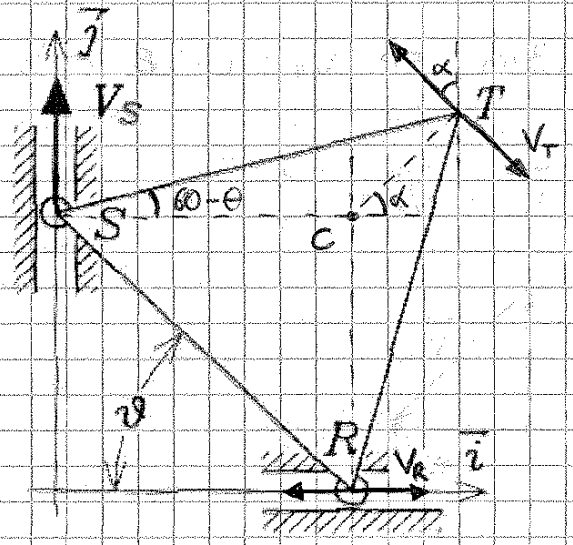
NUMERO: 504 DATA: 10/04/2013

A P P U N T I

STUDENTE: Guarracino

MATERIA: Meccanica Applicata alle Macchine
Prof. Ferraresi

1.51



$l = 0{,}5\ m$

$V_S = 0{,}3\ m/s$

$V_{T\vec{j}} = 0$

$a_S = 0$

θ ?

V_T ?

a_R ?

$SC = l\cos\theta$

$RC = l\sin\theta$

$$CT = \sqrt{l^2 + SC^2 - 2\,l\,SC\cos(60-\theta)}$$

$$= \sqrt{l^2 + l^2\cos^2\theta - 2l^2\cos\theta\cos(60-\theta)}$$

$$l\sin(60-\theta) = CT\cdot\sin\alpha$$

$$\vec{V_T} = V_T\vec{j} + V_T\vec{\imath} \qquad V_T\vec{j} = \vec{V_T}\cos\alpha = 0$$

$$\cos\alpha = 0 \Rightarrow \alpha = \frac{\pi}{2} \Rightarrow \sin\alpha = 1$$

$$l\sin(60-\theta) = \sqrt{l^2 + l^2\cos^2\theta - 2l^2\cos\theta\cos(60-\theta)}$$

$$l^2\sin^2(60-\theta) = l^2 + l^2\cos^2\theta - 2l^2\cos\theta\cos(60-\theta)$$

$$\sin^2(60-\theta) = 1 + \cos^2\theta - 2\cos\theta\cos(60-\theta)$$

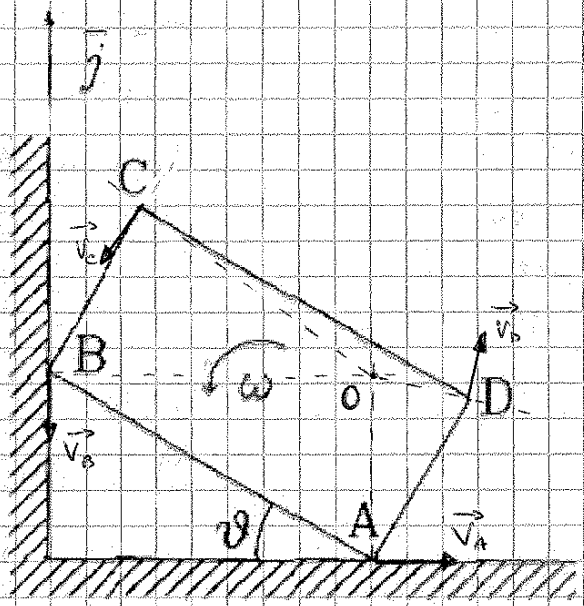
$$1 - \cos^2(60-\theta) = 1 + \cos^2\theta - 2\cos\theta\cos(60-\theta)$$

$$\cos^2(60-\theta) + \cos^2\theta - 2\cos\theta\cos(60-\theta) = 0$$

$$\left(\cos(60-\theta) - \cos\theta\right)^2 = 0 \Rightarrow \cos\theta = \cos(60-\theta)$$

$$\theta = 60 - \theta \Rightarrow \theta = 30°$$

1.6



$\omega = 3\ rad/s$

$\theta = 30°$

$AB = 1\ m$

$AD = 0,5\ m$

$\vec{V}_A$?

V_C ?

$\alpha_{\vec{V}_C/\vec{j}}$?

$V_A = \omega \cdot AO = \omega \cdot AB \sin\theta = 1,5\ m/s \Rightarrow \vec{V}_A = V_A \vec{i} = (1,5\vec{i})\ m/s$

$OC = \sqrt{OB^2 + BC^2 - 2OB \cdot BC \cdot \cos(90-\theta)}$

$= \sqrt{AB^2\cos^2\theta + BC^2 - 2AB \cdot BC \cdot \cos\theta \cdot \sin\theta} = 0,75\ m$

$V_C = \omega \cdot OC = 2,26\ m/s$

$x_0 = AB \cdot \cos\theta = 0,87\ m$

$y_0 = AB \cdot \sin\theta = 0,5\ m$

$x_C = BC \cdot \cos(90-\theta) = 0,25\ m$

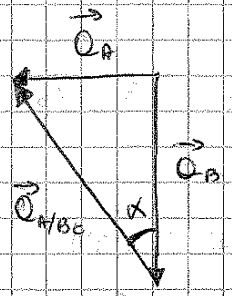
$y_C = y_0 + BC \cdot \sin(90-\theta) = 0,93\ m$

$r_{OC} = \dfrac{x - x_C}{x_0 - x_C} = \dfrac{y - y_C}{y_0 - y_C} \Rightarrow \dfrac{x - 0,25}{0,62} = \dfrac{y - 0,93}{-0,43}$

$0,11 - 0,43x = 0,62y - 0,58$

$y = -0,7x + 0,69$

$\alpha = \text{arctg}(-0,7) = 144,9°$



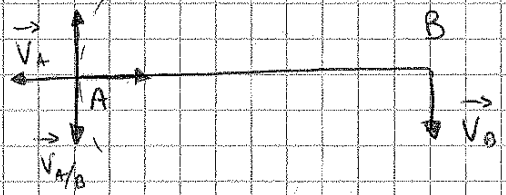
$a_A = \tg\alpha \cdot a_B = 436{,}9\ m/s^2$

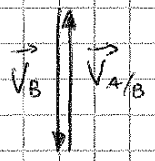
$\vec{a}_A = (-436{,}9\,\vec{i})\ m/s^2$

2) $\theta = 180°$

$\vec{v}_B = (-10\,\vec{j})\ m/s$

$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$

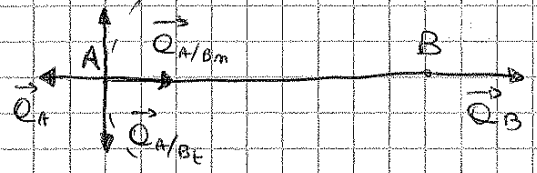




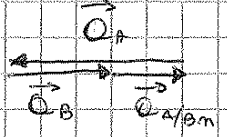
$v_B = v_{A/B} = 10\ m/s \Rightarrow v_A = 0$

$\omega_2 = 40\ rad/s$

$\vec{a}_B = \vec{a}_{B/Bm} = (1000\,\vec{i})\ m/s^2$



$\vec{a}_A = \vec{a}_B + \vec{a}_{A/Bm} + \vec{a}_{A/Bt}$



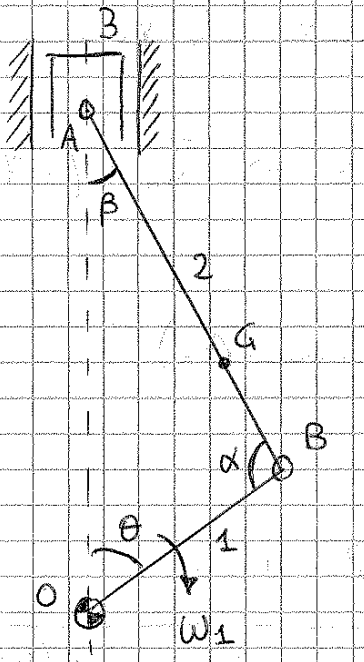
$a_A = a_B + a_{A/Bm} \Rightarrow a_{A/Bt} = 0$

$a_{A/Bm} = 400\ m/s^2 \Rightarrow \vec{a}_A = (-1400\,\vec{i})\ m/s^2$

$$a_B = \frac{a_{B/OM}}{\cos\alpha} = \frac{\omega_1^2 \cdot OB}{\cos\alpha} = 1{,}48 \text{ m/s}^2$$

$$\dot{\omega}_2 = \frac{a_{B/AB}}{AB} = \frac{a_B}{AB} = 1{,}48 \text{ rad/s}^2$$

1.9



$OB = 0{,}0425$ m

$AB = 0{,}107{,}5$ m

$AG = 0{,}075$ m

$\theta = 60°$

$\omega_1 = 1500$ rpm $= 157$ rad/s

$\dot{\omega}_1 = 0$

ω_2? $\dot{\omega}_2$?

$\vec{V}_G$? $\vec{a}_A$?

$$V_B = \omega_1 \cdot OB = 6{,}67 \text{ m/s}$$

$$AB^2 = AO^2 + OB^2 - 2AO \cdot OB \cdot \cos\theta$$

$$AO^2 - 2AO \cdot OB \cdot \cos\theta + OB^2 - AB^2 = 0$$

$$AO = \frac{OB\cos\theta + \sqrt{OB^2\cos^2\theta - OB^2 + AB^2}}{1} = 0{,}122 \text{ m}$$

$$AO^2 = OB^2 + AB^2 - 2AB \cdot OB\cos\alpha$$

$$\frac{OB^2 + AB^2 - AO^2}{2OB \cdot AB} = \cos\alpha \Rightarrow \alpha = 100°, \quad \beta = 20°$$

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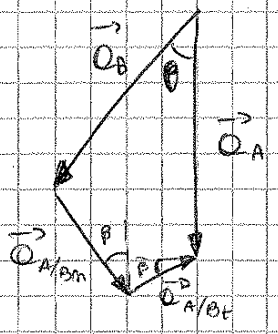
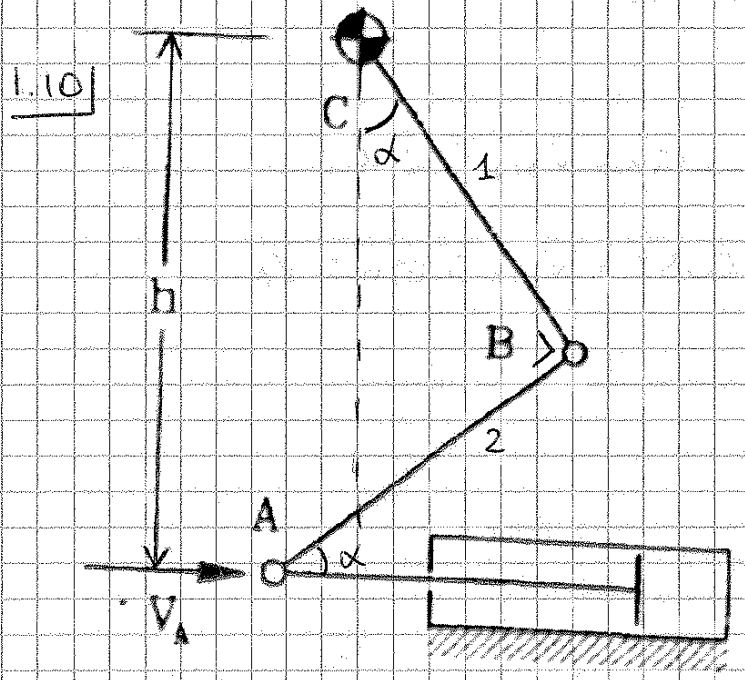
$$a_B \cdot \sin\theta = a_{A/Bn} \sin\beta + a_{A/Bt} \cos\beta$$

$$\Rightarrow a_{A/Bt} = 922{,}9 \text{ m/s}^2$$

$$\omega_2 = \frac{a_{A/Bt}}{AB} = 3584{,}9 \text{ rad/s}^2$$



$$a_A = a_B \cos\theta + a_{A/Bn} \cos\beta - a_{A/Bt} \sin\beta = 318{,}1 \text{ m/s}^2$$



$V_A = 0{,}5$ m/s

$a = 0{,}125$ m

$h = 0{,}175$ m

$\vec{\omega}_1$?

