v \rightarrow \infty$
 $r \rightarrow \infty \Rightarrow v \rightarrow 0$



$$\oint_{s-e} \vec{v} \cdot \vec{n} d\sigma = \int_{\sigma} \underbrace{\nabla \cdot \vec{v}}_{=0} d\sigma = 0$$



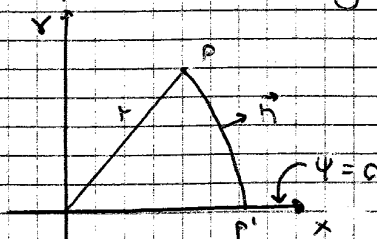
$$\oint_{\text{Circonf.}} \vec{v} \cdot \vec{n} d\sigma = Q = \int_{\sigma} \underbrace{\nabla \cdot \vec{v}}_{=0} d\sigma$$



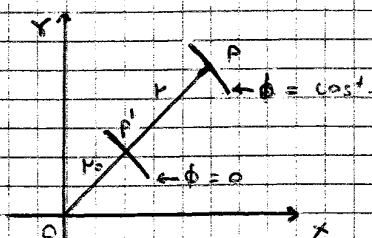
$r \rightarrow 0$
 $\sigma \rightarrow 0$

$$\oint_{\text{Circonf.}} \vec{v} \cdot \vec{n} d\sigma = Q = \int_{\sigma} \nabla \cdot \vec{v} d\sigma$$

In tutto il campo si ha che $\nabla \cdot \vec{v} = 0$, tranne nell'origine, che è un punto di singolarità.

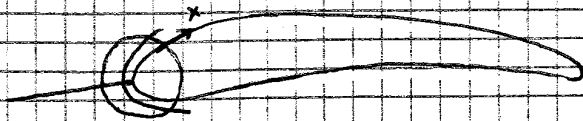


$$\begin{aligned} \Phi_P &= \int_0^P \vec{v} \cdot \vec{n} d\sigma = \int_0^{P'} \underbrace{\vec{v} \cdot \vec{n}}_{=0} d\sigma + \int_{P'}^P \vec{v} \cdot \vec{n} d\sigma = \\ &= \int_0^\theta \frac{Q}{2\pi r} r d\theta = \frac{Q}{2\pi} \theta \end{aligned}$$



$$\begin{aligned} \Phi_P &= \int_0^P \vec{v} \cdot \vec{n} d\sigma = \int_0^{P'} \underbrace{\vec{v} \cdot \vec{n}}_{=0} d\sigma + \int_{P'}^P \vec{v} \cdot \vec{n} d\sigma = \\ &= \int_{r_0}^r \frac{Q}{2\pi r} dr = \frac{Q}{2\pi} \int_{r_0}^r \frac{1}{r} dr = \frac{Q}{2\pi} (\ln r - \ln r_0) = \\ &= \frac{Q}{2\pi} \ln \frac{r}{r_0} \end{aligned}$$

Bordo di attacco regolare



$$U_e(x) = \delta x$$

$$\delta = \left(\frac{dU_e}{dx} \right)_{x=0}$$



$$\Theta^2(x) = \frac{0,95V}{U_e^2(x)} \int_0^x U_e^5(x') dx' = \frac{0,95V}{\delta^2 x^2} \int_0^x \delta^5 x'^5 dx' = \frac{0,95V}{\delta^2} \frac{\delta^5}{6} \frac{x^6}{x^2} = \frac{0,95}{6} \frac{V}{\delta}$$

$$\Theta(x) = \sqrt{\frac{0,95}{6}} \sqrt{\frac{V}{\delta}} = \Theta_0$$

$$\lambda_0 = \frac{\Theta_0^2}{V} \left(\frac{dU_e}{dx} \right)_{x=0} = \frac{\Theta_0^2}{V} \delta = \frac{0,95}{6} \frac{V}{\delta} \frac{\delta}{V} = 0,075$$

$$P(\lambda) = 0,22 + 1,57 \lambda_0 - 1,8 \lambda_0^2 = 0,328 = P_0$$

$$H(\lambda) = 2,67 - 3,75 \lambda_0 + 5,27 \lambda_0^2 = 2,358 = H_0$$

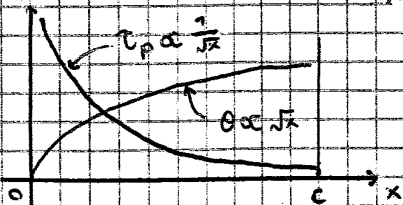
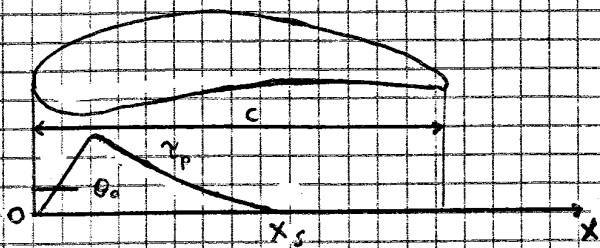
$$\delta^*(0) = H_0 \Theta_0 = 2,358 \sqrt{\frac{0,95}{6}} \sqrt{\frac{V}{\delta}}$$

$$C_0 = \frac{1}{2} C_{e0} Re_0 \rightarrow C_{f0} = \frac{2P_0}{Re_0} \rightarrow \infty$$

$$Re_{e0} = \frac{U_e \Theta_0}{\nu}$$

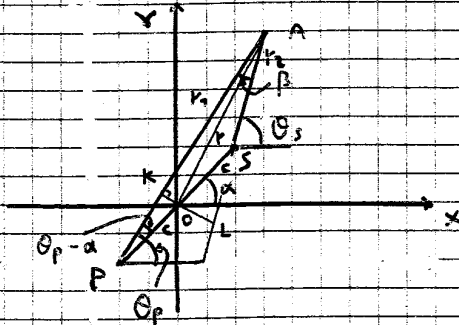
$$C_f = \frac{\tau_p}{\frac{1}{2} \rho U_e^2} \rightarrow \tau_p = \frac{1}{2} \rho U_e^2 C_f = \frac{1}{2} \rho \delta^2 x^2 \frac{2P_0}{\frac{U_e \Theta_0}{\nu}} =$$

$$= \frac{1}{2} \rho \delta^2 x^2 (2P_0) \nu \frac{1}{\delta x \Theta_0} = \frac{\rho \nu P_0}{\Theta_0} x = Kx$$



$U = \frac{\partial \phi}{\partial x}$	$u = \frac{\psi}{\partial y}$
$V = \frac{\partial \phi}{\partial y}$	$v = -\frac{\partial \psi}{\partial x}$
$V_r = \frac{\partial \phi}{\partial r}$	$V_r = \frac{1}{r} \frac{\partial \psi}{\partial \theta}$
$V_\theta = \frac{1}{r} \frac{\partial \phi}{\partial \theta}$	$V_\theta = -\frac{\partial \psi}{\partial r}$

Doppietta



$$\psi = \frac{Q}{2\pi} (\theta_s - \theta_p) = \frac{Q}{2\pi} \beta$$

$$\beta + \theta_p = \theta_s \rightarrow \theta_s - \theta_p = \beta$$

$$k_0 = c \sin(\theta - \alpha)$$

$$KL = 2c \sin(\theta - \alpha)$$

$$2c \sin(\theta - \alpha) = \beta r$$

$$\beta = \frac{2c \sin(\theta - \alpha)}{r}$$

$$\psi = \lim_{\substack{c \rightarrow 0 \\ Q \rightarrow \infty}} \frac{Q}{2\pi} \frac{2c \sin(\theta - \alpha)}{r}$$

