NUMERO : 80

DATA : 18/04/2011

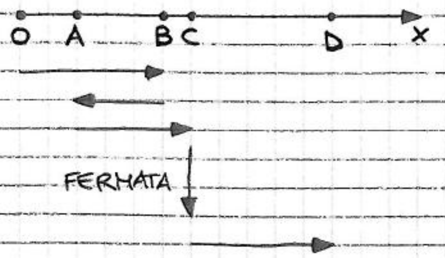
A P P U N T I

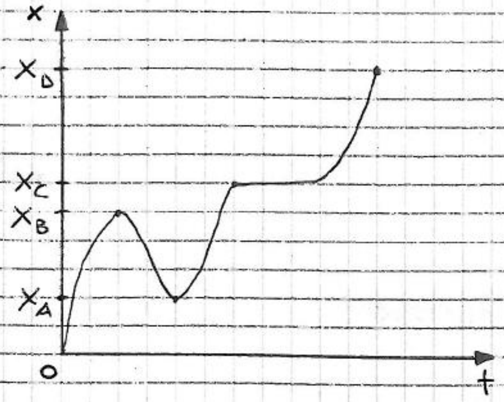
STUDENTE : Pignato

MATERIA : Fisica 1
Prof. Penna

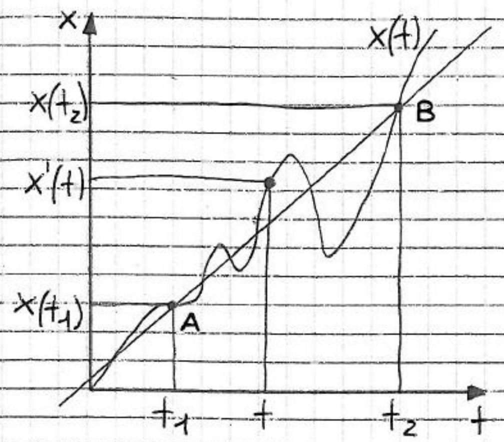
LEGGE ORARIA $x(t)$

La legge oraria descrive il moto di un corpo





• VELOCITÀ MEDIA



$$v_m = \frac{x(t_2) - x(t_1)}{t_2 - t_1}$$

la retta per AB ($x'(t)$) è la legge oraria di un corpo che si muove con velocità $= v_m$

$$\frac{x'(t) - x(t_1)}{x(t_2) - x(t_1)} = \frac{t - t_1}{t_2 - t_1} \longrightarrow x'(t) - x(t_1) = \frac{t - t_1}{t_2 - t_1}\left[x(t_2) - x(t_1)\right]$$

$$\longrightarrow x'(t) = x(t_1) + \underbrace{\frac{x(t_2) - x(t_1)}{t_2 - t_1}}_{v_m}(t - t_1) \longrightarrow x'(t) = x(t_1) + v_m(t - t_1)$$

• VELOCITÀ ISTANTANEA

$$v(t) = \lim_{t' \to t} \frac{x(t') - x(t)}{t' - t}$$

$$x(t') = x(t) + \underbrace{\frac{dx}{dt}}_{v(t)}(t' - t) + \frac{1}{2}\underbrace{\frac{d^2x}{dt^2}}_{a(t)}(t' - t)^2 + \dots$$

- **MOTO UNIFORMEMENTE ACCELERATO** : $a(t) = \text{cost} = a$

uso la ② $\quad v(t) = v(t_0) + a\int_{t_0}^{t} ds \longrightarrow v(t) = v(t_0) + a(t - t_0)$ ③

uso la ③ nella ①

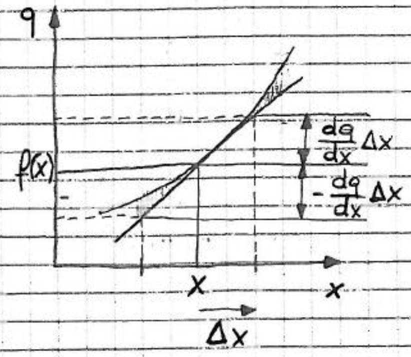
$$x(t) = x(t_0) + \int_{t_0}^{t} [v(t_0) + a(t-t_0)]ds = x(t_0) + v(t_0)(t-t_0) + a\int_{t_0}^{t}(t-t_0)ds =$$

$$\longrightarrow x(t) = x(t_0) + v(t_0)(t-t_0) + \frac{1}{2}a(t-t_0)^2$$

- **PROPAGAZIONE degli ERRORI**

- $q = f(x)$

1 variabile



misuro $x \pm \Delta x$ per trovare

$$q \pm \Delta q = q(x) \pm \left|\frac{dq}{dx}\right| \Delta x$$

questo perchè $\Delta q = \left|\frac{dq}{dx}\right| \Delta x$

$$q(x \pm \Delta x) = q(x) \pm \frac{dq}{dx}\Delta x$$

- $q = f(x,y,z)$

3 variabili

$$q(x \pm \Delta x, y \pm \Delta y, z \pm \Delta z) = q(x,y,z) \pm \frac{dq}{dx}\Delta x \pm \frac{dq}{dy}\Delta y \pm \frac{dq}{dz}\Delta z$$

$$\downarrow$$

$$q(x \pm \Delta x, y \pm \Delta y, z \pm \Delta z) = q(x,y,z) \pm \Delta q$$

$$\Delta q = \left|\frac{dq}{dx}\right|\Delta x + \left|\frac{dq}{dy}\right|\Delta y + \left|\frac{dq}{dz}\right|\Delta z$$ ERRORE ASSOLUTO

introducendo il valore assoluto, calcolo la MASSIMA deviazione positiva o negativa

$\frac{\Delta q}{q}$ ERRORE RELATIVO

$$\rightarrow g\left(\frac{\Delta L}{L} + 2\frac{\Delta T}{T}\right) = \Delta g$$

Dati: $\Delta L = 1\ mm$
$L = 1\ m$
$\Delta T = 0,2\ s$
$T = 2\ s$

$$\rightarrow 9,86\left(10^{-3} + 2\cdot 0,1\right) = 9,86 \cdot 0,201 = 1,98$$

errore relativo $\frac{\Delta g}{g} = 0,201 = \sim 20\%$

2) misuro il tempo pari a 10 oscillazioni

$t = 20\cdot 1\ s$

$\Delta t = \Delta T = 0,2\ s$ $\quad T' = \frac{t}{10} = 2,01\ s$ $\quad$ errore assoluto $\Delta T' = \frac{\Delta t}{10} = 0,02$

$$\rightarrow \Delta g = 9,77\left(10^{-3} + 2\cdot 0,0099\right) = 9,77 \cdot 0,02 = 0,20$$

$$\frac{\Delta g}{g} = \frac{0,20}{9,77} = 0,02 = \sim 2\%$$

es. 2 $\quad T = 2\pi\sqrt{\frac{I}{mgh}}$

Determinare l'errore relativo massimo del periodo, e il numero minimo di oscillazioni che bisogna effettuare, perchè $\frac{\Delta g}{g} \leq 0,02$