$\vec{\omega}_2$?

$$h = a\cos\alpha + a\sin\alpha$$

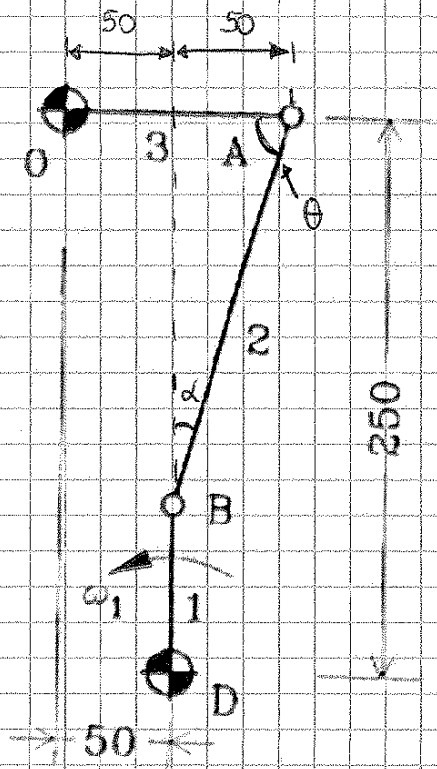
$$\cos\alpha + \sin\alpha = \frac{h}{a}$$

$$\cos^2\alpha + \sin^2\alpha + 2\cos\alpha\sin\alpha = \frac{h^2}{a^2}$$

$$1 + \sin 2\alpha = \frac{h^2}{a^2}$$

$$\alpha = \frac{\arcsin\left(\frac{h^2}{a^2} - 1\right)}{2} = 37°$$

1.11



$\omega_1 = 2\ rad/s$

$\dot{\omega}_1 = 0$

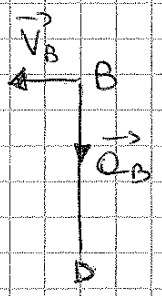
$OA = 0,1\ m$

$BD = 0,075\ m$

ω_2 ? ω_3 ?

$\dot{\omega}_2$? $\dot{\omega}_3$?

V_A ? a_A ?



$V_B = \omega_1 \cdot BD = 0,15\ m/s$

$a_B = a_{B/DM} = \omega_1^2 \cdot BD = 0,3\ m/s^2$

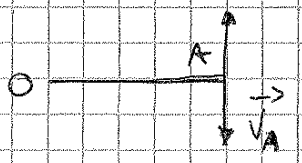
$$\begin{cases} AB \cdot \sin\theta = 0,175 \\ AB \cdot \cos\theta = 0,05 \end{cases}$$

$$AB = \frac{0,05}{\cos\theta} \Rightarrow 0,05 \cdot \operatorname{tg}\theta = 0,175$$

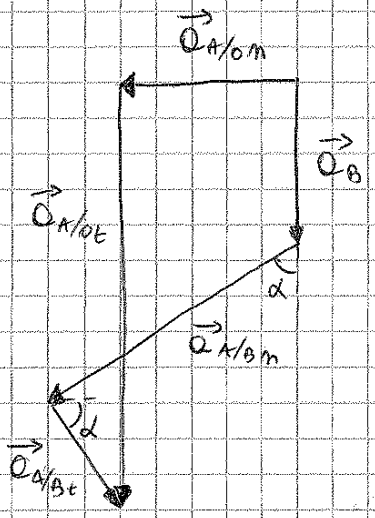
$$\theta = \operatorname{arctg}\left(\frac{0,175}{0,05}\right) = 74°$$

$AB = 0,182\ m$

$\alpha = 16°$



$V_A = \omega_3 \cdot OA$



$$a_{A/Om} = a_{A/Bn} \cdot \sin\alpha - a_{A/Bt} \cdot \cos\alpha$$

$$a_{A/Ot} = a_B + a_{A/Bn} \cdot \cos\alpha + a_{A/Bt} \cdot \sin\alpha$$

$$a_{A/Bt} = \frac{a_{A/Om} + a_{A/Bn} \sin\alpha}{\cos\alpha} = 0,06 \text{ m/s}^2$$

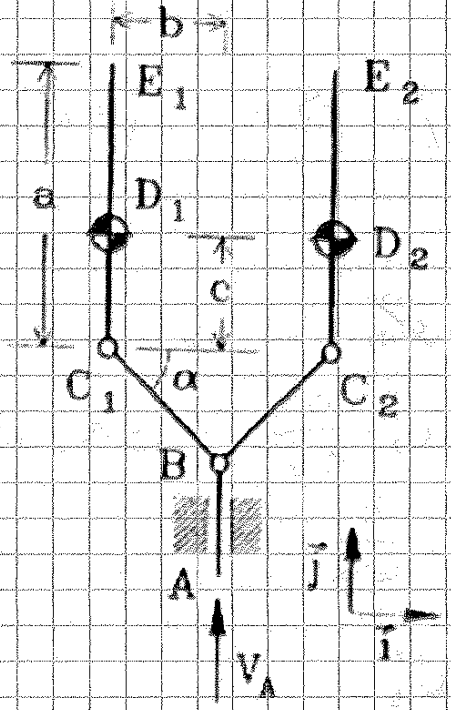
$$\Rightarrow \omega_2 = \frac{a_{A/Bt}}{AB} = 0,45 \text{ rad/s}^2 \quad \text{(oraria)}$$

$$a_{A/Ot} = 0,44 \text{ m/s}^2$$

$$\Rightarrow \omega_3 = \frac{a_{A/Ot}}{AO} = 4,41 \text{ rad/s}^2 \quad \text{(oraria)}$$

$$a_A = \sqrt{a_{A/Ot}^2 + a_{A/Om}^2} = 0,44 \text{ m/s}^2$$

1.12



$a = 0,5 \text{ m}$

$b = c = 0,2 \text{ m}$

$\alpha = 45°$

$V_A = 2 \text{ m/s}$

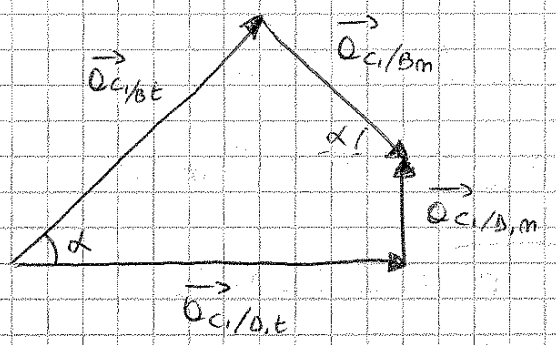
$a_A = 0$

$\vec{V}_{E_1}$?

a_{E_1} ?

$V_A = V_B = 2 \text{ m/s}$

$a_B = 0$



$$a_{C_1/D,t} = a_{C_1/Bm}\cos\alpha + a_{C_1/Bt}\cos\alpha = \cos\alpha\,(a_{C_1/Bm} + a_{C_1/Bt})$$

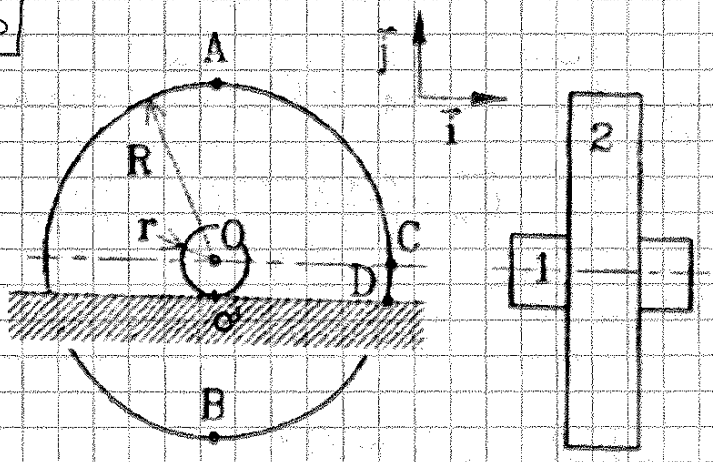
$$a_{C_1/Bt}\cdot\sin\alpha = a_{C_1/Bm} + a_{C_1/Bm}\cdot\sin\alpha$$

$$\Rightarrow \quad a_{C_1/Bt} = 56{,}6\ m/s^2$$

$$a_{C_1/D,t} = 60\ m/s^2$$

$$a_{E_1} = \sqrt{a_{E_1/D,m}^2 + a_{E_1/D,t}^2} = \sqrt{\omega_1^4(a-c)^2 + \dot{\omega}_1^2(a-c)^2} = 94{,}86\ m/s^2$$

1.13



$r = 0{,}05\ m$

$R = 0{,}25\ m$

$\vec{V}_0 = 0{,}8\,\vec{\imath}\ m/s$

$\vec{a}_0 = \vec{a}_{0t} = -1{,}4\,\vec{\imath}\ m/s^2$

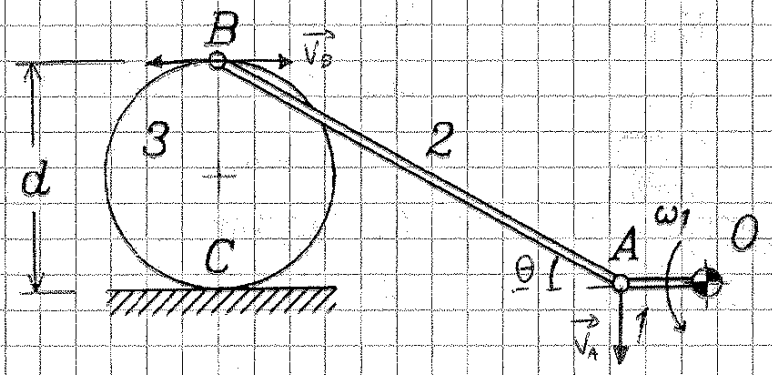
$\vec{V}_A\,?\ \vec{V}_B\,?\ \vec{V}_C\,?\ \vec{V}_D\,?$

$\vec{a}_A\,?\ \vec{a}_D\,?$

$$\omega = \frac{V_0}{r} = 16\ rad/s$$

$$\dot{\omega} = \frac{a_{0t}}{r} = 28\ rad/s^2$$

1.14



$AB = 0,2\,m$

$d = 0,1\,m$

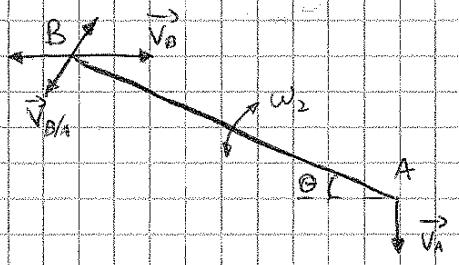
$AO = 0,04\,m$

$\omega_1 = 50\ rad/s$

$\vec{v}_B$?

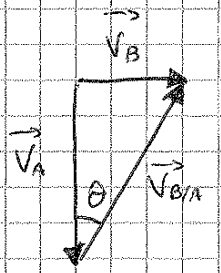
ω_2? ω_3?

$v_A = \omega_1 \cdot AO = 2\,m/s$



$\theta = \arcsin\left(\frac{d}{AB}\right) = 30°$

$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$

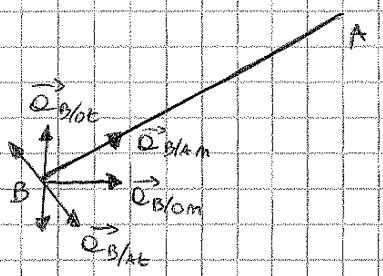


$v_{B/A} = \omega_2 \cdot AB$

$\vec{v}_B = v_A \operatorname{tg}\theta\, \vec{\imath} = 1,15\, \vec{\imath}\ m/s$

$v_{B/A} = \frac{v_A}{\cos\theta} = 2,3\,m/s \Rightarrow \omega_2 = 11,5\ rad/s$ (oraria)

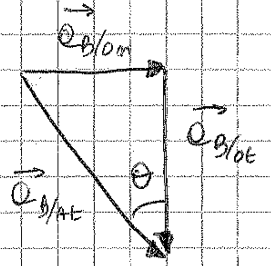
$\omega_3 = \frac{v_B}{d} = 11,5\ rad/s$ (oraria)



$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/At} + \vec{a}_{B/an}$$

$$\vec{a}_{B/om} + \vec{a}_{B/at} = \vec{a}_{B/At}$$

$$a_{B/At} = \dot{\omega}_2 \cdot AB$$

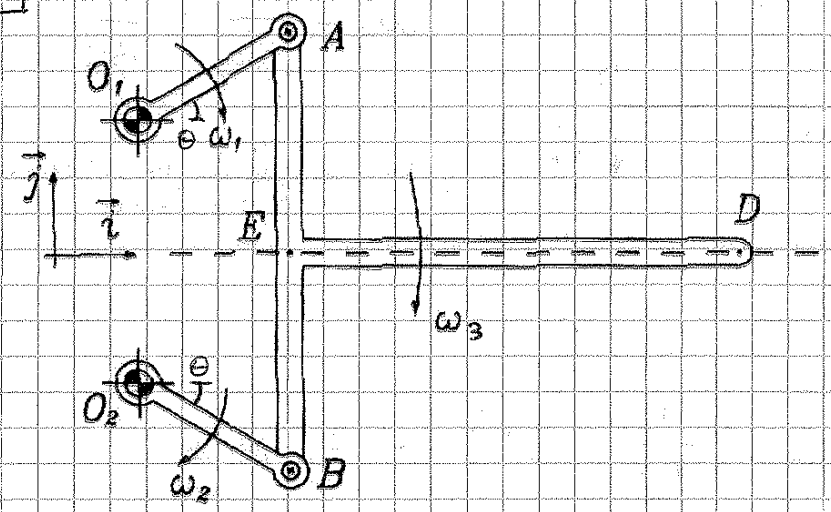


$$a_{B/At} = \frac{a_{B/om}}{\sin\theta} = \frac{\omega_1^2 \cdot R}{\sin\theta} = 10\ m/s^2$$

$$\Rightarrow \dot{\omega}_2 = \frac{a_{B/At}}{AB} = 12,5\ rad/s^2$$

$$a_B = a_{B/At} = 10\ m/s^2$$

1.16)



$O_1A = O_2B = 0,2m$

$AE = EB = 0,25m$

$AB = 0,5m$

$ED = 0,5m$

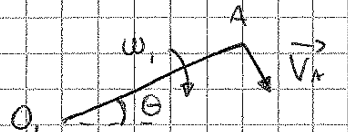
$O_1O_2 = 0,3m$

$\omega_1 = 10\ rad/s$

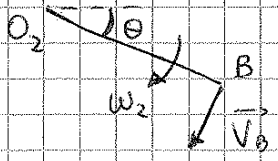
ω_2? ω_3?