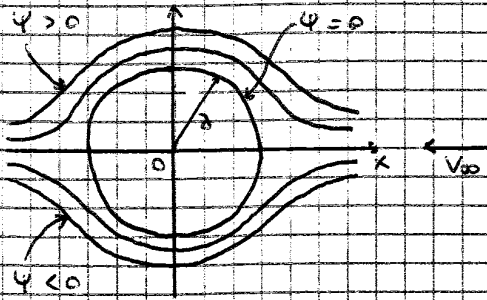
$M = 2cQ =$ Momento della doppietta

$$\psi = \frac{M}{2\pi r} \sin(\theta - \alpha)$$

In maniera analoga si trova che

$$\phi = -\frac{M}{2\pi r} \cos(\theta - \alpha)$$

<p>Corrente uniforme</p> $\psi = Uy - vx$ $\phi = Ux + Vy$	<p>Vortice irrotazionale</p> $\psi = -\frac{\Gamma}{2\pi} \ln \frac{r}{r_0}$ $\phi = \frac{\Gamma}{2\pi} \theta$
<p>Sorgente / pozzo</p> $\psi = \frac{Q}{2\pi} \theta$ $\phi = \frac{Q}{2\pi} \ln \frac{r}{r_0}$	<p>Doppietta</p> $\psi = \frac{M}{2\pi r} \sin(\theta - \alpha)$ $\phi = -\frac{M}{2\pi r} \cos(\theta - \alpha)$



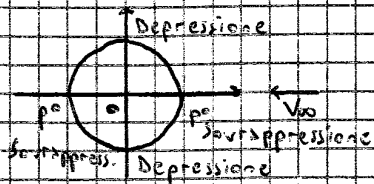
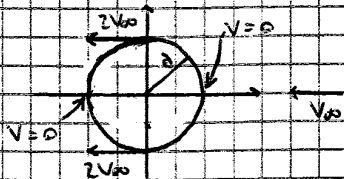
$$\psi = V_{\infty} r \left(\frac{r^2}{2r^2} - 1 \right) = V_{\infty} \left(\frac{r^2}{r^2} - 1 \right) r \sin \theta$$

$$V_r = \frac{1}{r} \frac{\partial \psi}{\partial \theta} = V_{\infty} \left(\frac{r^2}{r^2} - 1 \right) \cos \theta$$

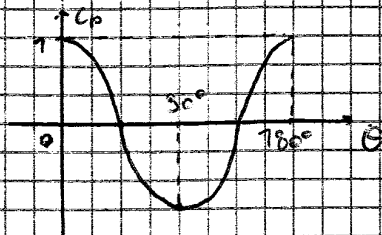
$$V_t = - \frac{\partial \psi}{\partial r} = V_{\infty} \left(\frac{r^2}{r^2} + 1 \right) \sin \theta$$

$r = a$

$$\begin{cases} V_r = 0 \\ V_t = 2V_{\infty} \sin \theta \end{cases}$$



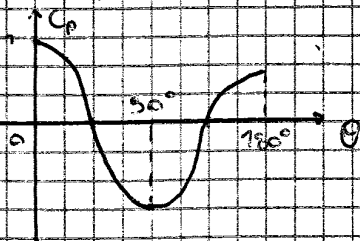
Non c'è portanza
Non c'è resistenza → Paradosso di d'Alembert



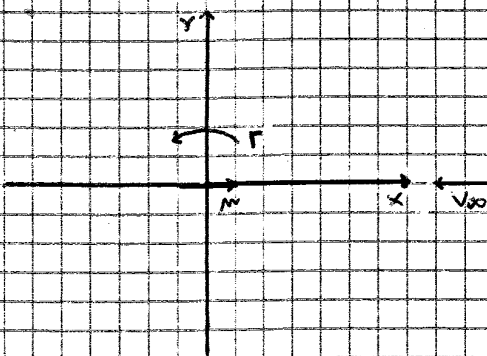
$$C_p = \frac{p - p_{\infty}}{\frac{1}{2} \rho V_{\infty}^2}$$

← Caso ideale

Nel caso reale si crea una scia turbolenta a valle del cilindro nella quale si ha $V \approx 0$ e $p \approx p_{\infty}$.



Vortice + doppietta + corrente uniforme



$$\psi = V_{\infty} r \left(\frac{r^2}{r^2} - 1 \right) \sin \theta - \frac{\Gamma}{2\pi} \ln \frac{r}{r_0}$$

$r = a$

$$\psi = V_{\infty} r \left(\frac{r^2}{r^2} - 1 \right) \sin \theta - \frac{\Gamma}{2\pi} \ln \frac{r}{a}$$

$$V_r = \frac{1}{r} \frac{\partial \psi}{\partial \theta} = V_{\infty} \left(\frac{r^2}{r^2} - 1 \right) \cos \theta$$

$$V_t = - \frac{\partial \psi}{\partial r} = V_{\infty} \left(\frac{r^2}{r^2} + 1 \right) \sin \theta + \frac{\Gamma}{2\pi r}$$

$r = a$

$$\begin{cases} V_r = 0 \\ V_t = 2V_{\infty} \sin \theta + \frac{\Gamma}{2\pi a} \end{cases}$$

Pressioni sul cilindro

$$C_p = 1 - \frac{v^2}{V_\infty^2}$$

$$0 < r < 4\pi V_\infty \delta$$

$$V_t = 2V_\infty \sin \theta + \frac{\Gamma}{2\pi a}$$

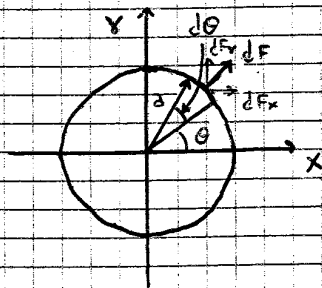
$$C_p = 1 - \frac{V_t^2}{V_\infty^2} = 1 - \frac{[2V_\infty \sin \theta + \Gamma/(2\pi a)]^2}{V_\infty^2} = 1 - \left(2 \sin \theta + \frac{\Gamma}{2\pi a V_\infty} \right)^2 =$$

$$= \underbrace{1 - 4 \sin^2 \theta}_{\text{Cil. non rotante}} - 2 \frac{\Gamma}{\pi a V_\infty} \sin \theta - \underbrace{\left(\frac{\Gamma}{2\pi a V_\infty} \right)^2}_{\text{Costante rispetto a } \theta}$$

$$\Delta C_p = C_p^- - C_p^+$$

$$C_p = \frac{p - p_\infty}{\frac{1}{2} \rho V_\infty^2} \rightarrow p - p_\infty = \frac{1}{2} \rho V_\infty^2 C_p = \frac{1}{2} \rho V_\infty^2 \left(-2 \frac{\Gamma}{\pi a V_\infty} \sin \theta \right)$$

$$p - p_\infty = - \frac{\rho V_\infty \Gamma}{\pi a} \sin \theta$$



$$\frac{d\theta}{d\theta} = 1$$

$$dF = (p - p_\infty) a d\theta$$

$$dF = - \frac{\rho V_\infty \Gamma}{\pi a} \sin \theta d\theta$$

$$dF_x = -dF \cos \theta = \frac{\rho V_\infty \Gamma}{\pi} \sin \theta \cos \theta d\theta$$

$$dF_r = -dF \sin \theta = \frac{\rho V_\infty \Gamma}{\pi} \sin^2 \theta d\theta$$

$$D = F_x = \frac{\rho V_\infty \Gamma}{\pi} \int_0^{2\pi} \sin \theta \cos \theta d\theta = \frac{\rho V_\infty \Gamma}{\pi} \left[\frac{\cos 2\theta}{4} \right]_0^{2\pi} = 0$$

$$L = F_r = \frac{\rho V_\infty \Gamma}{\pi} \int_0^{2\pi} \sin^2 \theta d\theta = \frac{\rho V_\infty \Gamma}{\pi} \left[\frac{\theta}{2} - \frac{\sin \theta \cos \theta}{2} \right]_0^{2\pi} = \rho V_\infty \Gamma$$

$$\boxed{D = 0} \rightarrow \text{Paradosso di d'Alembert}$$

$$\boxed{L = \rho V_\infty \Gamma} \rightarrow \text{Teorema di Kutta - Joukowski}$$

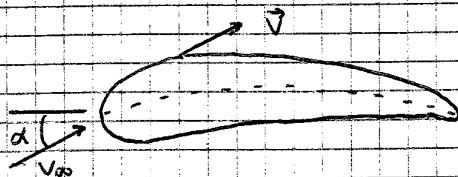
Teoria delle piccole perturbazioni (o dei profili sottili)

ϕ lineare

ψ lineare

si vuole linearizzare anche C_p .