$\frac{\Delta m}{m} = 0,001$ $\qquad \frac{\Delta g}{g} = 2\frac{\Delta T}{T} + \frac{\Delta I}{I} + \frac{\Delta m}{m} + \frac{\Delta h}{h}$

$\frac{\Delta h}{h} = 0,007$

$\frac{\Delta T}{T} = 0,001 \rightarrow \frac{\Delta t}{t} = \frac{\Delta T}{nT} = 0,001$

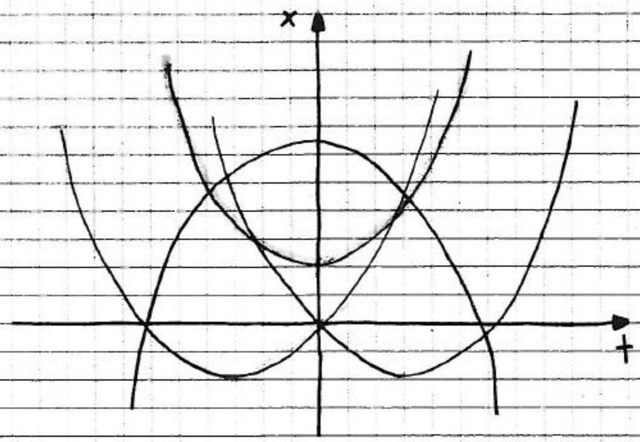
$\frac{\Delta I}{I} = 0,01$

esempi di moto uniformemente accelerato

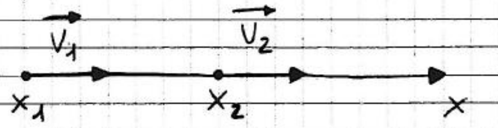
$V_0 = 0$ $\quad$ $X_0 = 0$ $\quad$ $a > 0$

$a > 0$ $\quad$ $V_0 > 0$

$a < 0$ $\quad$ $V_0 < 0$



RELAZIONE tra V FINALE/INIZIALE



$\vec{V} = V \cdot \vec{u}_x$

$$V(t) = V_0 + at$$

$$x(t) = x_0 + V_0 t + \frac{a}{2}t^2$$

$$\begin{cases} t_1, \quad x(t_1) = x_1, \quad V(t_1) = V_1 \\ t_2, \quad x(t_2) = x_2 = x_1 + V_1(t_2 - t_1) + \frac{a}{2}(t_2 - t_1)^2 \\ \quad V(t_2) = V_2 = V_1 + a(t_2 - t_1) \end{cases} =$$

$$= \begin{cases} x_2 - x_1 = V_1(t_2 - t_1) + \frac{a}{2}(t_2 - t_1)^2 \\ V_2 - V_1 = a(t_2 - t_1) \longrightarrow t_2 - t_1 = \frac{(V_2 - V_1)}{a} \end{cases}$$

$$x_2 - x_1 = V_1 \frac{(V_2 - V_1)}{a} + \frac{a}{2}\frac{(V_2 - V_1)^2}{a^2} = \frac{2(V_1 V_2 - V_1^2) + (V_2 - V_1)^2}{2a} = \frac{V_2^2 - V_1^2}{2a}$$

$$\boxed{a = \frac{1}{2}\frac{V_2^2 - V_1^2}{x_2 - x_1}}$$ ACCELERAZIONE nel MOTO UNIFORMEMENTE ACCELERATO

$T = \frac{2\pi}{\omega}$ periodo del PENDOLO $\longrightarrow x(t+T) = x(t)$

$$x(t+T) = A \operatorname{sen}\left[\omega(t+T)+\varphi\right] = A \operatorname{sen}\left[\omega t + \varphi + \underbrace{\omega T}_{2\pi}\right] = x(t)$$

velocità

$$v(t) = \frac{dx}{dt} = A\omega \cos(\omega t + \varphi)$$

$\downarrow$

accelerazione

$$a(t) = \frac{dv}{dt} = -A\omega^2 \operatorname{sen}(\omega t + \varphi) \longrightarrow a(t) = -\omega^2 x(t)$$

$\downarrow$

$$\frac{d^2x}{dt^2} + \omega x = 0$$

es.

$t=0$, $x(0) = x_0$, $v(0) = v_0$

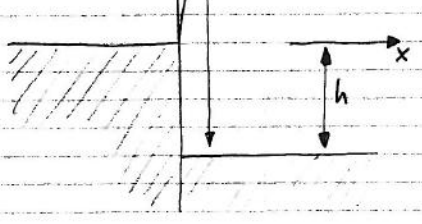
- $x_0 = A \operatorname{sen}\varphi$
- $v_0 = A\omega\cos\varphi$

$\cos^2\varphi + \operatorname{sen}^2\varphi = 1$

$$\frac{v_0}{\omega} = A\cos\varphi \longrightarrow x_0^2 + \frac{v_0^2}{\omega^2} = A^2\operatorname{sen}^2\varphi + A^2\cos^2\varphi = A^2$$

$$\frac{x_0}{v_0} = \frac{1}{\omega}\operatorname{tg}\varphi \longrightarrow \operatorname{tg}\varphi = \omega\frac{x_0}{v_0}$$

es. 1 MOTO VERTICALE (moto uniformemente accelerato)



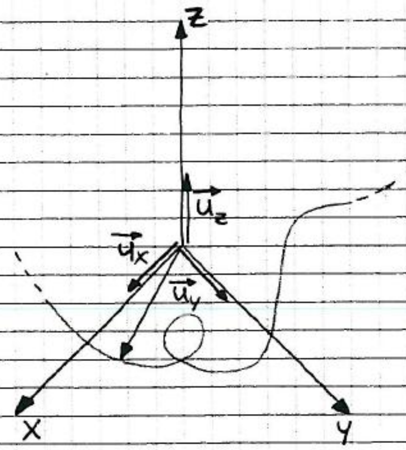
$x = \text{cost}$

$h = 100\,m$

$v_0 = 98\,m/s$

$a = -g$

• CINEMATICA TRIDIMENSIONALE



Un vettore nello spazio ha come componenti:

$$\vec{r} = (x\vec{u}_x, y\vec{u}_y, z\vec{u}_z)$$

ed è individuato come la somma vettoriale delle singole componenti

Un vettore che si muove nello spazio disegna una CURVA

$$\vec{r}(t) = x(t)\,\hat{u}_x + y(t)\,\hat{u}_y + z(t)\,\hat{u}_z$$ VETTORE POSIZIONE