$\vec{V}_D$?

$$\theta = \arcsin\left(\frac{\frac{AB - O_1O_2}{2}}{O_1A}\right) = 30°$$

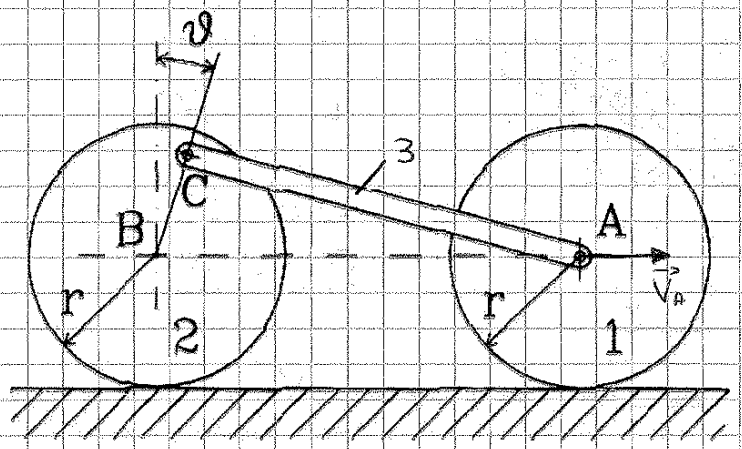


$$V_A = \omega_1 \cdot OA = 2m/s$$



$$V_B = \omega_2 \cdot O_2B$$

1.17]



$V_A = 5\ m/s$

$\omega_A = 0$

$AC = 0,8\ m$

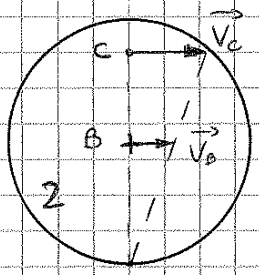
$r = 0,25\ m$

$BC = 0,2\ m$

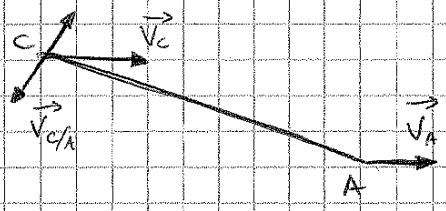
$\theta = 0$

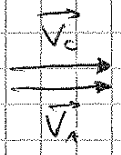
ω_3?

V_C? V_B?



$$\vec{V}_C = \vec{V}_A + \vec{V}_{C/A}$$





$V_{C/A} = \omega_3 \cdot AC$

$\vec{V}_C = \vec{V}_A$

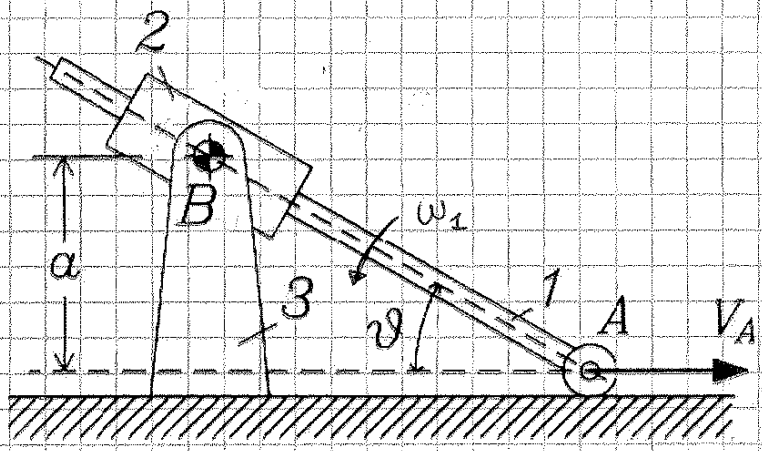
$\Rightarrow V_C = 5\ m/s$

$\omega_3 = 0$

$$\omega_2 = \frac{V_C}{r + BC} = 11,1\ rad/s$$

$$V_B = \omega_2 \cdot r = 2,8\ m/s$$

1.19



$a = 0,25 m$

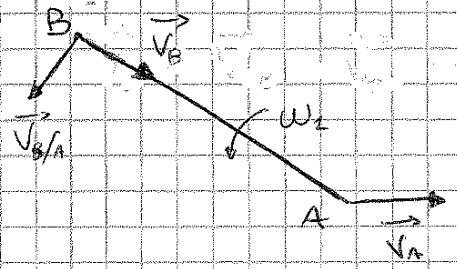
$V_A = 1 m/s$

$\vartheta = 30°$

ω_1 ?

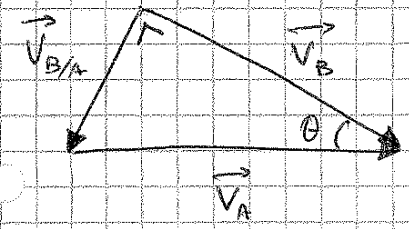
$V_{1/2}$?

$$AB = \frac{a}{\sin\vartheta} = 0,5 m$$



$$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$$

$$V_{B/A} = \omega_1 \cdot AB$$

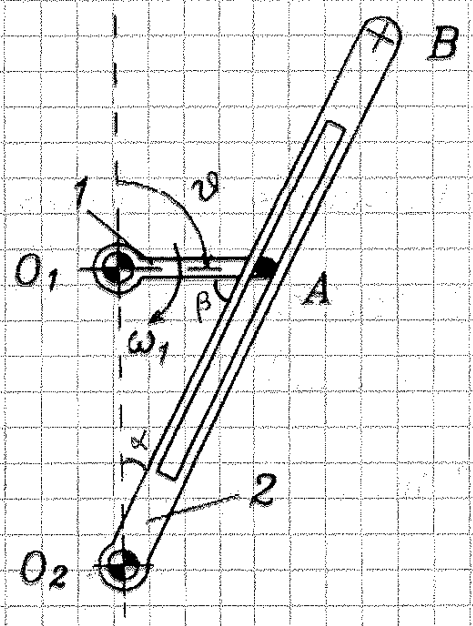


$$V_{B/A} = V_A \cdot \sin\vartheta$$

$$\Rightarrow \omega_1 = \frac{V_A \cdot \sin\vartheta}{AB} = 1 rad/s$$

$$V_{1/2} = V_B = V_A \cdot \cos\vartheta = 0,866 m/s$$

1.21



1) $\theta = 90°$

$\omega_1 = 1\ rad/s$

$\dot{\omega}_1 = 0$

$O_1A = 0,1\ m$

$O_1O_2 = 0,2\ m$

$O_2B = 0,4\ m$

$\omega_2 ?\ \dot{\omega}_2 ?\ V_B ?$

2) $\omega_2 = 0$

$\theta ?$

$$O_2A = \sqrt{O_1O_2^2 + O_1A^2 - 2\,O_1O_2 \cdot O_1A \cdot \cos(180-\theta)}$$

$$= \sqrt{O_1O_2^2 + O_1A^2 + 2\,O_1O_2 \cdot O_1A \cdot \cos\theta}$$

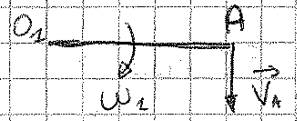
$$\frac{O_1A}{\sin\alpha} = \frac{O_1O_2}{\sin\beta} = \frac{O_2A}{\sin(180-\theta)} = \frac{O_2A}{\sin\theta}$$

$$\Rightarrow \sin\alpha = \frac{O_1A \sin\theta}{O_2A} \qquad \alpha = \arcsin\left(\frac{O_1A \sin\theta}{O_2A}\right)$$

$$\beta = 180 - \theta - \alpha$$

1) $\theta = 90°$

$$O_2A = \sqrt{O_1O_2^2 + O_1A^2} = 0,22\ m \qquad \alpha = 27° \quad \beta = 63°$$



$$V_A = \omega_1 \cdot O_1A = 0,1\ m/s$$

2) $\omega_2 = 0 \Rightarrow V_{At} = 0$

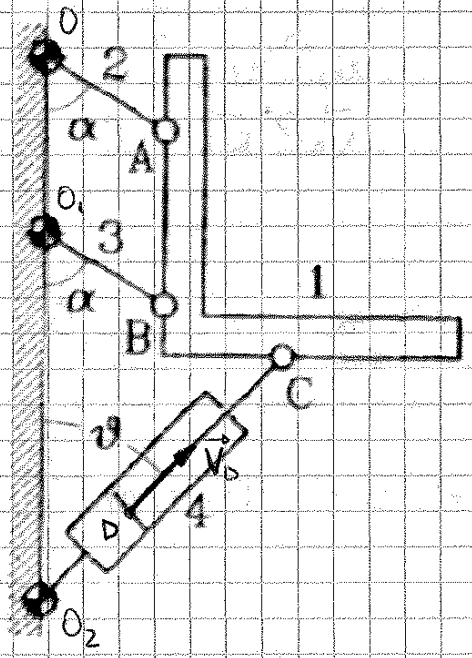
$$\Rightarrow V_A \cdot \sin\alpha = V_A \cdot \frac{O_1A \sin\theta}{O_2A}$$

$$= V_A \frac{O_1A \sin\theta}{\sqrt{O_1O_2^2 + O_1A^2 + 2O_1O_2 \cdot O_1A \cdot \cos\theta}} = 0$$

$$\Rightarrow \sin\theta = 0 \qquad \theta = 0$$

$$\theta = 180°$$

1.22



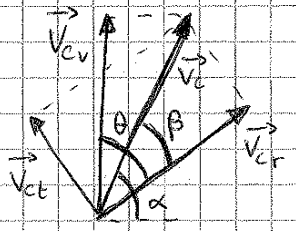
$V_D = 0{,}1$ m/s

$\theta = 45°$

$\alpha = 60°$

$\vec{V}_{Cv}$

$$\vec{V}_C = \vec{V}_{Cr} + \vec{V}_{Ct}$$



$V_{Cr} = V_D = 0{,}1$ m/s

$V_{Ct} = \omega_4 \cdot O_2C$

$\beta = \theta - (90 - \alpha) = 15°$

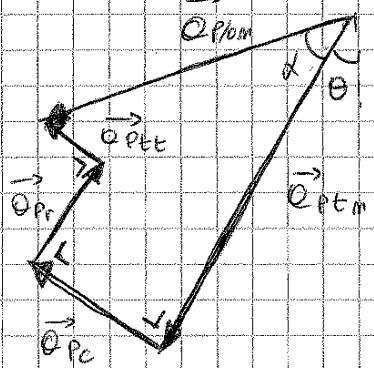
$V_{Cv} = V_C \cdot \cos(90 - \alpha)$

$$\Rightarrow \omega_2 = \frac{V_P \cdot \sin(\beta - 90 + \theta)}{O'P} = 40{,}63 \text{ rad/s}$$

(antioraria)

$$V_A = \omega_2 \cdot O'A = 32{,}5 \text{ m/s}$$

$$\vec{a}_P = \vec{a}_{P/Om} = \vec{a}_{Pr} + \vec{a}_{Ptm} + \vec{a}_{Ptt} + \vec{a}_{Pc}$$



$$a_{Ptt} = \dot{\omega}_2 \cdot O'P = a_{P/Om} \cdot \sin\alpha - a_{Pc}$$

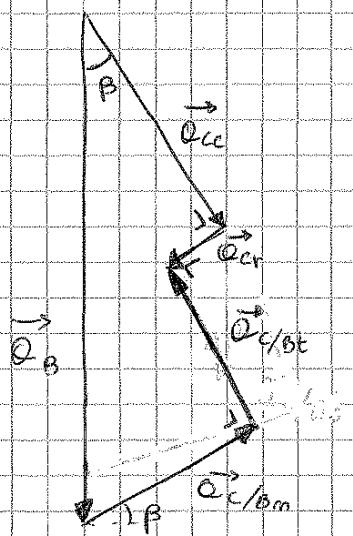
$$a_{Pc} = 2\omega_2 \cdot V_{Pr} = 1373{,}3 \text{ m/s}^2$$

$$\Rightarrow \dot{\omega}_2 = \frac{a_{P/Om} \sin\alpha - a_{Pc}}{O'P} = 520 \text{ rad/s}^2$$

$$a_{Am} = \omega_2^2 \cdot O'A = 1320{,}6 \text{ m/s}^2$$

$$a_{At} = \dot{\omega}_2 \cdot O'A = 416 \text{ m/s}^2$$

$$\Rightarrow a_A = \sqrt{a_{Am}^2 + a_{At}^2} = 1385 \text{ m/s}^2$$



$$a_{cc} = 2\,\omega_2 \cdot v_c = 1{,}7\ m/s^2$$

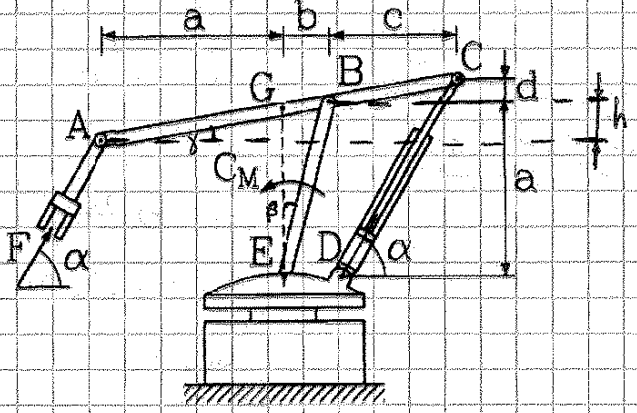
$$a_{c/Bt} = \dot{\omega}_2 \cdot BC$$

$$a_B \cdot \cos\beta = a_{cc} + a_{c/Bt}$$

$$a_{c/Bt} = a_B \cos\beta - a_{cc} = 4{,}07\ m/s^2$$

$$\Rightarrow \dot{\omega}_2 = \frac{a_{c/Bt}}{BC} = 6{,}25\ rad/s^2$$

2.7



$F = 100\ N$

$m_{AC} = 25\ Kg$

$a = 0{,}75\ m$

$b = 0{,}25\ m$

$c = 0{,}5\ m$

$d = 0{,}1\ m$

$\alpha = 60°$

F_{cil}?