2D, incomprimibile, irrotazionale

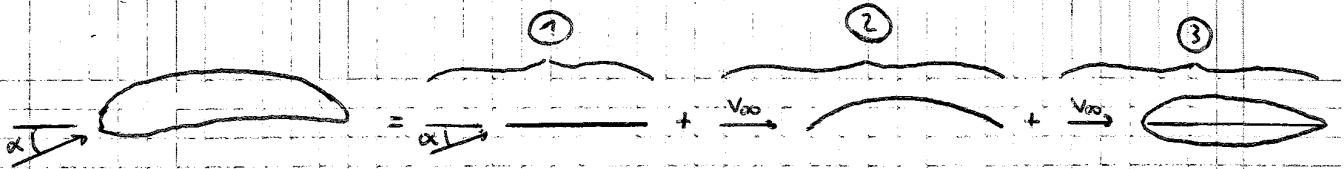


$$\vec{V} = \vec{V}_\infty + \vec{V}'$$

$$\vec{V}' = \vec{u}' + \vec{v}' + \vec{w}'$$

$$\left. \begin{matrix} u' \ll V_\infty \\ v' \ll V_\infty \\ w' \ll V_\infty \end{matrix} \right\} V' \ll V_\infty$$

- 1) Incidenza
2) Incurvamento
3) Spessore
- } Cause di disturbo

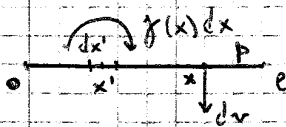


Distribuzione di sorgenti

$$U(x) = \frac{\gamma}{2\pi} \int_0^c \frac{q(x') dx'}{x-x'}$$

$$v^\pm(x) = \pm \frac{q(x)}{2}$$

Distribuzione di vortici



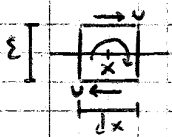
$\gamma(x) > 0 \rightarrow$ orario

$\Gamma > 0 \rightarrow$ Antiorario

$$dv = - \frac{\gamma(x') dx'}{2\pi(x-x')}$$

$$V = \frac{\Gamma}{2\pi r}$$

$$V(x) = \frac{1}{2\pi} \int_0^c \frac{\gamma(x') dx'}{x'-x}$$

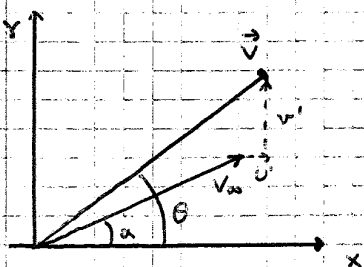
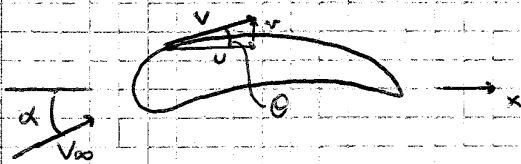


$$\gamma(x) dx = 2U dx$$

$$u^\pm(x) = \pm \frac{\gamma(x)}{2}$$

Condizione di tangenza

$$\vec{V} = \vec{V}_\infty + \vec{V}'$$



$$\frac{v}{U} = \tan \theta = \frac{dy}{dx}$$

$$\begin{cases} U = V_\infty \cos \alpha + U' \\ v = V_\infty \sin \alpha + v' \end{cases}$$

$$\alpha \ll 1 \text{ rad}$$

$$U' \ll V_\infty$$

$$v' \ll V_\infty$$

$$\begin{cases} U \approx V_\infty \\ v \approx V_\infty \alpha + v' \end{cases}$$

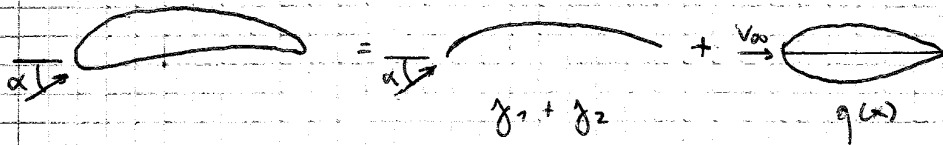
$$\frac{dy}{dx} = \frac{V_\infty \alpha + v'}{V_\infty} \rightarrow \boxed{\frac{dy}{dx} = \alpha + \frac{v'}{V_\infty}} \rightarrow \text{Equazione di tangenza}$$

$$C_p = 1 - \frac{V^2}{V_\infty^2} = 1 - \frac{U^2 + v^2}{V_\infty^2} = 1 - \frac{(V_\infty \cos \alpha + U')^2 + (V_\infty \sin \alpha + v')^2}{V_\infty^2}$$

$$= 1 - \frac{V_\infty^2 \cos^2 \alpha + U'^2 + 2V_\infty U' \cos \alpha + V_\infty^2 \sin^2 \alpha + v'^2 + 2V_\infty v' \sin \alpha}{V_\infty^2}$$

- 1) $\gamma_1(x)$
- 2) $\gamma_2(x)$
- 3) $q(x)$

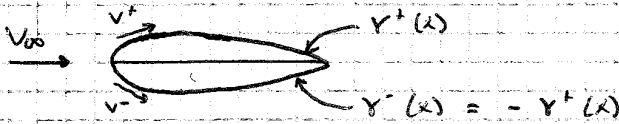
Si sommano i problemi 1 e 2: $\gamma_1(x) + \gamma_2(x)$. Così si hanno solo più 2 problemi.



$$L = \rho V_{\infty} \Gamma$$

$$\Gamma = \int_0^l \gamma(x) dx$$

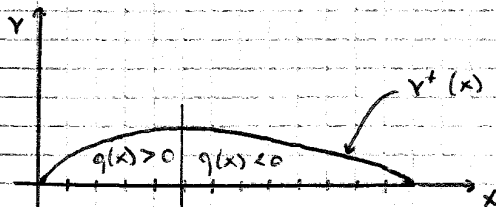
Analizziamo il problema 3:
profilo simmetrico senza incidenza



$$\frac{d\gamma}{dx} = \alpha + \frac{v^i}{V_{\infty}} = \frac{q(x)}{2V_{\infty}} \rightarrow \boxed{q(x) = 2V_{\infty} \frac{d\gamma}{dx}}$$

$$\boxed{v^i(x) = \frac{q(x)}{2}}$$

$$\boxed{U^i(x) = \frac{1}{2\pi} \int_0^l \frac{q(x') dx'}{x-x'}}$$



$$\frac{d\gamma^+}{dx} > 0 \rightarrow q(x) > 0$$

$$\frac{d\gamma^+}{dx} < 0 \rightarrow q(x) < 0$$

$$\int_0^l q(x) dx = 2V_{\infty} \int_0^l \frac{d\gamma}{dx} dx = 2V_{\infty} [\gamma(x) \Big|_0^l] = 0$$