• CURVE FONDAMENTALI:

1) CIRCONFERENZA (raggio R)

$$\vec{r} = x(t)\,\hat{u}_x + y(t)\,\hat{u}_y + \underbrace{z(t)}_{0}\,\hat{u}_z$$

$x(t) = R\cos(\omega t)$

$y(t) = R\,\text{sen}(\omega t)$

$$\vec{r} = R\cos(\omega t) + R\,\text{sen}(\omega t)$$

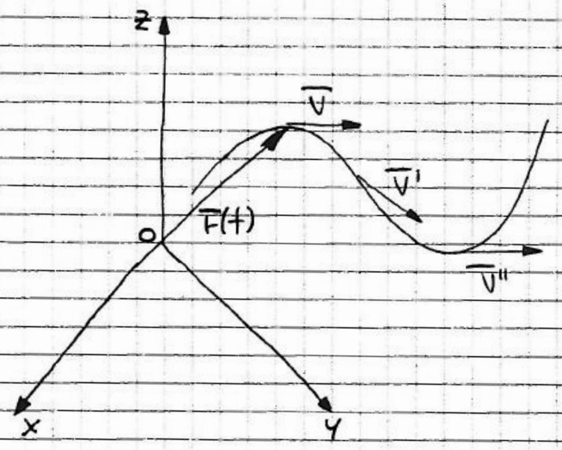
2) MOTO ELICOIDALE

$$\vec{r}(t) = R\cos(\omega t) + R\,\text{sen}(\omega t) + v_0 t$$

• VETTORE VELOCITÀ MEDIA

$$\vec{v}_m = \frac{\Delta\vec{r}}{t_2 - t_1} = \frac{\vec{r}(t_2) - \vec{r}(t_1)}{t_2 - t_1}$$

• ACCELERAZIONE MEDIA



$$\vec{a}_m = \frac{\overline{v}(t_2) - \overline{v}(t_1)}{t_2 - t_1}$$

$$\vec{a}(t) = a_x(t)\hat{u}_x + a_y(t)\hat{u}_y + a_z(t)\hat{u}_z$$

• ACCELERAZIONE ISTANTANEA

$$\vec{a}(t) = \lim_{t' \to t} \frac{\overline{v}(t_2) - \overline{v}(t_1)}{t_2 - t_1} = \frac{d\overline{v}(t)}{dt}$$

$$\begin{cases} a_x(t) = \dfrac{dv_x(t)}{dt} = \dfrac{d^2x(t)}{dt} \\ a_y(t) = \dfrac{dv_y(t)}{dt} = \dfrac{d^2y(t)}{dt} \\ a_z(t) = \dfrac{dv_z(t)}{dt} = \dfrac{d^2z(t)}{dt} \end{cases}$$

nota $\overline{a}(t) = \dfrac{d\overline{v}(t)}{dt} \longrightarrow \overline{v}(t) = \overline{v}(t_0) + \displaystyle\int_{t_0}^{t} \overline{a}(s)\,ds$ (2)

• MOTO UNIFORMEMENTE ACCELERATO 3D

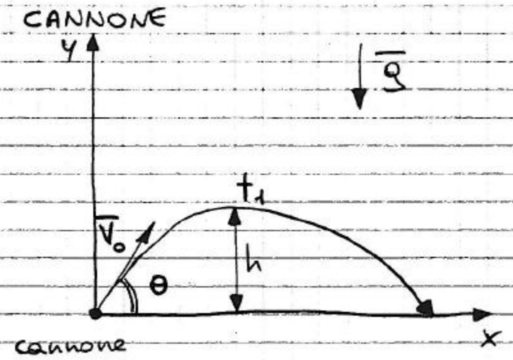
$\overline{a}(t) = a$ costante $\forall t$

applico (2)
$$\overline{v}(t) = \overline{v}(t_0) + \int_{t_0}^{t} \overline{a}\,ds = \overline{v}(t_0) + \overline{a}(t - t_0) \quad (3)$$

applico la (3) nella (1)
$$\overline{r}(t) = \overline{r}(t_0) + \int_{t_0}^{t} \left[\overline{v}(t_0) + (s - t_0)\overline{a}\right] ds = \overline{r}(t_0) + \overline{v}(t_0)(t - t_0) + \overline{a}\int_{t_0}^{t}(s - t_0)\,ds$$

$$\longrightarrow \overline{r}(t) = \overline{r}(t_0) + \overline{v}(t_0)(t - t_0) + \frac{1}{2}\overline{a}(t - t_0)^2$$

es. CANNONE



$\bar{a} = \bar{g} = -g\,\hat{u}_y$

$t_0 = 0$

$\bar{r}_0 = 0$

Il proiettile viene sparato con angolo θ e la velocità $v_0 = |\bar{v}_0| =$
Trovare l'altezza max raggiunta, il tempo impiegato (di salita), la gittata e il punto in cui atterra il proiettile

• DISTRIBUZIONE di PROBABILITÀ GAUSSIANA

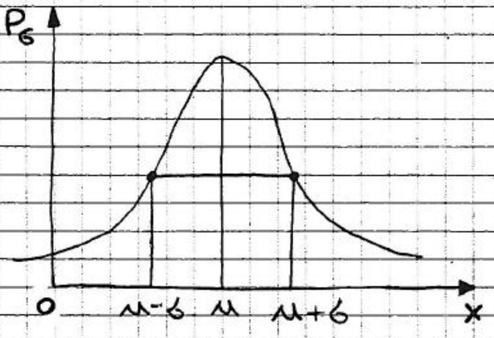
- analisi statistica dei dati
- costruzione di un istogramma

$$P_\sigma(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

σ^2 = varianza

μ = valor medio

σ = deviazione standard (misura la distribuzione statistica)



$\mu+\sigma$ e $\mu-\sigma$ danno un'indicazione su come si allunga e stringe la gaussiana

↓

Più piccola è σ, più stretta e piccola è la gaussiana

f = frequenza sperimentale $\longrightarrow$ rapporto tra il numero di volte in cui un certo evento accade, rispetto agli eventi misurati e presi in considerazione