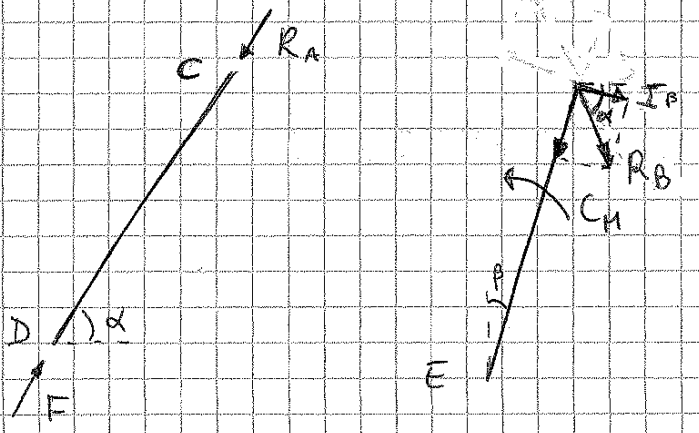
R_B?

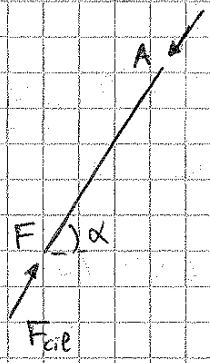
C_M?

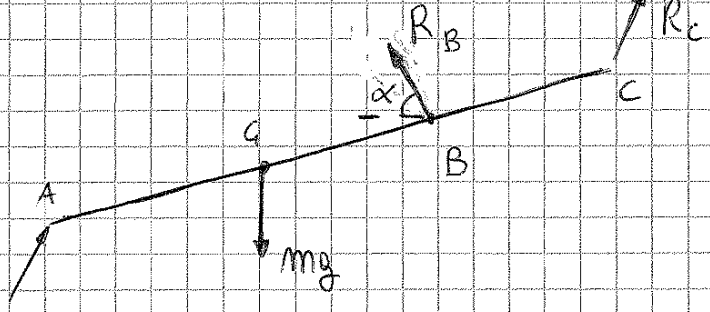
$$\beta = arctg\left(\frac{b}{a}\right) = 18{,}4°$$

$$\gamma = arctg\left(\frac{d}{c}\right) = 11{,}3°$$

$$h = (a+b)\,tg\,\gamma = 0{,}2\ m$$







$$R_A = F = 100\ N$$

$$R_C = F_{cil}$$

$$m\ddot{y}R + I_G\ddot{\theta} = 0 \qquad \Rightarrow m\ddot{y}R + I_G\frac{\ddot{y}-\ddot{x}}{R} = 0$$

$$\ddot{\theta} = \frac{\ddot{y}-\ddot{x}}{R}$$

$$m\ddot{y}R + I_G\frac{\ddot{y}-\ddot{x}}{R} = m\ddot{x}R - m\dot{x}R$$

$$m(\ddot{y}-\ddot{x})R + m\dot{x}R + \frac{I_G}{R}(\ddot{y}-\ddot{x}) = 0$$

$$-(\ddot{y}-\ddot{x})\left(mR + \frac{I_G}{R}\right) = m\dot{x}R$$

$$\ddot{x}-\ddot{y} = \frac{m\dot{x}R}{mR + I_G/R} = \frac{\dot{x}}{A} \qquad \text{con } A = \frac{mR}{mR + I_G/R}$$

$$S = \frac{1}{2}\ddot{x}t^2 \quad \Rightarrow \frac{t^2}{2} = \frac{S}{\dot{x}}$$

$$d = \frac{1}{2}(\ddot{x}-\ddot{y})t^2 = (\ddot{x}-\ddot{y})\frac{S}{\dot{x}} = \frac{\ddot{x}}{A}\cdot\frac{S}{\dot{x}} = \frac{S}{A}$$

$$S = d\cdot A = d\cdot\frac{mR}{mR + I_G/R} = d\cdot\frac{mR}{mR(1+1/2)} = \frac{2}{3}d$$

$$P = Kx\sqrt{\frac{K}{m}\left(x_0^2 - x^2\right)}$$

$$\frac{dP}{dx} = K\sqrt{\frac{K}{m}\left(x_0^2 - x^2\right)} + Kx\sqrt{\frac{K}{m}}\cdot\frac{1}{2}\cdot\frac{-2x}{\sqrt{x_0^2 - x^2}}$$

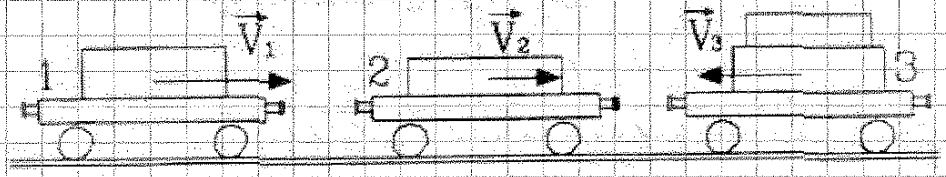
$$= K\sqrt{\frac{K}{m}}\left(\sqrt{x_0^2 - x^2} - \frac{x^2}{\sqrt{x_0^2 - x^2}}\right) = K\sqrt{\frac{K}{m}}\left(\frac{x_0^2 - x^2 - x^2}{\sqrt{x_0^2 - x^2}}\right)$$

$$= K\sqrt{\frac{K}{m}}\left(\frac{x_0^2 - 2x^2}{\sqrt{x_0^2 - x^2}}\right) = 0$$

$$\Rightarrow x_0^2 - 2x^2 = 0 \qquad x = \frac{x_0}{\sqrt{2}}$$

$$P_{MAX} = K\frac{x_0}{\sqrt{2}}\sqrt{\frac{K}{m}\frac{x_0^2}{2}} = K\frac{x_0^2}{2}\sqrt{\frac{K}{m}}$$

2.13



$V_1 = 2\ Km/h = 0,56\ m/s$

$V_2 = 1\ Km/h = 0,28\ m/s$

$V_3 = 1,5\ Km/h = 0,42\ m/s$

V_d ?

$E_{p\%}$?

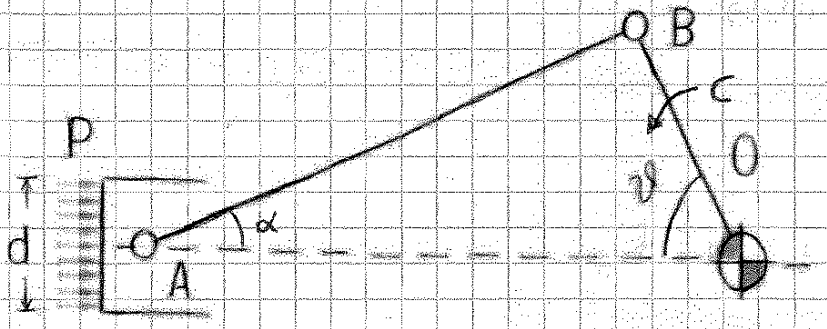
$m_1 = 65 \cdot 10^3\ Kg$

$m_2 = 50 \cdot 10^3\ Kg$

$m_3 = 75 \cdot 10^3\ Kg$

$$m_1 V_1 + m_2 V_2 - m_3 V_3 = (m_1 + m_2 + m_3) V_d$$

2.16

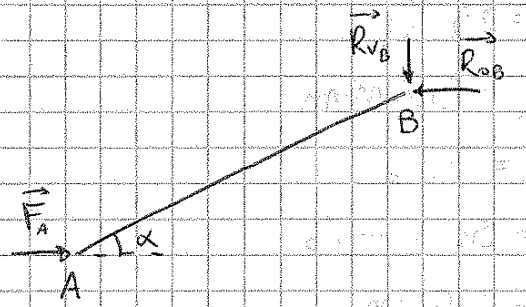


$p = 100\ kPa$
$d = 40\ mm$
$OB = 42,5\ mm$
$AB = 107,5\ mm$
$\theta = 60°$

R_O, R_A ?
R_B, C

$$\frac{OB}{\sin\alpha} = \frac{AB}{\sin\theta} \Rightarrow \alpha = 20,02°$$

$$F_A = p \cdot S = p \frac{\pi d^2}{4} = 125,6\ N$$

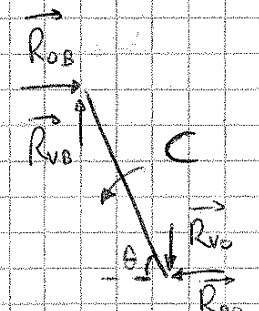


$F_A = R_{OB} = 125,6\ N$

A) $R_{OB} \cdot \cancel{AB} \sin\alpha - R_{VB} \cdot \cancel{AB} \cos\alpha = 0$

$$R_{VB} = \frac{R_{OB}\sin\alpha}{\cos\alpha} = R_{OB} \operatorname{tg}\alpha = 45,76N$$

$$R_B = \sqrt{R_{VB}^2 + R_{OB}^2} = 133,67\ N$$

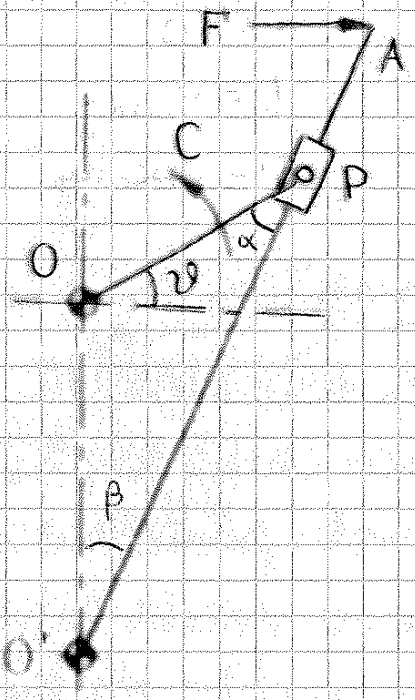


O) $R_{OB} \cdot \cancel{OB} \sin\theta + R_{VB} \cdot \cancel{OB} \cos\theta = C = 5,6\ Nm$

$R_{VB} = R_{VO} = 45,76\ N$
$R_{OB} = R_{OO} = 125,6\ N$

$R_O = R_B = 133,67\ N$

2.20



$OP = 0,3\ m$

$O'A = 0,8\ m$

$O'O = 0,4\ m$

$\theta = 25°$

$F = 100\ N$

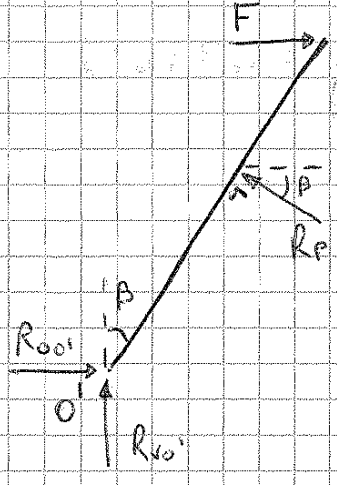
C?