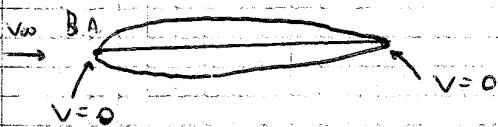
$$w^i(x) = w^o(x)$$

$$C_p = -2 \frac{w^i}{V_{\infty}} \rightarrow \boxed{C_p^+ = C_p^-}$$

$$L = 0$$

$$D = 0$$

$$M = 0$$



$$\vec{V} = \vec{V}_{\infty} + \vec{V}^i$$

$$\vec{V} = 0 \rightarrow \vec{V}_{\infty} = -\vec{V}^i$$

$$\begin{aligned}
 y(\theta) &= V^+ - V^- = V_{\infty} \left(1 + 2\beta \operatorname{sen} \theta + \alpha \frac{1 + \cos \theta}{\operatorname{sen} \theta} \right) - V_{\infty} \left(1 + 2\beta \operatorname{sen}(-\theta) + \alpha \frac{1 + \cos(-\theta)}{\operatorname{sen}(-\theta)} \right) = \\
 &= \cancel{V_{\infty}} + V_{\infty} 2\beta \operatorname{sen} \theta + V_{\infty} \alpha \frac{1 + \cos \theta}{\operatorname{sen} \theta} - \cancel{V_{\infty}} + V_{\infty} 2\beta \operatorname{sen} \theta + V_{\infty} \alpha \frac{1 + \cos \theta}{\operatorname{sen} \theta} = \\
 &= 4 V_{\infty} \beta \operatorname{sen} \theta + 2 V_{\infty} \alpha \frac{1 + \cos \theta}{\operatorname{sen} \theta}
 \end{aligned}$$

$$\boxed{y(\theta) = 2 V_{\infty} \left(2\beta \operatorname{sen} \theta + \alpha \frac{1 + \cos \theta}{\operatorname{sen} \theta} \right)} \rightarrow \text{Arco di cerchio}$$

$$\alpha \rightarrow A_0 = \alpha + \operatorname{cost}$$

$\rightarrow \operatorname{cost} = 0$ per lamina piana e arco di cerchio

$$2\beta \operatorname{sen} \theta \rightarrow \sum_{n=1}^{\infty} A_n \operatorname{sen}(n\theta)$$

Lamina piana: $A_0 = \alpha$, $A_1 = \dots = A_n = 0$
 Arco di cerchio: $A_0 = \alpha$, $A_1 = 2\beta$, $A_2 = \dots = A_n = 0$ } Importante

$$\boxed{y(\theta) = 2 V_{\infty} \left(A_0 \frac{1 + \cos \theta}{\operatorname{sen} \theta} + \sum_{n=1}^{\infty} A_n \operatorname{sen}(n\theta) \right)}$$

$$x = \frac{r}{2} (1 - \cos \theta)$$

$$x' = \frac{r}{2} (1 - \cos \theta')$$

$$dx' = \frac{r}{2} \operatorname{sen} \theta' d\theta'$$

$$\frac{1}{\pi} \int_0^{\pi} \frac{2 V_{\infty} \left[A_0 \frac{1 + \cos \theta'}{\operatorname{sen} \theta'} + \sum_{n=1}^{\infty} A_n \operatorname{sen}(n\theta') \right]}{\frac{r}{2} (1 - \cos \theta') - \frac{r}{2} (1 - \cos \theta')} \frac{r}{2} \operatorname{sen} \theta' d\theta' = V_{\infty} \left(\alpha - \frac{dy}{dx} \right)$$

$$\frac{1}{\pi} \int_0^{\pi} \frac{A_n (1 + \cos \theta') + \sum_{n=1}^{\infty} A_n \operatorname{sen} \theta' \operatorname{sen}(n\theta')}{1 - \cos \theta' - 1 + \cos \theta'} d\theta' = \alpha - \frac{dy}{dx}$$

$$\frac{dy}{dx} = \alpha - \frac{1}{\pi} \left[A_0 \int_0^{\pi} \frac{1 + \cos \theta'}{\cos \theta' - \cos \theta} d\theta' + \sum_{n=1}^{\infty} A_n \int_0^{\pi} \frac{\operatorname{sen} \theta' \operatorname{sen}(n\theta')}{\cos \theta' - \cos \theta} d\theta' \right]$$

Glauert ha risolto i 2 integrali

$$G_n = \int_0^{\pi} \frac{\cos(n\theta')}{\cos \theta' - \cos \theta} d\theta' = \pi \frac{\operatorname{sen}(n\theta)}{\operatorname{sen} \theta}$$

$$G_n^* = \int_0^{\pi} \frac{\operatorname{sen} \theta' \operatorname{sen}(n\theta')}{\cos \theta' - \cos \theta} d\theta' = -\pi \cos(n\theta)$$

$$\frac{dy}{dx} = \alpha - \frac{1}{\pi} \left[A_0 (G_0 + G_n) + \sum_{n=1}^{\infty} A_n G_n^* \right]$$

$$\frac{dy}{dx}(\theta) = \alpha - \frac{1}{\pi} \left[\pi A_0 - \pi \sum_{n=1}^{\infty} A_n \cos(n\theta) \right]$$

$$\boxed{\frac{dy}{dx}(\theta) = \alpha - A_0 + \sum_{n=1}^{\infty} A_n \cos(n\theta)}$$

$$\int_0^{\pi} \frac{dy}{dx}(\theta) d\theta = (\alpha - A_0) \int_0^{\pi} d\theta + \sum_{n=1}^{\infty} A_n \int_0^{\pi} \cos(n\theta) d\theta$$

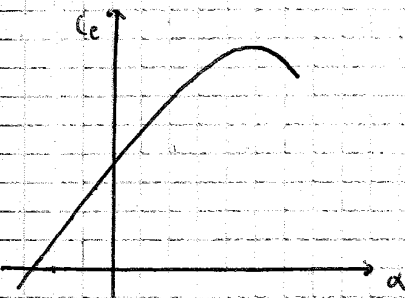
$$\frac{1}{\pi} \int_0^{\pi} \frac{dy}{dx}(\theta) d\theta = \alpha - A_0 \rightarrow \boxed{A_0 = \alpha - \frac{1}{\pi} \int_0^{\pi} \frac{dy}{dx}(\theta) d\theta}$$

α ideale

$$\begin{aligned}
 &= 2 \int_0^\pi \left[A_0 (1 + \cos \theta) + \sum_{n=1}^{\infty} A_n \sin \theta \sin(n\theta) \right] d\theta = \\
 &= 2 \left[\underbrace{A_0 \int_0^\pi (1 + \cos \theta) d\theta}_{=\pi} + \sum A_n \underbrace{\int_0^\pi \sin \theta \sin(n\theta) d\theta}_{= \pi/2 \text{ se } n=1} \right] = \\
 &\qquad\qquad\qquad = 0 \text{ se } n \neq 1 \\
 &= 2 \left[A_0 \pi + A_1 \frac{\pi}{2} \right]
 \end{aligned}$$

$$C_e = 2\pi \left[A_0 + \frac{A_1}{2} \right]$$

$$C_e = C_e' (\alpha - \alpha_0)$$



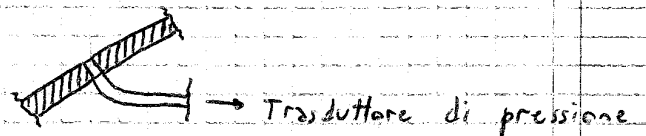
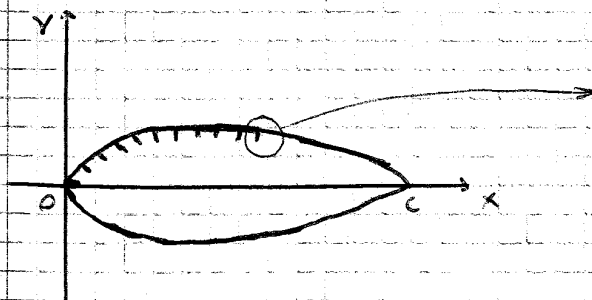
$$C_e = 2\pi \left[\alpha - \alpha_{id} + \frac{A_1}{2} \right] = 2\pi \left[\alpha - \left(\alpha_{id} - \frac{A_1}{2} \right) \right]$$

$$C_e' = 2\pi$$

$$\alpha_0 = \alpha_{id} - \frac{A_1}{2}$$

Esercizio

NACA 0015 $\rightarrow \frac{t}{c} = 0,15$



Ci sono 11 prese di pressione, che vanno dal bordo d'attacco all'80% della corda. Le loro posizioni in percentuale di corda sono 0, 2,5, 5, 10, 20, 30, 40, 50, 60, 70, 80