$\frac{f_K}{\Delta x} \cdot \Delta x$ = altezza classe K per base

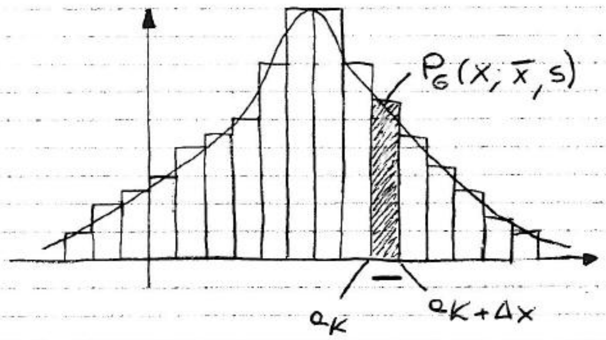
$K = 1, 2, 3, \dots, n$

frequenze sperimentali: $f_1 = N_1/N \; \dots \; f_n = N_n/N$

valor medio $\bar{x} = \frac{1}{N} \sum_{J=1}^{N} x_J$

varianza sperimentale $s^2 = \frac{1}{N-1} \sum_{J=1}^{N} (x_J - \bar{x})^2$

$N \to \infty$ le frequenze sperimentali tendono a quelle teoriche



$$\lim_{N \to \infty} f_K = \int_{a_K}^{a_K + \Delta x} P_G(x)\, dx$$

$$f_K = \frac{N_K}{N}$$

$$\lim_{N \to \infty} \bar{x} = \lim_{N \to \infty} \frac{1}{N} \sum_{J=1}^{N} x_J = \mu$$

$$\lim_{N \to \infty} s^2 = \lim_{N \to \infty} \frac{1}{N-1} \sum_{J=1}^{N} (x_J - \bar{x})^2 = \sigma^2$$

$$\overline{x} = \frac{1}{N}\sum_{J=1}^{N} x_J \qquad \langle (x-\mu)^2 \rangle = \int_{-\infty}^{\infty} (x-\mu)^2 P_G(x)\, dx = \sigma^2$$

$$S^2 = \frac{1}{N-1}\sum_J (x_J - \overline{x})^2$$

COSTRUZIONE della GAUSSIANA

1) Dati $x_1, x_2, \dots, x_N$

2) Costruzione dell'ISTOGRAMMA
(implica la valutazione delle frequenze sperimentali $f_K = \frac{N_u}{N}$)

$$\overline{x} = \frac{1}{N}\sum_{J=1}^{N} x_J \qquad S^2 = \frac{1}{N-1}\sum_{J=1}^{N} (x_J - \overline{x})^2$$

3) Costruzione della CURVA GAUSSIANA relativa all'istogramma

$$P_G(x) = \frac{1}{S\sqrt{2\pi}}\, e^{-\frac{(x-\overline{x})^2}{2S^2}}$$

4) Per $N \to \infty$
$P_G(x)$ sperimentale tende alla gaussiana teorica

$$\lim_{N\to\infty} \overline{x} = \mu$$

$$\lim_{N\to\infty} S^2 = \sigma^2$$

5) L'errore che si commette nel valutare μ con $\overline{x} = \frac{1}{N}\sum_{J=1}^{N} x_J$
(sperimentale)

$$\Delta\overline{x} = \frac{S}{\sqrt{N}}$$

① t_1 : $v_y(t_1) = v_{oy} - g t_1 = 0 \qquad t_1 = \dfrac{v_{oy}}{g}$

$$y(t_1) = v_{oy} t_1 - \frac{g}{2} t_1^2 = \frac{v_{oy}^2}{2g}$$

gittata : $y(t) = v_{oy} t - \dfrac{g}{2} t^2 = 0 \longrightarrow t\left(v_{oy} - \dfrac{g}{2} t\right) = 0 \qquad t_2 = \dfrac{2 v_{oy}}{g}$

$$x(t_2) = v_{ox} t_2 = \frac{2 v_{oy} v_{ox}}{g}$$

$$s = x(t_2) = \frac{2 v_{ox} v_{oy}}{g} = \frac{2 v_o \cos\theta \, v_o \operatorname{sen}\theta}{g} = \frac{2 v_o^2}{g} \cos\theta \operatorname{sen}\theta$$

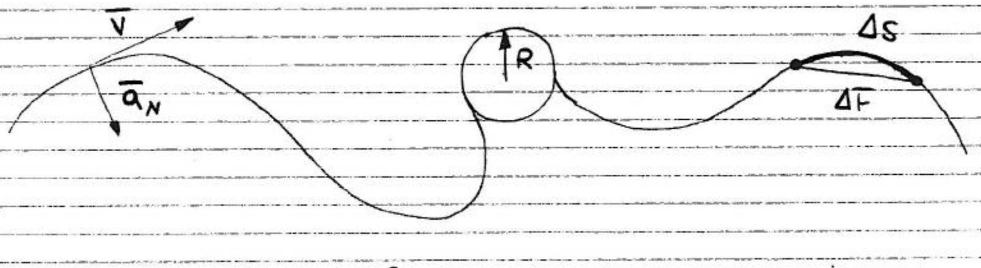
GITTATA $\quad s = \dfrac{v_o^2}{g} \operatorname{sen}(2\theta)$

gittata MAX : $\dfrac{d}{d\theta} s(2\theta) = \dfrac{v_o^2}{g} \cdot 2\cos(2\theta) = 0 \qquad d = \dfrac{\pi}{4}$

Considero un moto generico in 3 dimensioni:

Δs = arco tra P e P'

$\Delta t = t' - t$



Def.

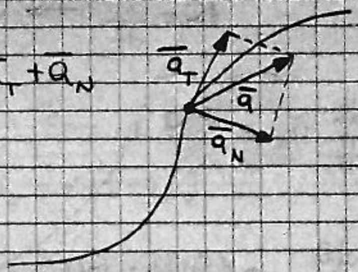
$$\vec{a} = \overline{a}_N + \overline{a}_T = \frac{dv}{dt} \hat{u}_T + \frac{v^2}{R} \hat{u}_N \qquad v = |\overline{v}|$$

$$\overline{v} = v \cdot \hat{u}_T \longleftrightarrow \hat{u}_T = \frac{\overline{v}}{v}$$

$\Delta S = R\theta = (S - S_0)$

$\theta = \dfrac{S - S_0}{R}$

$\vec{a} = \overline{a}_T + \overline{a}_N$



$\dfrac{d\theta}{dS} = \dfrac{1}{R}$

$$\vec{a} = \frac{dv}{dt}\hat{u}_T + v \underbrace{\frac{d\theta}{ds}}_{1/R} \underbrace{\frac{ds}{dt}}_{v} \hat{u}_N \longrightarrow \vec{a} = \frac{dv}{dt}\hat{u}_T + v^2 \frac{1}{R}\hat{u}_N$$

$\overline{a}_T$ influenza il modulo di $\overline{v}$

$\overline{a}_N$ influenza la direzione di $\overline{v}$

- CASI NOTEVOLI

1) MOTO UNIFORME

$v = cost \longrightarrow a_T = \dfrac{dv}{dt} = 0$

$\overline{v} = v \cdot \hat{u}_T$ costante $\longrightarrow a_N = \dfrac{v^2}{R}$