R_o?

$$PO' = \sqrt{PO^2 + OO'^2 - 2 \cdot OO' \cdot PO \cos(90-\theta)}$$

$$= \sqrt{PO^2 + OO'^2 + 2 \cdot OO' \cdot PO \sin\theta} = 0,59\ m$$

$$\frac{PO}{\sin\beta} = \frac{PO'}{\sin(90+\theta)} = \frac{OO'}{\sin\alpha} \Rightarrow \begin{matrix} \alpha = 37,7° \\ \beta = 27,3° \end{matrix}$$



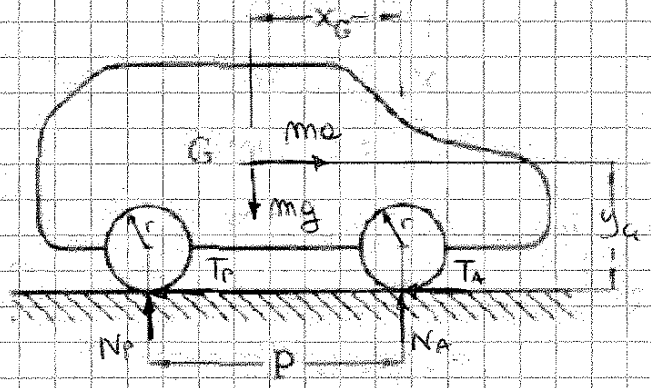
O') $R_P \cdot O'P - F \cdot O'A \cdot \cos\beta = 0$

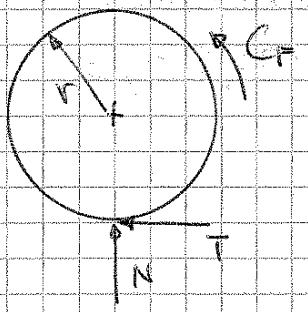
$$\Rightarrow R_P = F \frac{O'A}{O'P} \cos\beta = 119,92\ N$$

$$mg\frac{L}{2} = m\ddot{x}\frac{L}{2} + m\frac{L^2}{12}\ddot{x}\frac{2}{L} \quad \Rightarrow \ddot{x} = \frac{g}{1+\frac{1}{3}} = \frac{3}{4}g$$

$$R'_{AV} = R'_A = mg - m\ddot{x} = mg\left(1 - \frac{3}{4}\right) = \frac{mg}{4}$$

3.8





$p = 2,5\,m$
$x_G = 1,4\,m$
$y_G = 0,8\,m$
$r = 0,32\,m$
$m = 1000\,kg$
$f = 0,4$
$V_0 = 27,8\,m/s$

x ?
C_P ?
C_A ?

$$\begin{cases} C_A = T_A \cdot r \\ C_P = T_P \cdot r \\ T_A = f N_A \\ T_P = f N_P \\ N_A + N_P = mg \\ T_A + T_P = ma \\ N_P p + ma y_G = mg x_G \end{cases}$$

$$f(N_P + N_A) = ma$$

$$fmg = ma \Rightarrow a = fg = 3,9\,m/s^2$$

$$N_P = \frac{mg x_G - mfg y_G}{p} = 4238\,N$$

$$T_1 = T_2 + ma$$

$$mg(x_G + u) + may_G = N_1 p$$

$$T_1 = f_a N_1$$

$$T_2 = f_a N_2$$

$$N_1 = \frac{mg(x_G + u) + may_G}{p} = 6610,6\ N$$

$$N_2 = mg - N_1 = 3199,4\ N$$

$$T_2 = N_2 \frac{u}{r} = 400\ N$$

$$T_1 = T_2 + ma = 3400\ N$$

1) $f_a = \frac{T_1}{N_1} = 0,51$

$T_2 \leq f_a N_2 \Rightarrow \frac{T_2}{N_2} = 0,12 \leq f_a$ aderenza verificata

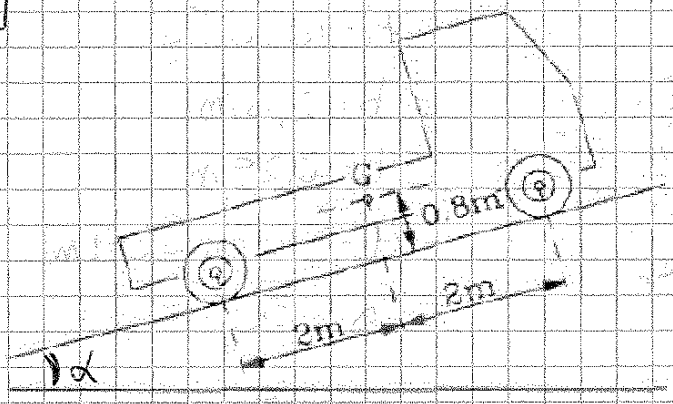
2) $f_a = \frac{T_2}{N_2} = 0,12$

$T_1 \leq f_a N_1 \Rightarrow \frac{T_1}{N_1} = 0,51 > f_a$ aderenza non verificata

$\Rightarrow f_a = 0,51$

$$C_M = T_1 r + N_1 u = 1352,4\ Nm$$

3.11



$h = 0,8\ m$

$b = 2\ m$

$V_0 = 0\ m/s$

$a = cost$

$V_g = 40\ Km/h$

$= 11,1\ m/s$

$x = 50\ m$

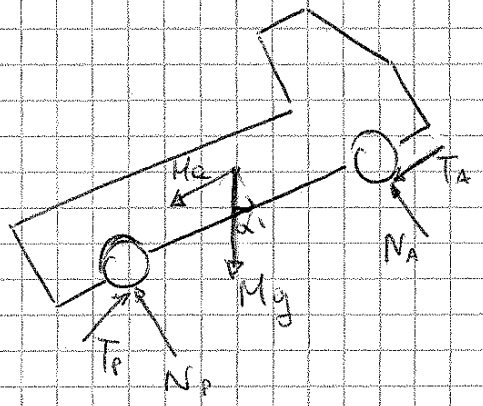
$M = 5000\ Kg$

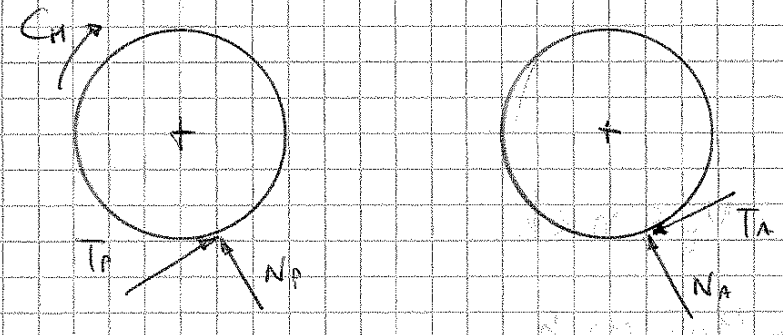
N_P, T_P

N_A, T_A ?

g_{min}

$$\alpha = \operatorname{arctg}\left(\frac{15}{100}\right) = 8,5°$$



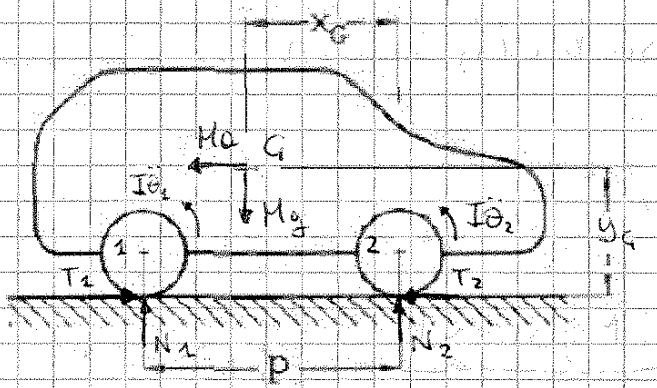


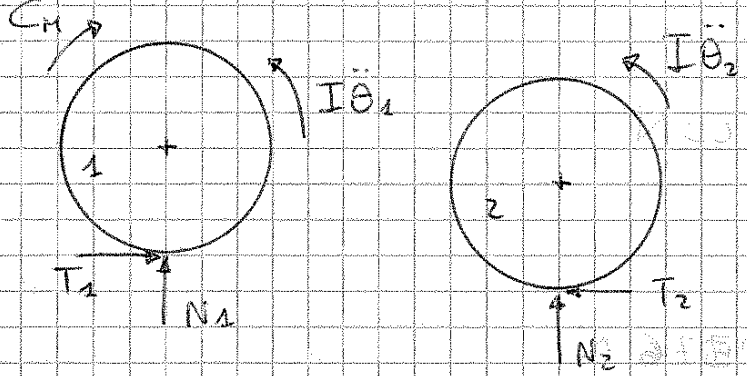
$$T_P \frac{h}{2} = C_M$$

$$T_A \frac{h}{2} = 0$$

$$T_P = T_A + Ma + Mg \sin\alpha$$

3.12





$M = 1360\ Kg$

$p = 2,3\ m$

$D = 0,65\ m$

$x_G = 1,3\ m$

$y_G = 0,72\ m$

$m = 10\ Kg$

$\rho = 0,2\ m$

$f_0 = 1$

C_M

a ?

N_A, T_A

N_P, T_P

$I = m\rho^2 = 0,4\ Kgm^2$

$T_1 = f_0 N_1$

$Mg = 2N_1 + 2N_2$

$2T_1 = 2T_2 + Ma$

$2I\ddot{\theta}_2 + May_G + Mgx_G - 2N_1 p + 2I\ddot{\theta}_1 = 0$

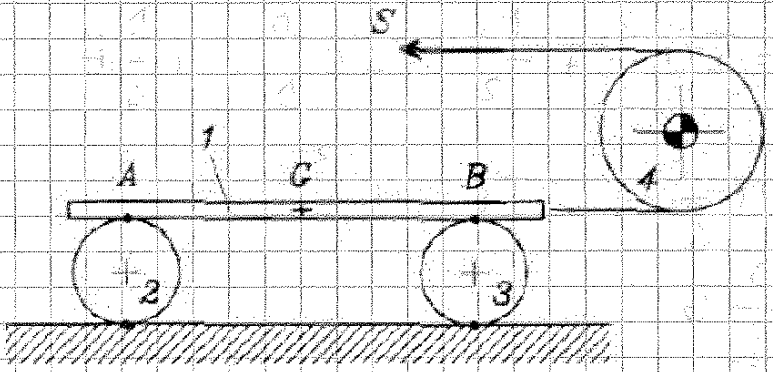
$a = \frac{D}{2}\ddot{\theta}_1$

$C_M = 2I\ddot{\theta}_1 + 2T_1\frac{D}{2}$

$I\ddot{\theta}_2 = T_2\frac{D}{2}$

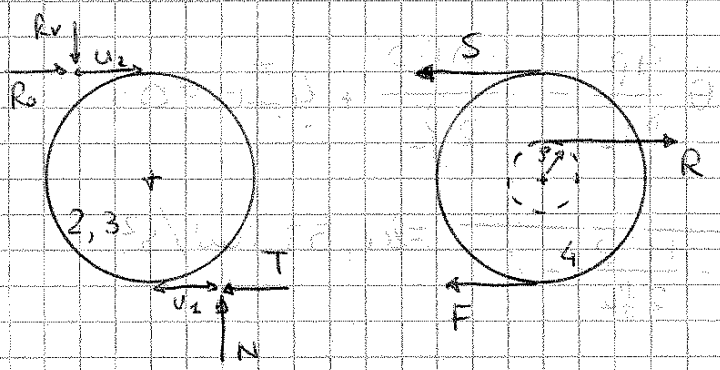
$a = \frac{D}{2}\ddot{\theta}_2$ (verificare l'aderenza, $T_2 \leq f_0 N_2$)

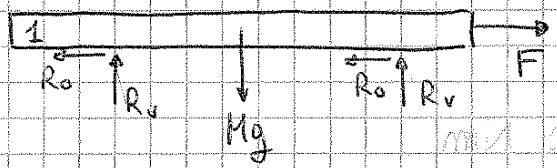
3,13



$M = 200\ Kg$
$d = 0,2\ m$
$u_1 = 0,01\ m$
$u_2 = 0,005\ m$
$D = 0,3\ m$
$r = 0,04\ m$
$\varphi = 30°$
$v = cost$

S?





$$\begin{cases} N = R_v \\ T = R_0 \\ N u_1 + R_v u_2 = (T + R_0)\frac{d}{2} \end{cases} \Rightarrow N(u_1 + u_2) = Td$$

$$\begin{cases} 2R_v = Mg \\ 2R_0 = F \end{cases} \Rightarrow \begin{matrix} 2N = Mg \\ 2T = F \end{matrix}$$

$$F\left(\frac{D}{2} + \rho\right) = S\left(\frac{D}{2} - \rho\right)$$

$$\rho = \sin\varphi \cdot r = 0,02\ m$$

$$T = \frac{2N(u+\beta)}{d}$$

$$N = \frac{F \sin\beta\, d}{2(u+\beta)}$$

$$\frac{F \sin\beta\, d}{2(u+\beta)} + F\cos\beta = Mg$$

$$F\cos\beta(\ell+u) + F\sin\beta\, h = Mg(a+u)$$

$$F = \frac{Mg(a+u)}{\cos\beta(\ell+u)+\sin\beta\, h}$$

$$\frac{Mg(a+u)}{\cos\beta(\ell+u)+\sin\beta\, h}\left(\frac{\sin\beta\, d}{2(u+\beta)} + \cos\beta\right) = Mg$$

$$\frac{Mg(a+u)\sin\beta\, d}{2(u+\beta)(\cos\beta(\ell+u)+\sin\beta\, h)} + \frac{Mg(a+u)\cos\beta}{\cos\beta(\ell+u)+\sin\beta\, h} = Mg$$

$$Mg(a+u)\sin\beta\, d + 2(u+\beta)Mg(a+u)\cos\beta = 2Mg(u+\beta)(\cos\beta(\ell+u)+\sin\beta\, h)$$

$$\sin\beta(ad+ud-2uh-2\beta h) = \cos\beta(2u\ell+2u^2+2\beta\ell+2u\beta-2au-2\beta a-2\beta u-2u^2)$$

$$tg\beta = \frac{2u\ell+2\beta\ell-2au-2\beta a}{ad+ud-2uh-2\beta h} \Rightarrow \beta = 2{,}84°$$

$$F = 444{,}7\ N$$

$$\begin{cases} N + F\sin\alpha = m_3 g\cos\beta \\ m_3\ddot{x} + T + m_3 g\sin\beta = F\cos\alpha \\ T = f N \end{cases}$$

$$F = \frac{m_3\ddot{x} + f m_3 g\cos\beta + m_3 g\sin\beta}{\cos\alpha + f\sin\alpha} = 4294\ N$$