$$c = 100 \text{ mm}$$

$$X = \frac{x}{c}$$

$$Y = \frac{y}{c}$$

$$\pm Y = \frac{t/c}{0,2} \left(0,2963 X^{0,5} - 0,126 X - 0,3516 X^2 + 0,2843 X^3 - 0,1075 X^4 \right)$$

$$C_p = -2 \frac{W^2}{V_\infty^2}$$

$$W^2 = \pm \frac{\gamma}{2}$$

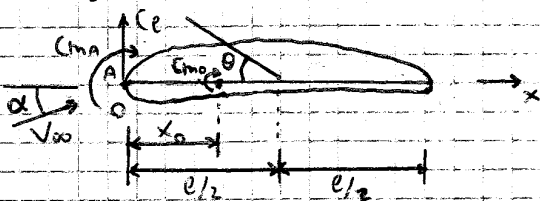
$$\Delta C_p = C_p^- - C_p^+ = \frac{2\gamma}{V_\infty}$$

$$C_e = 2\pi \left(A_0 + \frac{A_1}{2} \right)$$

$$\gamma(\theta) = 2V_\infty \left[A_0 \frac{1+\cos\theta}{\sin\theta} + \sum_{n=1}^{\infty} A_n \sin(n\theta) \right]$$

$$\frac{d\gamma}{dx}(\theta) = \alpha - A_0 + \sum_{n=1}^{\infty} A_n \cos(n\theta)$$

Valutazione del C_m



$$x = \frac{l}{2} (1 - \cos\theta) \rightarrow dx = \frac{l}{2} \sin\theta \cdot d\theta$$

$$\begin{aligned} C_{m\alpha} &= - \int_0^l \Delta C_p \frac{x}{l} \frac{dx}{l} = - \int_0^\pi \frac{2\gamma}{V_\infty} \frac{l}{2} (1 - \cos\theta) \frac{l}{2} \sin\theta d\theta = \\ &= - 2 \frac{V_\infty}{V_\infty} \frac{l}{V_\infty} \int_0^\pi \left(A_0 \frac{1+\cos\theta}{\sin\theta} + \sum_{n=1}^{\infty} A_n \sin(n\theta) \right) \frac{l}{2} (1 - \cos\theta) \frac{l}{2} \sin\theta d\theta = \\ &= - \int_0^\pi \left[A_0 (1 - \cos^2\theta) + \sum_{n=1}^{\infty} A_n \sin(n\theta) (1 - \cos\theta) \sin\theta \right] d\theta = \\ &= - \left[A_0 \int_0^\pi (1 - \cos^2\theta) d\theta + \sum_{n=1}^{\infty} A_n \int_0^\pi \sin(n\theta) (1 - \cos\theta) \sin\theta d\theta \right] = \end{aligned}$$

$$= \pi/2$$

$$\begin{aligned} &= \pi/2 \text{ se } n=1 \\ &= -\pi/4 \text{ se } n=2 \\ &= 0 \text{ se } n \neq 1 \text{ e } n \neq 2 \end{aligned}$$

$$C_{m\alpha} = -A_0 \frac{\pi}{2} - A_1 \frac{\pi}{2} + A_2 \frac{\pi}{4}$$

$$C_{m0} = C_{m\alpha} + C_e \frac{x_0}{l}$$

$$C_{m0} = -A_0 \frac{\pi}{2} - A_1 \frac{\pi}{2} + A_2 \frac{\pi}{4} + (2\pi A_0 + \pi A_1) \frac{x_0}{l} =$$

$$= A_0 \left(-\frac{\pi}{2} + 2\pi \frac{x_0}{l} \right) + A_1 \left(-\frac{\pi}{2} + \pi \frac{x_0}{l} \right) + A_2 \frac{\pi}{4}$$

$$A_0 = \alpha - \alpha_{id}$$

$$C_{m0} = (\alpha - \alpha_{id}) \left(-\frac{\pi}{2} + 2\pi \frac{x_0}{l} \right) + A_1 (\dots) + A_2 \frac{\pi}{4}$$

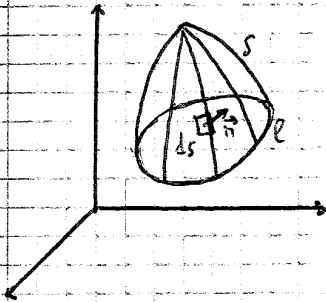
$$\frac{\partial C_{m0}}{\partial \alpha} = 0 = -\frac{\pi}{2} + 2\pi \frac{x_0}{l} \rightarrow \boxed{\frac{x_0}{l} = \frac{1}{4}} \rightarrow \text{Per } \dots \text{ pro fili subsonici}$$

Distribuzioni di singolarità superficiali

3D

Distribuzione di vortici

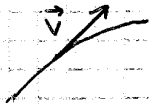
$$\Gamma = \oint_C \vec{v} \cdot d\vec{e}$$



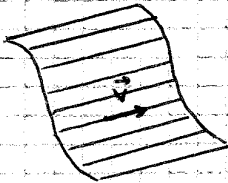
$$\vec{\omega} = \nabla \times \vec{v}$$

$$\Gamma = \int_S (\nabla \times \vec{v}) \cdot \vec{n} \, dS = \int_S \vec{\omega} \cdot \vec{n} \, dS$$

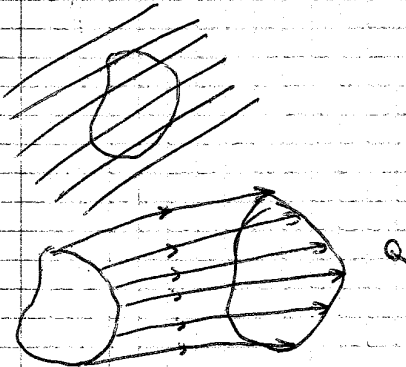
Linee di corrente



Superficie di corrente

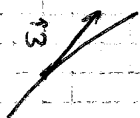


Tube di flusso



$$\vec{\omega} = \nabla \times \vec{v}$$

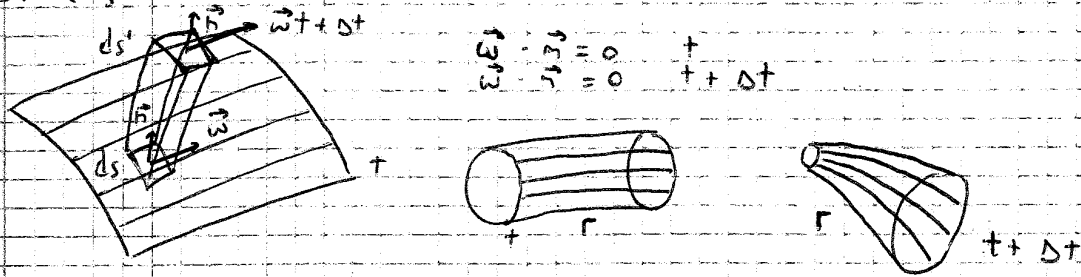
Linee vorticosi



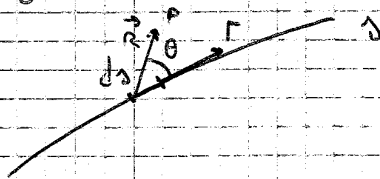
Teorema di Lagrange- Helmholtz.
 Per un fluido ideale ρ è costante oppure è in funzione di p ,
 $\frac{d\Gamma}{dt} = 0$

$$\Gamma = \oint_C \vec{v} \cdot d\vec{e} = \int_S \vec{\omega} \cdot \vec{n} ds$$

$$\frac{D}{Dt} \left(\int_S \vec{\omega} \cdot \vec{n} ds \right) = 0$$



Legge di Biot e Savart



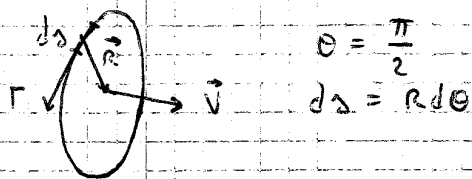
$$d\vec{v} = \frac{\Gamma}{4\pi} \frac{d\vec{s} \times \vec{R}}{R^3}$$

$$|d\vec{s} \times \vec{R}| = ds R \sin \theta$$

$$dV = \frac{\Gamma}{4\pi} \frac{ds R \sin \theta}{R^3}$$

$$dV = \frac{\Gamma}{4\pi R^2} \sin \theta ds$$

Vortice ad anello circolare

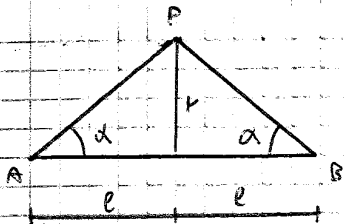


$$dV = \frac{\Gamma}{4\pi R^2} R d\theta$$

$$V = \int_0^{2\pi} \frac{\Gamma}{4\pi R} d\theta = \frac{\Gamma}{4\pi R} 2\pi = \frac{\Gamma}{2R}$$