In questo caso l'accelerazione ha il solo ruolo di cambiare la posizione del corpo

2) MOTO RETTILINEO

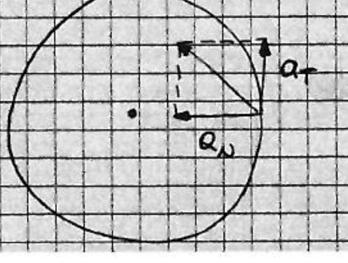
$a_N = \dfrac{v^2}{R} = 0$

$a_T = \dfrac{dv}{dt}$

3) MOTO CIRCOLARE — uniforme / vario

a) UNIFORME

$R = cost$



4) MOTO CIRCOLARE UNIFORMEMENTE ACCELERATO

$\alpha = cost \longrightarrow a_T = R\alpha = cost$

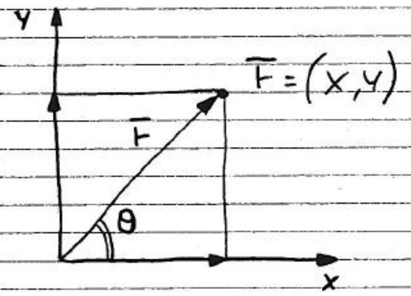
$$\omega(t) = \omega(t_0) + \int_{t_0}^{t} \alpha \, dt \longrightarrow \omega(t) = \omega_0 + \alpha(t - t_0)$$

$$\theta(t) = \theta(t_0) + \int_{t_0}^{t} \omega(t) \, dt \longrightarrow \theta(t) = \theta_0 + \int_{t_0}^{t} [\omega_0 + \alpha(t - t_0)] dt =$$

$$\longrightarrow \theta(t) = \theta_0 + \omega_0 (t - t_0) + \frac{\alpha}{2}(t - t_0)^2$$

$$a_N = \frac{v^2}{R} = R\omega^2 = R[\omega_0 + \alpha(t - t_0)]^2$$

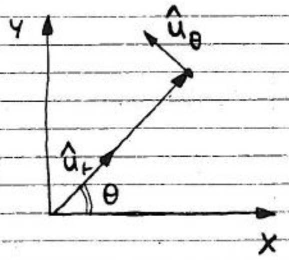
- COORDINATE POLARI



$$\begin{cases} r = |\vec{r}| = \text{raggio} = \sqrt{x^2 + y^2} \\ \theta = \text{angolo} \end{cases}$$

Legame tra coordinate cartesiane e polari:

$$\begin{cases} x = r \cos\theta \\ y = r \operatorname{sen}\theta \end{cases} \qquad \theta = tg \frac{y}{x}$$



$\vec{r} = r \hat{u}_r \qquad \hat{u}_r = \hat{u}_r(\theta)$

$\hat{u}_\theta = \hat{u}_z \wedge \hat{u}_r$

$$\vec{r} = x + y = r \underbrace{(\cos\theta \, \hat{u}_x + \operatorname{sen}\theta \, \hat{u}_y)}_{\hat{u}_r}$$

es. ALENIA

Moto armonico con periodo $T = 4,4 s$, si trova in $x(0) = 0,28 m$ e $v(0) = -2,5 m/s$

Calcolare la A, ω, φ e V_{MAX}, a_{MAX}

$x(t) = A \sin(\omega t + \varphi)$ $\qquad$ $v(t) = A\omega \cos(\omega t + \varphi)$

$\omega = \frac{2\pi}{T} = 0,455\pi$ $\qquad$ $\varphi = \text{arctg}\left(\omega \frac{x(0)}{v(0)}\right) = -0,16 + \overset{1}{n}\pi = 2,98$

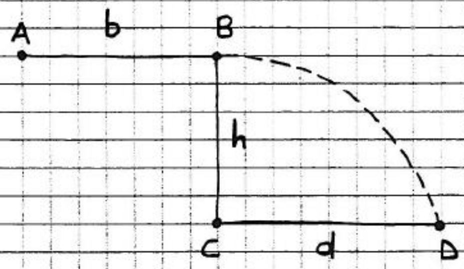
$A = \frac{0,28}{0,16} = 1,75$ $\qquad$ $V_{MAX} = A\omega = 2,5 m/s$ $\qquad$ $a_{MAX} = +5,1 m/s^2$

es. 2 Un punto percorre il tratto A-B (orizzontale) con:

$V_A = V_1$ $\quad$ $V_2 < V_1$ $\quad$ $a = -KV$

$V_B = V_2$ $\quad$ $K = 2,3 s^{-1}$

$\overline{AB} = b = 2,14 m$ $\qquad$ $\overline{BC} = h = 1,5 m$ $\qquad$ $\overline{CD} = d = 1,35 m$



Calcolare V_1

$t_0 = 0$

$$V_y(t) = V_y(t_0) + \int_{t_0}^{t} a(t)\, dt$$

$$\underbrace{y(t)}_{0} = \underbrace{y(t_0)}_{h} + \underbrace{V_y(t_0)(t - t_0)}_{0} - \frac{g}{2}(t - t_0)^2 \longrightarrow 0 = h - \frac{g}{2} t_R^2$$

$$t_h = \sqrt{\frac{2h}{g}} \qquad V_2 = \frac{s}{t} = \frac{s}{\sqrt{\frac{2h}{g}}} \longrightarrow \frac{dv}{dt} = -Kv(t) \longrightarrow$$

$$\longrightarrow \frac{dv}{v} = -K dt \longrightarrow \int_{V_1}^{v(t)} \frac{dv}{v} = -K \int_0^t dt \longrightarrow \ln \frac{v(t)}{V_1} = -Kt \longrightarrow$$

• DINAMICA

Il mondo fisico circostante è un sistema complesso di corpi in continua evoluzione per effetto delle INTERAZIONI presenti (o passate) tra i suoi costituenti, a tutte le scale spaziali disponibili

DIMENSIONI:

universo $> 10^{26}$ m
galassia $\sim 5 \cdot 10^{20}$ m
sole-plutone $\sim 5,9 \cdot 10^{12}$ m
sole $\sim 6,69 \cdot 10^{5}$ Km
terra $\simeq 6,37 \cdot 10^{3}$ Km

oggetti quotidiani ~ 1 m
atomo $\sim 1\,\mathring{A} = 10^{-10}$ m
nucleo $\sim A^{1/3} \cdot 1,4 \cdot 10^{-15}$
elettrone $\sim 2,8 \cdot 10^{-15}$ m