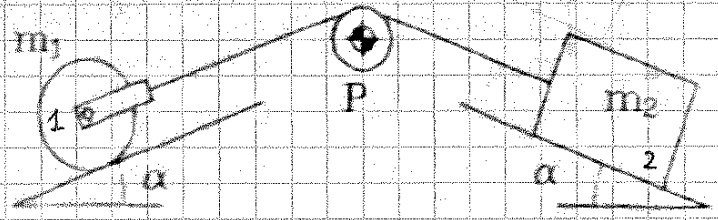
$$N_P = \frac{I\ddot{\theta} + m_1 g b\cos\beta + m_1 g h\sin\beta + m_1\ddot{x}h + Fe\cos\alpha + F(a+b+c)\sin\alpha}{a+b}$$

$$= 18\cdot 10^3\ N$$

$$N_A = m_1 g\cos\beta + F\sin\alpha - N_P = 5{,}78\cdot 10^3\ N$$

$$T_P = m_1\ddot{x} + m_1 g\sin\beta + F\cos\alpha = 15{,}95\cdot 10^3\ N$$

3.16)

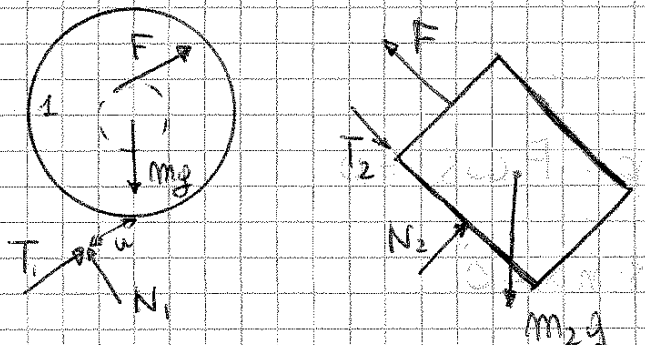


$\alpha = 20°$
$r = 0{,}2\ m$
$a = 0{,}02\ m$
$f = 0{,}3$
$m_1 = 60\ Kg$
$I = 1{,}2\ Kg\,m^2$
$r_p = 0{,}04\ m$
$f_p = 0{,}4$

1) $V = cost \rightarrow m_2$?

2) $m_2 = 20\ Kg \rightarrow a$? $f_{a\,min}$?

1)



$$\begin{cases} N_1 = m_1 g \cos\alpha \\ T_1 + m_1 a + F = m_1 g \sin\alpha \\ m_1 a \cdot r + F(r+\rho) + I\ddot{\theta}_1 + m_1 g \cos\alpha \cdot u = m_1 g \sin\alpha \cdot r \\ a = r\ddot{\theta}_1 \\ T_1 = f_{a_{min}} N_1 \\ N_2 = m_2 g \cos\alpha \\ T_2 + m_2 a + m_2 g \sin\alpha = F \\ T_2 = f N_2 \end{cases}$$

$$f m_2 g \cos\alpha + m_2 a + m_2 g \sin\alpha = F$$

$$m_1 a r + (f m_2 g \cos\alpha + m_2 a + m_2 g \sin\alpha)(r+\rho) + I\frac{a}{r} + m_1 g \cos\alpha \cdot u = m_1 g \sin\alpha \cdot r$$

$$a = \frac{-(f m_2 g \cos\alpha + m_2 g \sin\alpha)(r+\rho) - m_1 g \cos\alpha \cdot u + m_1 g \sin\alpha \cdot r}{m_1 r + m_2 (r+\rho) + I/r}$$

$$= 0{,}13\ m/s^2$$

$$f_{a_{min}} = \frac{T_1}{N_1} = \frac{m_1 g \sin\alpha - m_1 a - f m_2 g \cos\alpha - m_2 a - m_2 g \sin\alpha}{m_1 g \cos\alpha}$$

$$= 0{,}124$$

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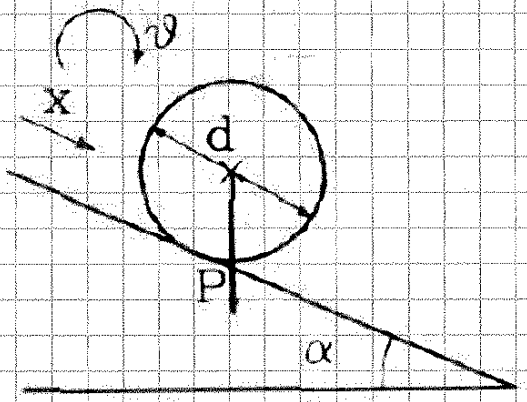
$$\frac{\sin\beta}{\cos\beta} = \cos\beta \rightarrow \operatorname{tg}\beta = f \qquad \beta = 11,3°$$

$$T'_{min} = 2303\ N$$

3.18



$V_0 = 0$

$d = 1\ m$

$M = 10^4\ Kg$

$f_a = 0,2$

$f = 0,15$

$u = 0,02\ m$

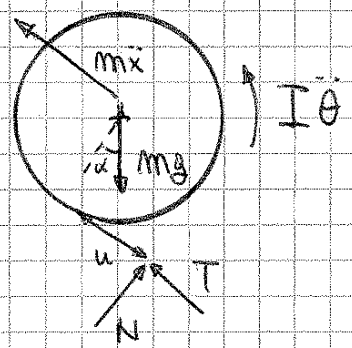
1) $\alpha = 10°$

2) $\alpha = 45°$

$x = 200\ m$

t?

m_{fin}?



$$N = mg\cos\alpha$$

$$T + m\ddot{x} = mg\sin\alpha$$

$$I\ddot{\theta} + mg\cos\alpha \cdot u + m\ddot{x}\frac{d}{2} = mg\sin\alpha\frac{d}{2}$$

a) aderenza $\rightarrow \ddot{x} = \frac{d}{2}\ddot{\theta}$

b) strisciamento $\rightarrow T = fN$

$$I = m\frac{d^2}{8}$$

2) $\alpha = 45°$ → strisciamento

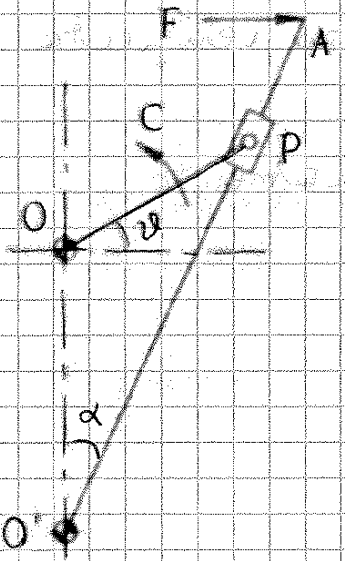
$$\ddot{x} = g(\sin\alpha - f\cos\alpha) = 5,9\ m/s^2$$

$$t = \sqrt{\frac{2x}{\ddot{x}}} = 8,2\ s$$

$$\ddot{\theta} = \frac{m g \sin\alpha \frac{d}{2} - m\ddot{x}\frac{d}{2} - m g \cos\alpha \cdot u}{m\frac{d^2}{8}} = 3\ rad/s^2$$

$$n_{giri} = \frac{\ddot{\theta} t^2}{4\pi} = 16,3\ giri$$

3.20



$OP = 0,3\ m$

$O'A = 0,8\ m$

$O'O = 0,4\ m$

$\theta = 25°$

$F = 100\ N$

$f = 0,5$

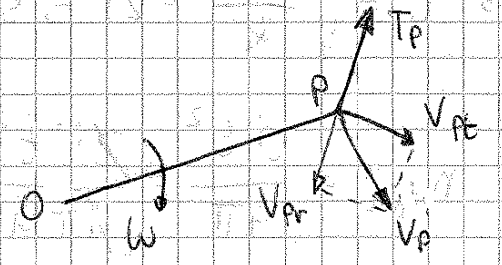
C_o?

C_A?

$$O'P = \sqrt{OP^2 + OO'^2 - 2OP \cdot OO' \cdot \cos(90+\theta)} = 0,6\ m$$

$$\frac{O'P}{\sin(90+\theta)} = \frac{OP}{\sin\alpha} \Rightarrow \alpha = 27°$$

1) ω_1 oraria:



$$C = N_p \cdot OP(\cos\alpha \cdot \sin\theta + \sin\alpha\cos\theta) + T_p\, OP(\cos\alpha\cos\theta - \sin\alpha\sin\theta)$$

$$= 39\ Nm$$

$$mg\frac{D_4}{2} = (T_1 + T_2)\frac{D_3}{2}$$

$$2Fa = T_1\left(\frac{a}{f} + h\right)$$

$$2Fa = T_2\left(\frac{a}{f} - h\right)$$

$$T_1 = \frac{2Fa}{\frac{a}{f} + h}$$

$$\Rightarrow \quad T_2 = \frac{2Fa}{\frac{a}{f} - h}$$

$$T_1 + T_2 = 2Fa\left(\frac{1}{\frac{a}{f}+h} + \frac{1}{\frac{a}{f}-h}\right) = 2Fa\left(\frac{f}{a+fh} + \frac{f}{a-fh}\right)$$

$$= 2faF\left(\frac{a - fh + a + fh}{a^2 - f^2h^2}\right) = \frac{4a^2fF}{a^2 - f^2h^2}$$

$$\frac{4a^2fF}{a^2-f^2h^2} = mg\frac{D_4}{D_3} \Rightarrow F = mg\frac{D_4}{D_3}\frac{a^2-f^2h^2}{4a^2f} = 81,5\ N$$

$T_1 D = F b$

$\frac{T_2}{T_1} = e^{f\theta^*} \rightarrow T_2 = T_1 e^{f\theta^*}$

$M_{f_0} = F \frac{b}{D} \left(e^{f\theta^*} - 1\right) \frac{D}{2} = 119,3 \text{ Nm}$

$T_1 = \frac{Fb}{D} = 500 \text{ N}$

$T_2 = T_1 e^{f\theta^*} = 1096,6 \text{ N}$

$R_{00} = R_{00_v} = T_1 + T_2 = 1596 \text{ N}$

2) ω antioraria $\rightarrow T_1 > T_2$

$M_{f_A} = (T_1 - T_2) \frac{D}{2}$

$T_1 D = F b \Rightarrow T_1 = \frac{Fb}{D}, \quad T_2 = \frac{T_1}{e^{f\theta^*}} = \frac{Fb}{D} \frac{1}{e^{f\theta^*}}$

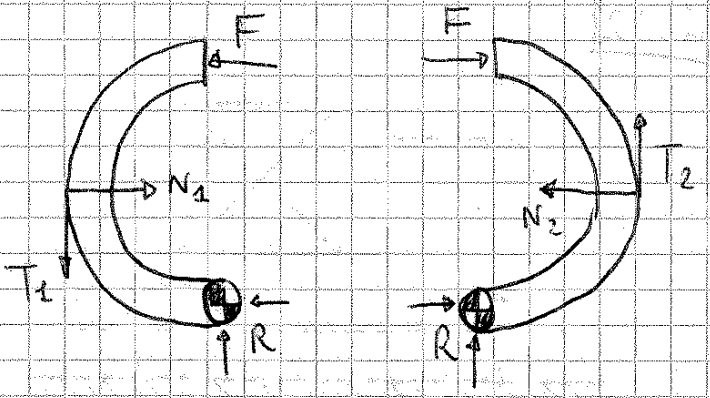
$\frac{T_1}{T_2} = e^{f\theta^*}$

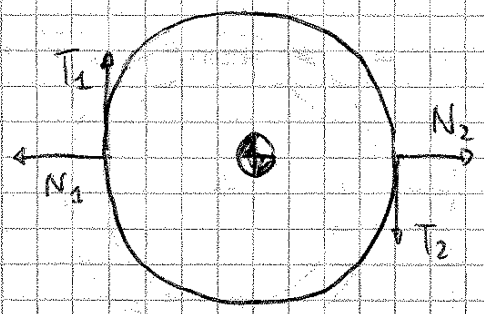
$M_{f_A} = \frac{Fb}{D} \left(1 - \frac{1}{e^{f\theta^*}}\right) \frac{D}{2} = 54,4 \text{ Nm}$