Vortice rettilineo di lunghezza finita



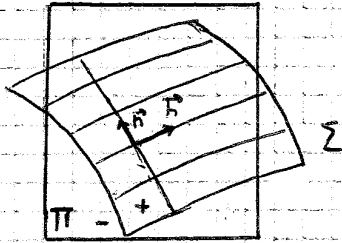


$$\beta = \alpha$$

$$V_P = \frac{\Gamma}{4\pi r} \cdot 2 \cos \alpha = \frac{\Gamma}{2\pi r} \cos \alpha$$

$$\frac{r}{l} = 0,1 \rightarrow \cos \alpha = 0,995 \rightarrow \text{Errore del } 5\%$$

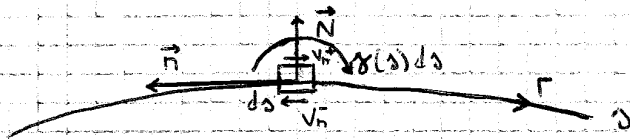
Superfici vorticose



$$\vec{N} = \vec{r} \times \vec{n}$$

Superficie vorticosa + Superficie di corrente

$$\vec{V} = V_n \vec{n} + V_h \vec{h}$$



$$\Gamma = \oint_{\sigma} \vec{V} \cdot d\vec{s}$$

$$\delta(\sigma) ds = V_n^- ds - V_n^+ ds$$

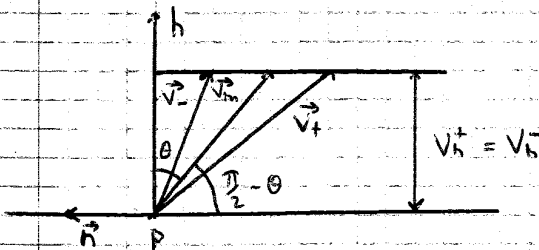
$$V_n^- - V_n^+ = \gamma(\sigma)$$

$$V_h^+ = V_h^-$$

$$\left(p + \frac{1}{2} \rho V^2 \right)_+ = \left(p + \frac{1}{2} \rho V^2 \right)_-$$

$$p^+ + \frac{1}{2} \rho V_+^2 = p^- + \frac{1}{2} \rho V_-^2$$

$$p^- - p^+ = \frac{1}{2} \rho (V_+^2 - V_-^2) = \frac{1}{2} \rho (\vec{V}_+ + \vec{V}_-) (\vec{V}_+ - \vec{V}_-)$$



$$\vec{V}_m = \frac{\vec{V}_+ + \vec{V}_-}{2}$$

$$\vec{V}_+ - \vec{V}_- = - (V_n^+ - V_n^-) \vec{n} = \gamma(\sigma) \cdot \vec{n}$$

$$p^- - p^+ = \rho \vec{V}_m \cdot [(V_n^+ - V_n^-) \vec{n}] = \rho V_m (V_n^+ - V_n^-) \cos(\pi/2 - \theta) =$$

$$= \rho \underbrace{V_m \gamma(\sigma) \sin \theta}_{|\vec{V}_m \times \gamma(\sigma) \vec{n}|}$$

$$\boxed{(p^- - p^+) \vec{N} = \rho \vec{V}_m \times \gamma(\sigma) \vec{r}} \rightarrow 3D \text{ (Teorema di Kutta - Joukowski locale)}$$

$$\text{In } 2D : L = \rho V_{\infty} \Gamma \text{ (Teorema di Kutta - Joukowski)}$$

$$d_0 = 0 - \frac{-4h}{2} = -2h = -0,075 \text{ rad} = -4,30^\circ$$

$$C_{e,did} = 2\pi \left(A_0 + \frac{A_1}{2} \right) = 2\pi \left(\cancel{d_0} - \cancel{d_{id}} + \frac{24h}{2} \right) = 4\pi h = 0,471$$

$$C_{mo} = -\frac{\pi}{4} (A_1 - A_2) = -\frac{\pi}{4} (4h - 0) = -\pi h = -0,178$$

$$C_{p,did} = -2 (d - d_{id}) \frac{1 + \cos\theta}{\sin\theta} = -2 \sum_{n=1}^{\infty} A_n \sin(n\theta) = -2 (4h \sin\theta) = -8h \sin\theta$$

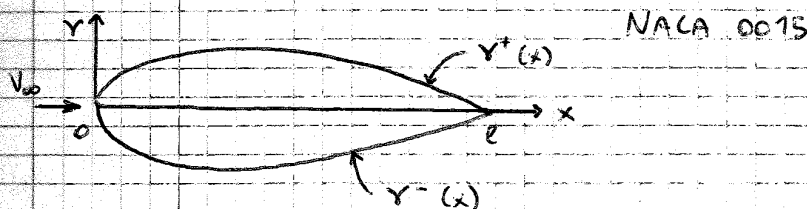
$$x = \frac{e}{2} (1 - \cos\theta) \rightarrow \cos\theta = -\frac{2x}{e} + 1$$

$$C_{p,did} = -8h \sin\theta = -8h \sqrt{1 - \cos^2\theta}$$

$$C_{p,did}(x) = -8h \sqrt{1 - \left(1 - \frac{2x}{e}\right)^2}$$

$$C_p = 0 \text{ per } x = 0 \text{ e } x = e.$$

Esercizio (non ci sarà all'esame)



$$q(x) = 2V_{\infty} \frac{dy}{dx}$$

$$W'(x) = \frac{1}{2\pi} \int_0^e \frac{q(x') dx'}{x-x'}$$

$$C_p = -2 \frac{W'}{V_{\infty}}$$

$$W'(x) = \frac{V_{\infty}}{\pi} \int_0^e \frac{\frac{dy}{dx}(x') dx'}{x-x'}$$

$$C_p = -\frac{2}{\pi} \int_0^e \frac{\frac{dy}{dx}(x') dx'}{x-x'}$$

$$X = \frac{x}{e} \quad Y = \frac{y}{e}$$

$$Y = \pm \frac{\tau}{0,20} (0,2569 \sqrt{X} - 0,1260 X - 0,3516 X^2 + 0,2843 X^3 - 0,1015 X^4)$$

$$\tau = 15\% \rightarrow \text{spessore percentuale (NACA 0015)}$$

$$\frac{dy}{dx} = \frac{d(Y/e)}{d(x/e)} = \frac{dY}{dX}$$

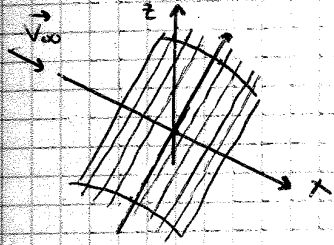
$$C_p = -\frac{2}{\pi} \int_0^1 \frac{dY}{dX} \frac{1}{X-X'} dX'$$

$$\frac{dY}{dX} = \frac{\tau}{0,20} \left(\frac{0,2569}{2\sqrt{X}} - 0,1260 - 2 \cdot 0,3516 X + 3 \cdot 0,2843 X^2 - 4 \cdot 0,1015 X^3 \right)$$

$$C_p(X) = -\frac{2}{\pi} \frac{\tau}{0,20} \left[\int_0^1 \frac{0,12845}{\sqrt{X'}(X-X')} dX' - \int_0^1 \frac{0,1260}{X-X'} dX' - \int_0^1 \frac{0,7032X'}{X-X'} dX' + \int_0^1 \frac{0,8523X'^2}{X-X'} dX' - \int_0^1 \frac{0,4060}{X-X'} X'^3 dX' \right]$$

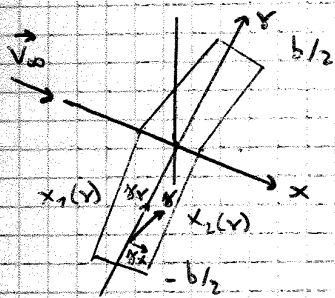
$\frac{\delta z}{\delta x} = \frac{w'}{v_0}$ → Condizione di tangenza per il caso 3D.