LEGGI della DINAMICA

1) Legge di INERZIA

2) Legge di NEWTON

3) Legge di AZIONE-REAZIONE

• LEGGE di INERZIA

Una particella LIBERA si muove con VELOCITÀ COSTANTE (cioè con ACCELERAZIONE NULLA).
Per particella libera si intende una particella non soggetta ad interazioni di alcun tipo con l'ambiente circostante

$$\bar{v} = \begin{cases} 0 \\ \\ cost \end{cases} \longleftrightarrow \bar{a} = \frac{d\bar{v}}{dt} = 0$$

Per accertare lo stato di moto bisogna disporre di un sistema di riferimento sicuramente fermo, o con velocità costante, da cui studiare il moto del corpo, cioè un SISTEMA di RIFERIMENTO INERZIALE

- PRINCIPIO di AZIONE-REAZIONE

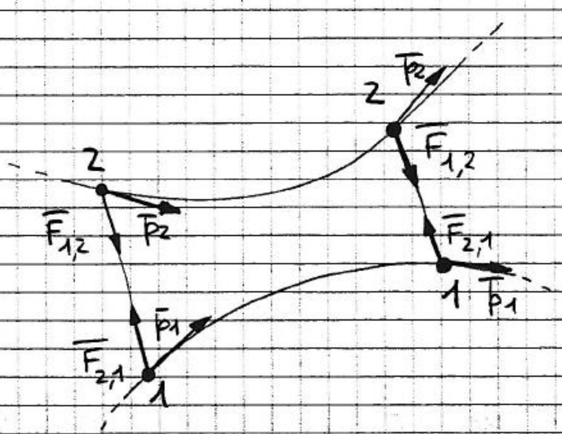
Se un corpo A esercita una forza $\vec{F}_{A-B}$ su un corpo B, allora il corpo B reagisce esercitando una forza $\vec{F}_{B-A}$ tale che

$$\vec{F}_{B-A} = -\vec{F}_{A-B}$$

- CONSERVAZIONE della QUANTITÀ di MOTO

$\vec{p} = m \cdot \vec{v}$ QUANTITÀ di MOTO

$$\bar{F} = m \cdot a = \frac{d\bar{p}}{dt} \qquad \left(\frac{d\bar{p}}{dt} = m\frac{d\bar{v}}{dt} = m\bar{a}\right)$$



$t' > t$

$\forall t$

$$\bar{F}_{2,1} = -\bar{F}_{1,2}$$

$$\bar{F}_{2,1} = m_1 \bar{a}_1 = \frac{d\bar{p}_1}{dt}$$

$$\bar{F}_{1,2} = m_2 \bar{a}_2 = \frac{d\bar{p}_2}{dt}$$

dalla 1° equazione trovo $\frac{d\bar{p}_1}{dt} = -\frac{d\bar{p}_2}{dt}$ $\quad \frac{d}{dt}\left(\bar{p}_1 + \bar{p}_2\right) = 0$

$\bar{p}_1 + \bar{p}_2 = cost$

$$\int_{t_0}^{t_1} \frac{d\bar{p}_1}{dt}dt = \int_{t_0}^{t_1} -\frac{d\bar{p}_2}{dt}dt \longrightarrow \bar{p}_1(t_1) - \bar{p}_1(t_0) = -\left[\bar{p}_2(t_1) - \bar{p}_2(t_0)\right]$$

$$\downarrow$$

$$\bar{p}_1(t_1) + \bar{p}_2(t_1) = \bar{p}_1(t_0) + \bar{p}_2(t_0)$$

2) FORZA ELETTROSTATICA

$$F = \frac{Q_1 Q_2}{4\pi \varepsilon_0 r^2}$$

3) FORZA DEBOLE $r \sim 10^{-18}\ m$

4) FORZA FORTE $r \sim 10^{-15}\ m$

5) FORZA PESO (forza gravitazionale a livello terrestre)

$\bar{F} = m\bar{g}$

• FORZE EFFETTIVE

1) FORZA D'ATTRITO RADENTE

$F_a = \mu N$ N = reazione vincolare

2) FORZA DI REAZIONE VINCOLARE (impenetrabilità di un corpo)

3) FORZA ELASTICA

$F(x) = k(x - \lambda)$

4) FORZA VISCOSA (forza di ARCHIMEDE)

$\bar{F}(\bar{v}) = -b\bar{v}$ $b = mk$

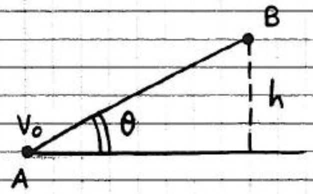
es. 3 Hp: $v(0) = v_0 = 4{,}2\ m/s$ Th: t_{AB}

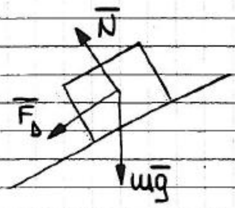
$\theta = 30°$ $\mu_D \,/\, v_B = 0$

$h = 0{,}4\ m$

$\mu_D = 0{,}2$

$\overline{N} = -m\overline{g}\cos\theta$





$\overline{F} = \overline{F}_D + m\overline{g}\,\text{sen}\,\theta$

$m \cdot a = \mu_D m g\cos\theta + m g\,\text{sen}\,\theta$

$\longrightarrow a = 6{,}6\ m/s^2$

$AB = \dfrac{h}{\text{sen}\theta} = 2h \longrightarrow 2h = v_0 t_f - \dfrac{a}{2} t_f^2 \longrightarrow t_{AB} = 0{,}23\ s$

$\begin{cases} v_B = v_0 - \tilde{a}t = 0 \\ AB = v_0 t - \dfrac{1}{2}\tilde{a}t^2 \end{cases}$ $\tilde{a} = \dfrac{v_0^2}{4h}$ $\mu = \left(\dfrac{v_0^2}{4h} - g\,\text{sen}\,\theta\right)\dfrac{1}{g\cos\theta} = 0{,}72$

- RISULTANTE delle FORZE

$\overline{R} = \overline{F}_1 + \overline{F}_2 + \ldots + \overline{F}_n = \sum_i \overline{F}_i$

Un corpo può subire l'azione contemporanea di molte forze (Principio di sovrapposizione delle forze)

- EQUILIBRIO STATICO

Un corpo è in equilibrio statico se

$\vec{R} = \sum_i \vec{F}_i = 0 \longrightarrow \begin{cases} R_x = \sum_i F_{x_i} = 0 \\ R_y = \sum_i F_{y_i} = 0 \\ R_z = \sum_i F_{z_i} = 0 \end{cases}$