$T_1 = 500 \text{ N} \Rightarrow R_{0A} = R_{0A_v} = T_1 + T_2 = 728 \text{ N}$

$T_2 = 228 \text{ N}$

$$\begin{cases} N_c = mg \\ T_c = m\ddot{x} \\ Mg + N_c = N_p + N_A \\ M\ddot{x} + T_c = T_p + \cancel{T_A} \\ T_c = f_{c_{min}} N_c \end{cases}$$





$$X = \frac{V_0^2}{2\ddot{x}} = 32,2\ m$$

$$t = \frac{V_0}{\ddot{x}} = 4,6\ s$$

$$\begin{cases} \frac{C_F}{\cancel{2}} = \left(T_1 + T_2\right)\frac{d}{\cancel{2}} \\ N_1 a = 2Fa + T_1 \frac{d}{2} \\ N_2 a = T_2 \frac{d}{2} = 2Fa \\ T_1 = f N_1 \rightarrow T_1\left(\frac{a}{f} - \frac{d}{2}\right) = 2Fa \\ T_2 = f N_2 \rightarrow T_2\left(\frac{a}{f} + \frac{d}{2}\right) = 2Fa \end{cases}$$

$$T_1 = 2Fa\left(\frac{2f}{2a - df}\right)$$

$$T_2 = 2Fa\left(\frac{2f}{2a + df}\right)$$

$$\rightarrow T_1 + T_2 = 2Fa\left(\frac{2f}{2a - df} + \frac{2f}{2a + df}\right)$$

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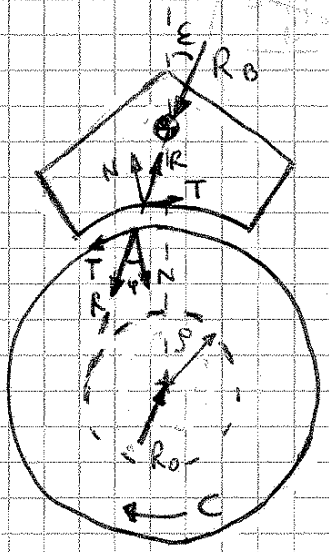


$$R_{Bv} = R_{Av} + P$$

$$R_{Bo} = R_{Ao}$$

$$R_{Bo} \cdot h + P(a+b) = R_{Bv} \cdot a$$

$$C = T \cdot \frac{d}{2}$$

$$R_B = \frac{T}{\sin\varphi}$$

$$\varphi = \operatorname{arctg} f = 21,8°$$

$$R_{Bv} = R_B \cos\varepsilon = \frac{T}{\sin\varphi}\cos\varepsilon$$

$$R_{Bo} = R_B \sin\varepsilon = \frac{T}{\sin\varphi}\sin\varepsilon$$

$$\rho = \frac{d}{2}\sin\varphi = 0,04\ m$$

$$\varepsilon = \operatorname{arcsin}\left(\frac{\rho}{h+\frac{d}{2}}\right) = 14,5°$$

$$\frac{T}{\sin\varphi}\sin\varepsilon \cdot h + P(a+b) = \frac{T}{\sin\varphi}\cos\varepsilon \cdot a$$

$$T = \frac{P(a+b)}{a\frac{\cos\varepsilon}{\sin\varphi} - h\frac{\sin\varepsilon}{\sin\varphi}} = 125,6\ N$$

$$C = T \cdot \frac{d}{2} = 13,8\ Nm$$

$$R_{Av} = \frac{T}{\sin\varphi}\cos\varepsilon - P = 227,7\ N$$

$$R_B = \frac{T_1}{\sin\varphi} \Rightarrow T_1\left(-\frac{\sin\varepsilon}{\sin\varphi}\cdot c + \frac{\cos\varepsilon}{\sin\varphi}\cdot b\right) = F(a+b)$$

$$T_1 = 37\ N$$

$$R_D = \frac{T_2}{\sin\varphi} \Rightarrow T_2\left(\frac{\sin\varepsilon}{\sin\varphi}\cdot c + \frac{\cos\varepsilon}{\sin\varphi}\cdot b\right) = F(a+b)$$

$$T_2 = 34,6\ N$$

$$R_{AV} = R_B \sin\varepsilon = \frac{T_1}{\sin\varphi}\sin\varepsilon = 31,1\ N$$

$$R_{AO} = F - R_B\cos\varepsilon = F - \frac{T_1}{\sin\varphi}\cos\varepsilon = -86,2\ N$$

$$R_{CV} = R_D \sin\varepsilon = \frac{T_2}{\sin\varphi}\sin\varepsilon = 29,1\ N$$

$$R_{CO} = -R_D\cos\varepsilon + F = -\frac{T_2}{\sin\varphi}\cos\varepsilon + F = -74,1\ N$$

$$C_M = (T_1 + T_2)\,r = 21,5\ Nm$$

$$R_A = \sqrt{R_{AV}^2 + R_{AO}^2} = 91,6\ N$$

$$R_C = \sqrt{R_{CV}^2 + R_{CO}^2} = 79,6\ N$$

$$\begin{cases} \omega_0 = \ddot{\theta}\, t \\ v_0 = \omega_0 \dfrac{D_1}{2} \\ v_0 = \ddot{x}\, t \end{cases}$$

$$T_2 = \frac{P(a+b)}{a} = 2533,3\,N$$

$$T_1 = T_2\, e^{\delta\theta^*} = 4506,3\,N$$

$$C_\delta = (T_1 - T_2)\frac{D_2}{2} = 789,2\ Nm$$

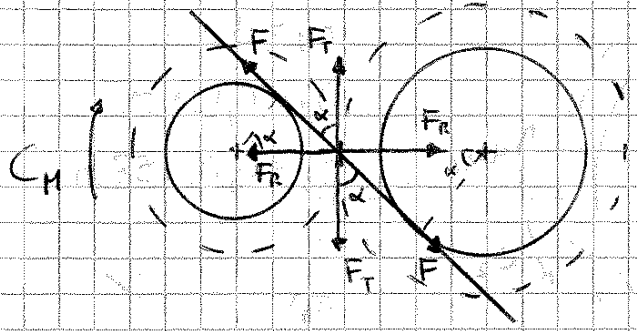
$$\ddot{\theta} = \frac{\omega_0}{t}$$

$$\ddot{x} = \frac{v_0}{t} = \frac{\omega_0 D_1}{2t} \quad \Rightarrow \quad T = mg + m\frac{\omega_0 D_1}{2t}$$

$$I\frac{\omega_0}{t} + T_2\frac{D_2}{2} + \left(mg + m\frac{\omega_0 D_1}{2t}\right)\left(\frac{D_1}{2} + e_1 - e_2\right) = T_1\frac{D_2}{2}$$

$$t = \frac{I\omega_0 + \dfrac{m\omega_0 D_1}{2}\left(\dfrac{D_1}{2} + e_1 - e_2\right)}{T_1\dfrac{D_2}{2} - T_2\dfrac{D_2}{2} - mg\left(\dfrac{D_1}{2} + e_1 - e_2\right)} = 1,1\ s$$

5.5



$C_M = 150\ Nm$

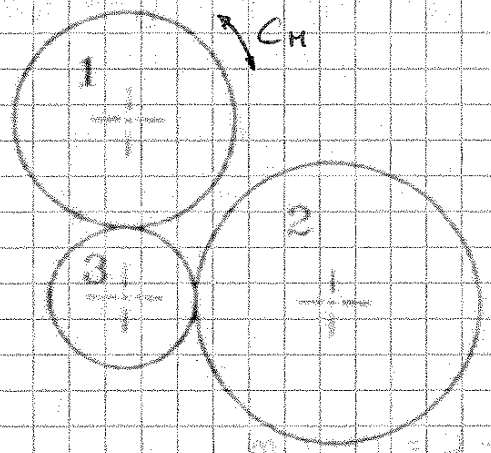
$\alpha = 20°$

$r_1 = 0,06\ m$

$$C_M = F_T \cdot r_1$$

$$F = \frac{F_T}{\cos\alpha} \Rightarrow F = \frac{C_M}{r_1 \cos\alpha} = 2660\ N$$

5.6



$\eta = 1$

$z_3 = 40$

$\omega_3 = 36\ rpm = 37,7\ rad/s$

$m = 6\ mm$

$\alpha = 20°$

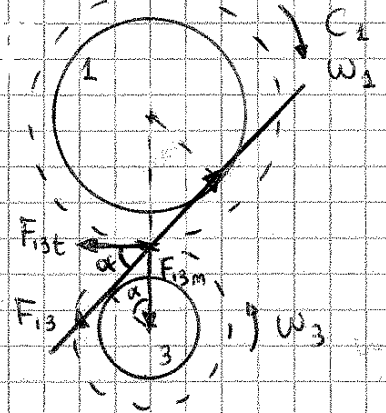
$P = 5\ CV = 3675\ W$

$$m = \frac{2r_3}{z_3} \Rightarrow r_3 = \frac{m z_3}{2} = 0,12\ m$$

R_{30} ?

R_{3A}

1) ω_1 oraria

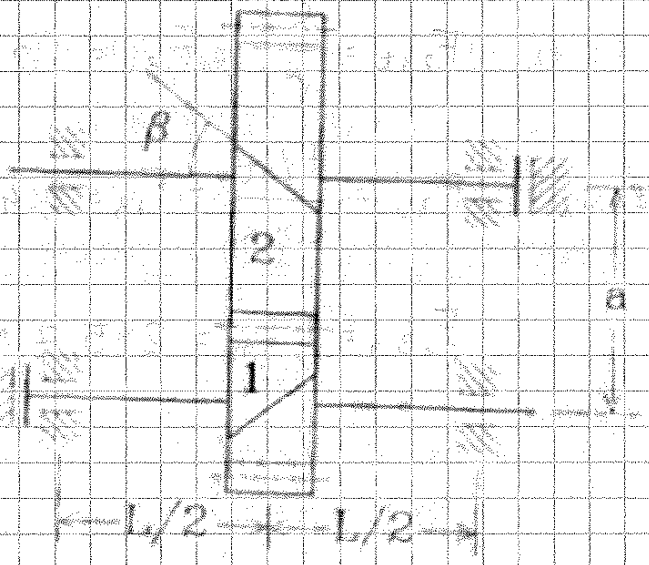


$$\eta = 1 = \frac{C_3 \omega_3}{C_1 \omega_1} = \frac{C_3 \omega_3}{P} \Rightarrow C_3 = 97,5\ Nm$$

$$F_{13t} = \frac{C_3}{r_3} = 812,3\ N$$

$$F_{13} = \frac{F_{13t}}{\cos\alpha} = 864,5\ N$$

5.7



$i = 2$

$a = 0,156\ m$

$\alpha_n = 20°$

$m_n = 2,75\ mm$

$z_1 = 37$

$L = 0,076\ m$

$P_M = 1\ CV = 735\ W$

$\omega_1 = 720\ rpm$
$= 75,4\ rad/s$

$\eta = 1$

r_1, r_2
β ?
R_{MAX}

$$\begin{cases} tg\,\alpha_n = tg\,\alpha \cos\beta \\ m_n = m \cos\beta \end{cases}$$

$$z_2 = i \cdot z_1 = 74$$

$$C_1 = \frac{P_M}{\omega_1} = 9,7\ Nm$$

$$\eta = \frac{C_2}{C_1 i} \Rightarrow C_2 = C_1 i \eta = 19,5\ Nm$$

$$\omega_2 = \frac{\omega_1}{i} = 37,7\ rad/s$$

$$\begin{cases} i = \frac{r_2}{r_1} \\ r_1 + r_2 = a \end{cases}$$

$$r_1 = \frac{a}{i+1} = 0,052\ m$$

$$r_2 = a - r_1 = 0,104\ m$$

$$R_A + R_B = F_R$$

$$\overset{\curvearrowright}{A}) \quad R_B \cdot L + F_A \cdot \rho_1 = F_R \cdot \frac{L}{2}$$

$$\Rightarrow R_B = \frac{F_R}{2} - F_A \cdot \frac{\rho_1}{L} = 9,4\ N$$

$$R_A = F_3 - R_B = 60,2\ N$$

$$R_A' + R_B' = F_T$$

$$\overset{\curvearrowright}{A}) \quad F_T \cdot \frac{L}{2} = R_B' \, L$$

$$\Rightarrow R_B' = \frac{F_T}{2} = R_A' = 93,8\ N$$

$$R_{A\,TOT} = \sqrt{R_A^2 + R_A'^2} = 111,5\ N \qquad \Rightarrow R_{MAX} = R_{A\,TOT} = 111,5\ N$$

$$R_{B\,TOT} = \sqrt{R_B^2 + R_B'^2} = 94,3\ N$$

5.8] $C_1 = 250\ Nm \quad i = 50 \quad d_p = 0,2\ m \quad \eta = 0,75$

F_T?

$$C_2 = C_1 \eta i = 9375\ Nm$$

$$F_T = C_1 \cdot \frac{2}{d_p} = 93750\ N$$

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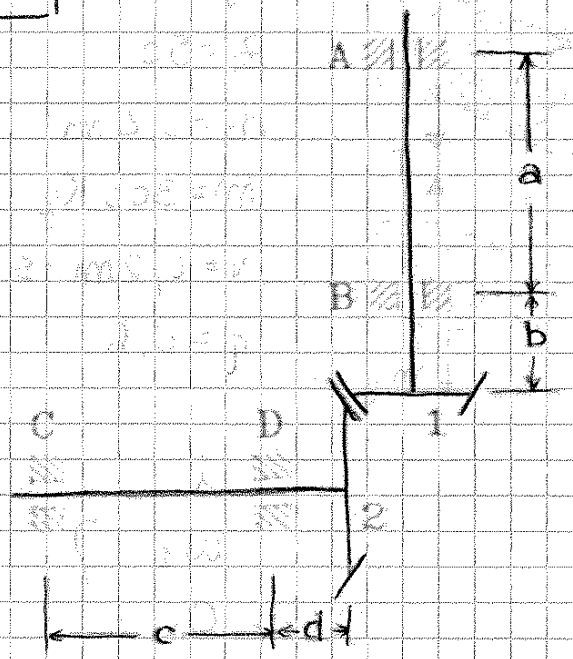
5.10



$W_u = 20 \cdot 10^3 \, W$

$\omega_1 = 1500 \, rpm$

$= 157 \, rad/s$

$i = 2$

$z_1 = 14$

$m = 5 \, mm$

$\alpha = 20°$

$a = 0,2 \, m$

$b = 0,1 \, m$

R_A, R_B ?