$\frac{\delta y}{\delta x} = \alpha + \frac{v'}{v_0}$ → Condizione di tangenza per il caso 2D.



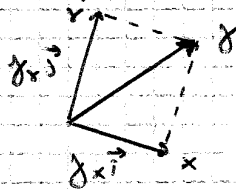
2D

I filetti vorticosi hanno solo direzione j_r .

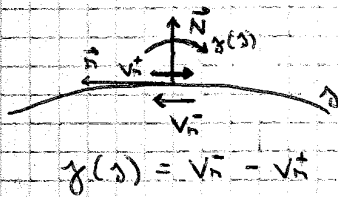


3D

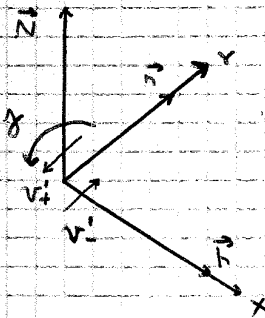
I filetti vorticosi hanno direzione \vec{j} scomponibile in j_r e j_x .



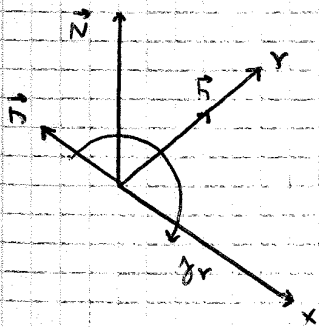
$\vec{j} = \vec{j}_B = \vec{j}_x + \vec{j}_r$



$\vec{j}(s) = v_1^- - v_1^+$



$\vec{j}_x = v_1^- - v_1^+$



$\vec{j}_r = -w_1^- + w_1^+$

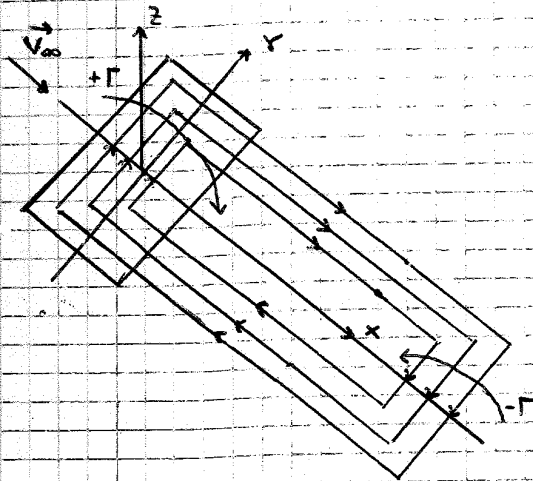
$$(p^- - p^+) \vec{N} = \rho \vec{v}_m \times \gamma(\Delta) \vec{R} = 0$$

$$\vec{v}_m \parallel \gamma \vec{R}$$

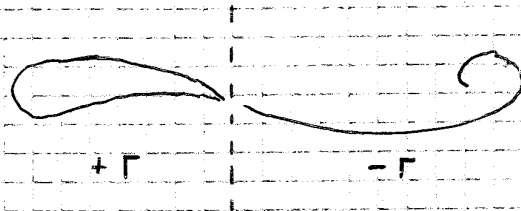
$$\vec{v}_\infty \parallel \gamma \vec{R}$$

$$\gamma \vec{R} \parallel \text{Asse } x$$

Nella scia $\gamma \vec{R} = \gamma x \vec{i}$



$$\Gamma_{tot} = 0$$



$\vec{\gamma}$ solenoidale

$$\vec{\omega} = \nabla \times \vec{v}$$

$$\nabla \cdot \vec{\omega} = \nabla \cdot (\nabla \times \vec{v}) = 0$$

$$\nabla \cdot (\gamma \vec{R}) = 0$$

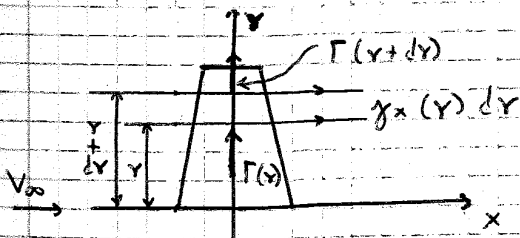
$$\frac{\partial \gamma_x}{\partial x} + \frac{\partial \gamma_y}{\partial y} = 0$$

Nella scia $\vec{\gamma} = \gamma_x \vec{i}$ e $\gamma_y = 0$.

$$\frac{\partial \gamma_x}{\partial x} = 0 \rightarrow \gamma_x = \gamma_x(y) \text{ (Nella scia)}$$

Nella parte aderente invece $\gamma_y = \gamma_y(x, y)$

Nella scia $\vec{\gamma} = \gamma_x(y) \vec{i}$



\vec{R} è solenoidale

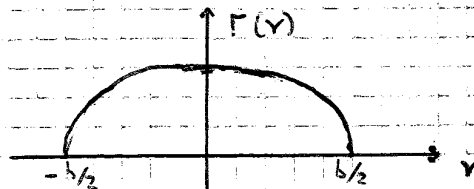
$$\Gamma(y) = \Gamma(y+dy) + \gamma_x(y) dy$$

$$\Gamma(y+dy) = \Gamma(y) + \frac{d\Gamma(y)}{dy} dy + \dots$$

$$\Gamma(y+dy) \cong \Gamma(y) + \frac{d\Gamma(y)}{dy} dy$$

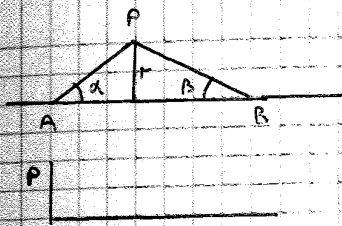
~~$$\Gamma(y) = \Gamma(y) + \frac{d\Gamma(y)}{dy} dy + \gamma_x(y) dy$$~~

$$\boxed{\gamma_x(y) = - \frac{d\Gamma(y)}{dy}}$$



$$\gamma_x(\pm \frac{b}{2}) \text{ MAX}$$

$$\gamma_x(0) = 0$$



$$V_p = \frac{\Gamma}{4\pi r} (\cos \alpha + \cos \beta)$$

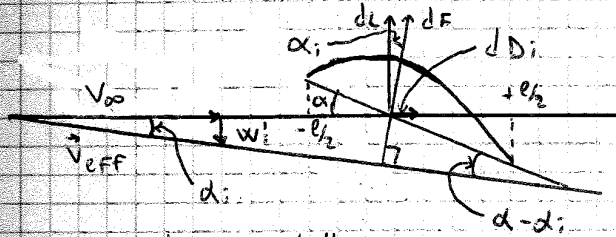
$$\alpha = \frac{\pi}{2}, \quad \beta = 0 \quad \rightarrow \quad V = \frac{\Gamma}{4\pi r}$$

$$dw'_i = - \frac{\gamma_x(y') dy'}{4\pi(y'-y)}$$

$$W'_i(y) = - \int_{-b/2}^{+b/2} \frac{\gamma_x(y') dy'}{4\pi(y'-y)}$$

$$\gamma_x(y) = - \frac{d\Gamma(y)}{dy}$$

$$W'_i(y) = \frac{1}{4\pi} \int_{-b/2}^{+b/2} \frac{\frac{d\Gamma(y')}{dy'}}{y'-y} dy'$$