$\downarrow$

$m \cdot \overline{a} = 0 \longrightarrow \overline{R}^{a} = 0$

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Quiete :

$$\overline{F} + \overline{F_a} + \overline{N} + m\overline{g} = \overline{R} = 0$$

$$x)\ -F\hat{u}_x + F_a\hat{u}_x = 0$$

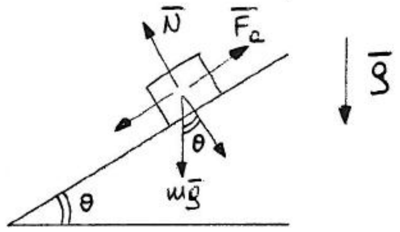
$$y)\ N\hat{u}_y - mg\hat{u}_y = 0$$

$$\longrightarrow \begin{cases} F_a = F \\ N = mg \end{cases} \longrightarrow F = F_a \leq F_{a_{MAX}}$$

② ATTRITO DINAMICO : μ_D

$$F_a = \mu_D N = \mu_D mg$$

③ PIANO INCLINATO



Quiete :

$$\begin{cases} -mg\cos\theta + N = 0 \\ F_a - mg\overset{\text{sen}}{\cancel{\cos}}\theta = 0 \end{cases} \qquad F_a = \mu_S N = \mu_S mg\cos\theta$$

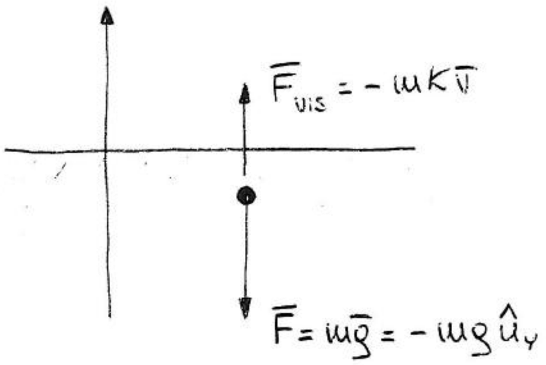
- FORZA VISCOSA

$$\begin{rcases} \vec{F} = -mK\vec{v} = \vec{F}(\overline{v}) \\ \vec{F} = m\vec{a} \end{rcases} \longrightarrow -mK\overline{v} = m\frac{d\overline{v}}{dt}$$

es.1 Moto smorzato nel campo di gravità

$$\begin{rcases} \overline{R} = m\overline{a} \\ \overline{R} = \overline{F} + m\overline{g} = -mK\overline{v} + m\overline{g} \end{rcases} \longrightarrow m\frac{d\overline{v}}{dt} = m\overline{g} - mK\overline{v}$$

es. Discesa di un corpo nell'ACQUA

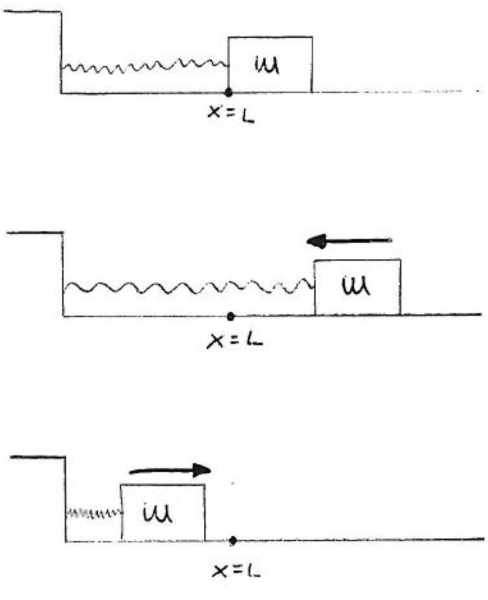


$$\cancel{m}\frac{dv_y}{dt} = -\cancel{m}g - K\cancel{m}v_y$$

$$v(\infty) = \frac{F}{Km} = -\frac{\cancel{m}g}{K\cancel{m}} = -\frac{g}{K}$$

$$K = \begin{cases} \text{acqua } 10^{-2}\ s^{-1}\ (20°C) \\ \text{aria } 1{,}81 \cdot 10^{-4}\ s^{-1}\ (20°C) \end{cases}$$

- FORZA ELASTICA

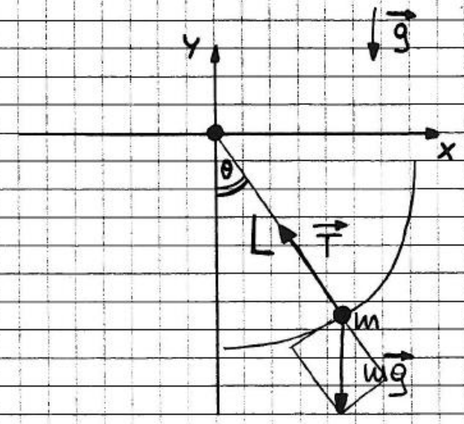


$$F(x) = -\underbrace{K}_{\text{COSTANTE ELASTICA}}(x - \overbrace{L}^{\text{LUNGHEZZA A RIPOSO}})$$

$$F = m \cdot a \longrightarrow -K(x-L) = \frac{d^2x}{dt^2}m \longrightarrow \frac{d^2x}{dt^2} + \frac{K}{m}x = \frac{K}{m}L$$

$x(t) = L + A\sin(\omega t + \varphi) \longrightarrow$ soluzione che se sostituita sopra da $\frac{K}{m}L$.

• PENDOLO SEMPLICE



$$\overline{R} = m\overline{a}$$

$$\overline{R} = \overline{T} + m\overline{g}$$

$$\downarrow$$

$$m(\overline{a}_T + \overline{a}_N) = \overline{T} + m\overline{g} = \overline{T} - mg\cos\theta\,\hat{u}_N - mg\operatorname{sen}\theta\,\hat{u}$$

$$\begin{cases} m\overline{a}_T = -mg\operatorname{sen}\theta\,\hat{u}_T \\ m\overline{a}_N = (\overline{T} - mg\cos\theta)\,\hat{u}_N = 0 \end{cases} \longrightarrow \cancel{m}\frac{dv}{dt}\hat{u}_T = -\cancel{m}g\operatorname{sen}\theta\,\hat{u}_T \longrightarrow \frac{dv}{dt} = -g\operatorname{sen}\theta$$

Ricordando che:

$$v = R\omega = R\dot{\theta} = L\dot{\theta}$$

$$\downarrow$$

$$L\frac{d^2\theta}{dt^2} = -g\operatorname{sen}\theta \longrightarrow x(t) = \ddot{\theta} + \frac{g}{L}\operatorname{sen}\theta$$

$$\downarrow$$

$$\theta \ll \pi\ (\theta \simeq 0) \longrightarrow \operatorname{sen}\theta = \theta - \frac{1}{3!}\theta^3 + \dots \simeq 0$$

$$\downarrow$$

$$x(t) = \ddot{\theta} + \frac{g}{L}\theta$$

REGIME di PICCOLE OSCILLAZIONI

$$\theta(t) = A\operatorname{sen}(\omega t + \varphi) \qquad \omega^2 = \frac{g}{L} \qquad \omega = \sqrt{\frac{g}{L}} \qquad \omega = \frac{2\pi}{T}$$

$$\downarrow$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

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$$E_c = \frac{m}{2}\bar{v}^2 = \frac{\bar{p}^2}{2m}$$