$\delta_1 + \delta_2 = 90°$

$\frac{\sin \delta_2}{\sin \delta_1} = i \Rightarrow tg \, \delta_2 = i$

$\delta_2 = \arctg \, i = 63,4°$

$\delta_1 = 26,6°$

$\omega_2 = \frac{\omega_1}{i} = 78,5 \, rad/s$

$C_2 = \frac{W_u}{\omega_2} = 254,8 \, Nm$

$z_2 = i \cdot z_1 = 28$

$m = \frac{2r_2}{z_2} \Rightarrow r_2 = \frac{m z_2}{2} = 0,07 \, m$

5.11 $P_u = 30\,KW \quad \omega_1 = 1500\,rpm \quad i = 50 \quad \eta = 0,9$

$\omega_1 = 157\,rad/s$

C_1, C_2 ? P_p ?

$$\eta = \frac{P_u}{P_M} \Rightarrow P_M = \frac{P_u}{\eta} = 33,3\,KW$$

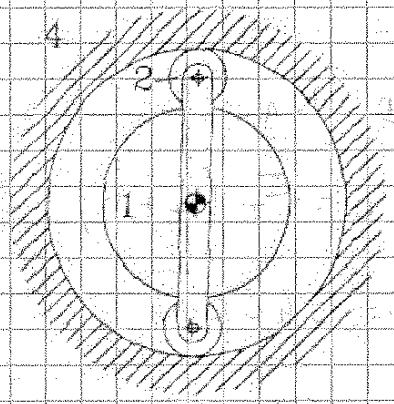
$$P_p = P_M - P_u = 3,3\,KW$$

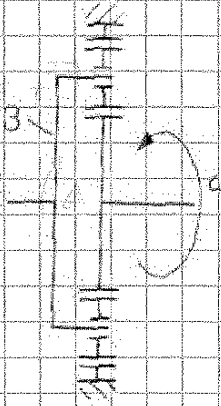
$$\frac{\omega_1}{\omega_2} = i \Rightarrow \omega_2 = \frac{\omega_1}{i} = 3,14\,rad/s$$

$$P_u = C_2 \omega_2 \Rightarrow C_2 = \frac{P_u}{\omega_2} = 9554\,Nm$$

$$C_1 = \frac{C_2}{\eta i} = 212,3\,Nm$$

5.13





$z_1 = 30$

$z_2 = 18$

$\omega_1 = 30\,rad/s$

Ω ?

$\omega_4 = 0$

$$\begin{cases} \frac{2r_1}{z_1} = \frac{2r_2}{z_2} = \frac{2r_4}{z_4} \Rightarrow z_1 + 2z_2 = z_4 \\ r_1 + 2r_2 = r_4 \end{cases}$$

$$-\frac{\omega_1 - \Omega}{\omega_2 + \Omega} = -\frac{z_2}{z_1}$$

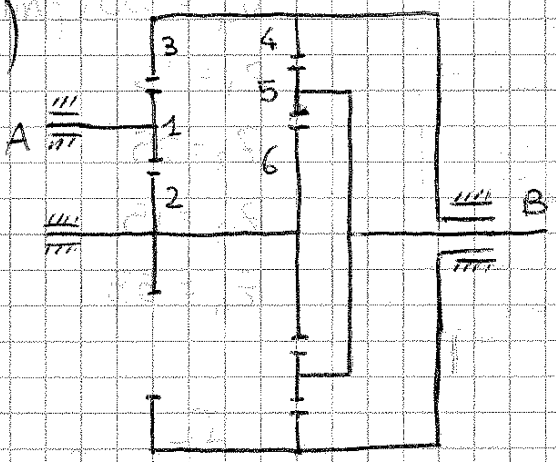
$$\omega_1 - \Omega = \frac{z_2}{z_1}(\omega_2 + \Omega)$$

$$\omega_2 = \frac{z_1}{z_2}\left(\omega_1 - \Omega\left(1 + \frac{z_2}{z_1}\right)\right) = 218,8 \text{ rpm}$$

$$i_{1/5} = \frac{\omega_1}{\Omega} = 30,1$$

Esercizi aggiuntivi

1)



$z_1 = 12$
$z_2 = 36$
$z_3 = 60$
$z_4 = 69$
$z_5 = 12$
$z_6 = 45$
$\omega_1 = 600$ rpm
$P_M = 3$ kW
$\eta = 1$

C_r?
i?

$$\frac{\omega_1}{\omega_3} = \frac{z_3}{z_1}$$

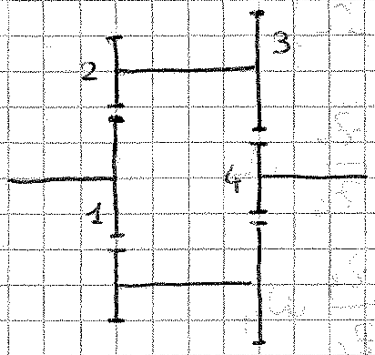
$$\Rightarrow \omega_3 = \omega_1 \frac{z_1}{z_3} = 120 \text{ rpm} = \omega_4$$

$$\frac{\omega_1}{\omega_2} = -\frac{z_2}{z_1}$$

$$\Rightarrow \omega_2 = -\omega_1 \frac{z_1}{z_2} = -200 \text{ rpm} = \omega_6$$

$$\delta_1 + \delta_2 = 90° \Rightarrow \delta_2 = 60°$$

$$\frac{\omega_i}{\omega_1} = \frac{\sin\delta_2}{\sin\delta_1} \Rightarrow \omega_1 = \omega_i \frac{\sin\delta_1}{\sin\delta_2} = 17,3 \text{ rpm}$$



$$\frac{\omega_1 - \omega_u}{\omega_4 - \omega_u} = -\frac{z_2}{z_1} \cdot \left(-\frac{z_4}{z_3}\right)$$

$$r_1 + r_2 = r_3 + r_4 \Rightarrow z_1 + z_2 = z_3 + z_4$$

$$\Rightarrow z_2 = z_4 = 45$$

$$\omega_4 = 0$$

$$\Rightarrow \frac{\omega_1 - \omega_u}{-\omega_u} = \frac{z_2 z_4}{z_1 z_3}$$

$$\omega_u \left(1 - \frac{z_2 z_4}{z_1 z_3}\right) = \omega_1$$

$$\omega_4 = \omega_1 \frac{z_1 z_3}{z_1 z_3 - z_2 z_4} = -2,16 \text{ rpm}$$

$$T_3 = T_4 = \frac{m_6 g}{2} = 7357,5\ N$$

$$T_2 = T_4 + m_4 g = 10300,5\ N$$

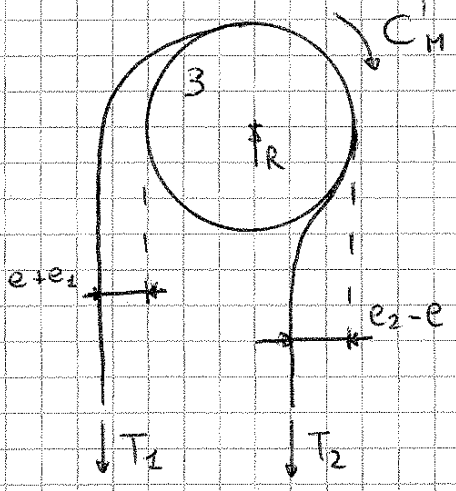
$$T_1 = T_3 + m_1 g = 13243,5\ N$$

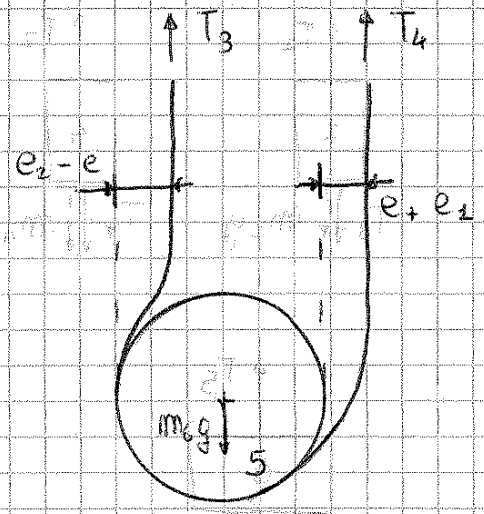
$$C_M = (T_1 - T_2) R_3 = 882,9\ Nm$$

$$\omega = \frac{V}{R_3} = 3,3\ rad/s$$

$$\frac{T_1}{T_2} = e^{f_{2min}\cdot\pi} \Rightarrow f_{2min} = \frac{1}{\pi} \ln\left(\frac{T_1}{T_2}\right) = 0,08$$

$$P_M = C_M \cdot \omega = 2943\ W$$

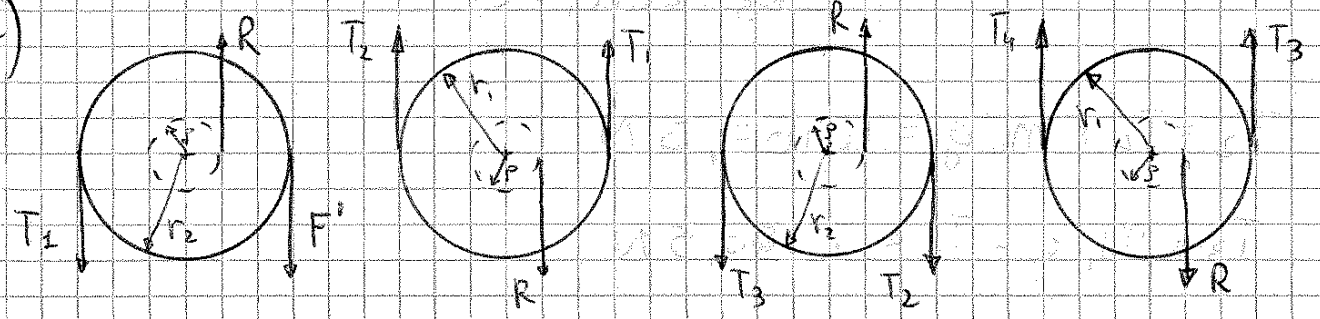




$$\begin{cases} C'_M = T_1 \cdot (R_3 + e + e_1) - T_2 (R_3 - e_2 + e) \\ T_1 = T_3 + m_1 g \\ T_2 = T_4 + m_4 g \\ T_3 + T_4 = m_6 g \\ T_3 (R_5 - e_2 + e) = T_4 (R_5 + e + e_1) \end{cases}$$

$$F = \frac{mg}{4} = 1962\ N$$

2)



$$\varphi = \operatorname{arctg} f = 16,7°$$

$$\rho = r_p \sin\varphi = 5,7 \cdot 10^{-3}\ m$$

$$\begin{cases} mg = T_1 + T_2 + T_3 + T_4 \\ F'(r_2 - \rho) = T_1(r_2 + \rho) \\ T_2(r_1 + \rho) = T_1(r_1 - \rho) \\ T_3(r_2 + \rho) = T_2(r_2 - \rho) \\ T_4(r_1 + \rho) = T_3(r_1 - \rho) \end{cases}$$

$$T_3 = T_4 \frac{r_1 + \rho}{r_1 - \rho}$$

$$T_2 = T_4 \frac{r_1 + \rho}{r_1 - \rho} \frac{r_2 + \rho}{r_2 - \rho}$$

$$T_1 = T_4 \left(\frac{r_1 + \rho}{r_1 - \rho}\right)^2 \frac{r_2 + \rho}{r_2 - \rho}$$

$$F' = T_4 \left(\frac{r_1 + \rho}{r_1 - \rho}\right)^2 \left(\frac{r_2 + \rho}{r_2 - \rho}\right)^2$$