α_i = Incidenza indotta
 D_i = Resistenza indotta

$$dF = \rho V_{eff} \Gamma(y) dy$$

$$dL = dF \cos \alpha_i = \rho V_{\infty} \Gamma(y) dy \approx dF$$

$$dD_i = dF \sin \alpha_i \approx dF \alpha_i = dL \alpha_i$$

$$L = \rho V_{\infty} \int_{-b/2}^{+b/2} \Gamma(y) dy$$

$$D_i = \rho V_{\infty} \int_{-b/2}^{+b/2} \Gamma(y) \alpha_i(y) dy$$

$$dL = \rho V_{\infty} \Gamma(y) dy$$

$$dL = \frac{1}{2} \rho V_{\infty}^2 c_e e dy$$

$$\rho V_{\infty} \Gamma(y) dy = \frac{1}{2} \rho V_{\infty}^2 c_e e dy$$

$$\Gamma(y) = \frac{1}{2} V_{\infty} c_e e$$

$$c_e = 2\pi (\alpha - \alpha_0) = c'_e (\alpha - \alpha_0)$$

$$c_e = k 2\pi (\alpha - \alpha_0)$$

$\alpha_0 = \alpha - \alpha_0 =$ Incidenza aerodinamica

$$c_e = k 2\pi \alpha_0$$

$$c_e = k 2\pi [(\alpha - \alpha_i) - \alpha_0] = k 2\pi (\alpha_a - \alpha_i)$$

$\alpha_a(y)$ $\alpha_i(y)$

$$\alpha_i = \arctan \left(- \frac{W'_i}{V_{\infty}} \right) \approx - \frac{W'_i}{V_{\infty}}$$

$$\alpha_i(y) = - \frac{1}{4\pi V_{\infty}} \int_{-b/2}^{+b/2} \frac{\frac{d\Gamma(y')}{dy'}}{y'-y} dy'$$

$$C_L = \frac{L}{\frac{1}{2} \rho V_{\infty}^2 S} = \frac{\rho V_{\infty} \pi / 4 \Gamma_0 b}{\frac{1}{2} \rho V_{\infty}^2 S} = \frac{\pi}{2} \frac{\Gamma_0 b}{V_{\infty} S}$$

$$\lambda = \frac{b^2}{S}$$

$$C_L = \frac{\pi}{2} \frac{\Gamma_0 b^2}{V_{\infty} S b} = \frac{\pi}{2} \frac{\Gamma_0}{V_{\infty} b} \lambda = \pi \alpha_i \lambda$$

$$\alpha_i = \frac{C_L}{\pi \lambda}$$

$$D_i = \rho V_{\infty} \int_{-b/2}^{b/2} \Gamma(y) \alpha_i(y) dy$$

$$D_i = \left(\rho V_{\infty} \int_{-b/2}^{b/2} \Gamma(y) dy \right) \alpha_i$$

L

$$C_{Di} = \alpha_i C_L$$

$$C_{Di} = \frac{C_L^2}{\pi \lambda}$$

Nell'ala ellittica non si usa svergolamento e c'è lo stesso profilo in tutte le sezioni.

$$\Gamma(y) = k \pi V_{\infty} \ell(y) [\alpha_0 - \alpha_i]$$

$$L = \rho V_{\infty} \int_{-b/2}^{b/2} \Gamma(y) dy = \rho V_{\infty} k \pi V_{\infty} (\alpha_0 - \alpha_i) \int_{-b/2}^{b/2} \ell(y) dy = \rho V_{\infty}^2 k \pi S (\alpha_0 - \alpha_i)$$

$$C_L = \frac{L}{\frac{1}{2} \rho V_{\infty}^2 S} = \frac{\rho V_{\infty}^2 k \pi S (\alpha_0 - \alpha_i)}{\frac{1}{2} \rho V_{\infty}^2 S} = 2 k \pi (\alpha_0 - \alpha_i)$$

$$C_L = k 2 \pi (\alpha_0 - \alpha_i) = k 2 \pi \left(\alpha_0 - \frac{C_L}{\pi \lambda} \right)$$

$$C_L \left(1 + k 2 \pi \frac{1}{\pi \lambda} \right) = 2 k \pi \alpha_0$$

$$C_L = \frac{k 2 \pi \alpha_0}{1 + \frac{2k}{\lambda}} = \frac{\underbrace{k 2 \pi}_{C_e'} (\alpha_0 - \alpha_i)}{1 + \frac{2k}{\lambda}} \left. \right\} \rightarrow \boxed{C_L' = \frac{k 2 \pi}{1 + \frac{2k}{\lambda}} = \frac{C_e'}{1 + \frac{2k}{\lambda}} = \frac{C_e'}{1 + C_e'/\pi \lambda}} \rightarrow \text{Ala ellittica}$$

Ala con distribuzione di portanza qualsiasi

$$\alpha_i = \frac{C_L}{\pi \lambda e}$$

e = Fattore di Oswald

e ≤ 1

Se e = 1 la distribuzione di portanza è ellittica.

$$C_{Di} = \frac{C_L^2}{e \pi \lambda}$$

$$C_D = C_{D0} + C_{Di} = C_{D0} + \frac{C_L^2}{e \pi \lambda} \rightarrow \text{Equazione polare dell'ala}$$

C_{D0} dipende dalla resistenza di forma.

$$C_{L2} = \lambda_2 \frac{2\pi K d_0}{\lambda_2 + 2K} = 0,623$$

Alc. di basso allungamento

$$\lambda < 10$$

$$C_L = \frac{C_e}{1 + C_e/\pi\lambda} = \frac{2\pi K}{1 + 2K/\lambda}$$

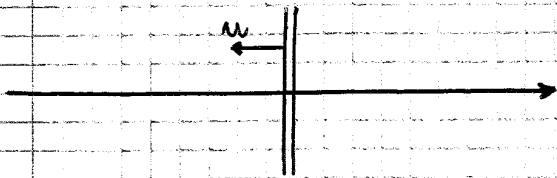
$$C_L = \frac{C_e \lambda}{\lambda + C_e/\pi} = C_e \frac{\lambda}{C_e/\pi + \lambda^2}$$

$$C_L = C_e \frac{\lambda}{C_e/\pi + \sqrt{(C_e/\pi)^2 + \lambda^2}} \rightarrow \text{Equazione di Helmholtz}$$

Supponiamo di essere nel caso comprimibile

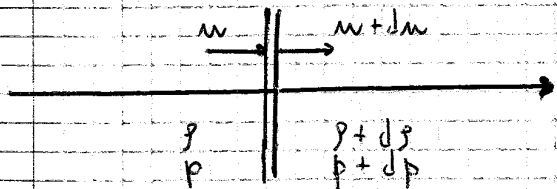
$$0,3 < M < 1$$

Velocità del suono



$$\frac{\partial}{\partial t} \neq 0 \rightarrow \text{Non stazionario}$$

Rendiamo il sistema di riferimento solidale con l'onda.



$$\frac{\partial}{\partial t} = 0 \rightarrow \text{Stazionario}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \rightarrow \rho u = (\rho + d\rho)(u + du) \quad (1)$$

$$\frac{\partial \rho}{\partial t} + \vec{v} \cdot \nabla \rho = -\frac{1}{\rho} \nabla p \rightarrow u \frac{du}{dx} = -\frac{1}{\rho} \frac{dp}{dx} \quad (2)$$

$$\frac{p}{\rho^k} = \text{cost.}$$

$$\rho u = \rho u + \rho du + u d\rho + d\rho du \rightarrow \text{trasc.}$$

$$u d\rho = -\rho du \rightarrow \frac{1}{\rho} = -\frac{du}{u d\rho} \quad (1)$$

$$\frac{1}{\rho} = -\frac{u du}{dp} \quad (2)$$

$$\frac{u du}{dp} = \frac{du}{u d\rho}$$

$$u^2 = \frac{dp}{d\rho} = c^2$$