ENERGIA CINETICA

- TEOREMA dell'ENERGIA CINETICA

$$W_{AB} = \int_{A\gamma}^{B} \bar{F}\cdot d\bar{r} = \int_A^B m\bar{a}\cdot d\bar{r} = \int_A^B m(\bar{a}_N + \bar{a}_T)\,\bar{v}\,dt = \int_{t_A}^{t_B} m\bar{a}_T\cdot\bar{v}\,dt =$$

$$\longrightarrow = \int_{t_A}^{t_B} m\bar{v}\cdot\frac{d\bar{v}}{dt}\,dt = \int_{t_A}^{t_B} m\frac{d}{dt}\left(\frac{1}{2}v^2\right)dt = \left[\frac{m}{2}v^2\right]_{t_A}^{t_B} = \frac{m}{2}\cdot\left[v^2(t_B) - v^2(t_A)\right] =$$

$$\downarrow$$

$$W_{AB} = E_k^B - E_k^A = \frac{m}{2}\left(v_B^2 - v_A^2\right)$$

es. ALENIA

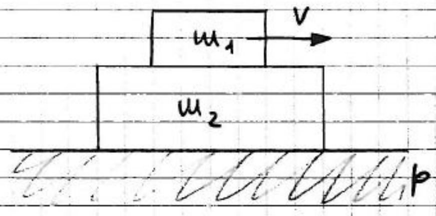
Hp: $m_1 = 2\,Kg$

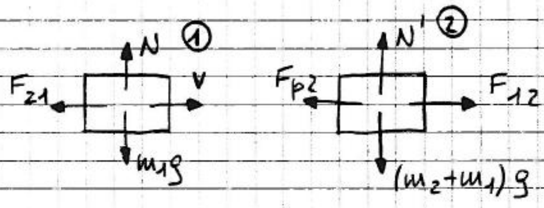
$m_2 = 3\,Kg$

$\mu_1 = 0{,}6$

$\mu_2 = 0{,}2$

$V = 3\,m/s$





① per muovere m_2:

$$F_{12} > F_{p2}$$

$$\downarrow$$

$$m_1 g\,\mu_1 > (m_1 + m_2)\,g\,\mu_2$$

② che distanza percorre m_1 rispetto a p prima di fermarsi

$m_1)\quad F = \cancel{m_1}a = -\mu_1\cancel{m_1}g \longrightarrow a = -\mu_1 g \longrightarrow x(t) = V_0 t - \frac{1}{2}at^2$

$m_2)\quad F = m_2 a_2 = m_1\mu_1 g - (m_1+m_2)\mu_2 g$

$$\begin{cases} V_1(t) = V_1^i - a_1 t \\ V_2(t) = a_2 t \end{cases} \longrightarrow V_1(t) = V_2(t) \longrightarrow t^* = 0{,}46\,s \quad V^* = 0{,}3\,m/s$$

$x_1(t^*) = 0{,}758$

$x_2(t^*) = 0{,}069\,m$

$x_{12} = 0{,}689$

FORMULE ESPONENZIALI - TRIGONOMETRICHE

1) $e^{i\alpha} = \cos\alpha + i \operatorname{sen}\alpha$

2) $\cos\alpha = \dfrac{e^{i\alpha} + e^{-i\alpha}}{2}$

3) $\operatorname{sen}\alpha = \dfrac{e^{i\alpha} - e^{-i\alpha}}{2i}$

$\rightarrow$ $x(0) = 0$, $v(0) = 0$ $\rightarrow$ $\begin{cases} \dfrac{g}{\omega^2} + A\operatorname{sen}(\varphi) = 0 \\ A\omega\cos(\varphi) = 0 \end{cases}$

$\varphi = \pm\dfrac{\pi}{2} \rightarrow A\operatorname{sen}(\varphi) = -\dfrac{g}{\omega^2}$

$\varphi = -\dfrac{\pi}{2}$

$A = \dfrac{g}{\omega^2}$

$$x(t) = \frac{g}{\omega^2}\operatorname{sen}\left(\omega t - \frac{\pi}{2}\right) + \frac{g}{\omega^2}$$

$$x_M = \frac{2g}{\omega^2}$$

$$v(t) = \frac{g}{\omega}\cos\left(\omega t - \frac{\pi}{2}\right) \qquad \omega t - \frac{\pi}{2} = 0 \rightarrow \bar{t} = \frac{\pi}{2\omega}$$

$$x(\bar{t}) = \frac{g}{\omega^2}\operatorname{sen}(0) + \frac{g}{\omega^2} = \frac{g}{\omega^2} \qquad v_M = \frac{g}{\omega}$$

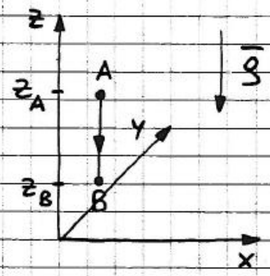
$$W_{AB} = \int_{A\gamma}^{B} \overline{F} \cdot d\overline{r} = \int_{A}^{B} (F_x\,dx + F_y\,dy + F_z\,dz) = \overline{F} \cdot \int_{A\gamma}^{B} d\overline{r} = \overline{F} \int_{A}^{B} (\hat{u}_x\,dx + \hat{u}_y\,dy + \hat{u}_z\,dz)$$

$$= F_x \int_{x_A}^{x_B} dx + F_y \int_{y_A}^{y_B} dy + F_z \int_{z_A}^{z_B} dz = F_x (x_B - x_A) + F_y (y_B - y_A) + F_z (z_B - z_A) =$$

$$= \vec{F} \cdot (\overline{r}_B - \overline{r}_A) = -\left[U(\overline{r}_B) - U(\overline{r}_A)\right]$$

$$W_{AB} = U(\overline{r}) = -\vec{F} \cdot \vec{r} + C$$

- CASO PARTICOLARE
  caduta di un corpo



$$\vec{F} = m\overline{g} = -mg\,\hat{u}_z$$

$$W_{AB} = -\left[U(\overline{r}_B) - U(\overline{r}_A)\right] = U(\overline{r}) = -m\overline{g} \cdot \overline{r} + C = +mgz + C = U(z) =$$

$$= -\left[U(z_B) - U(z_A)\right] = -mg\,(z_B - z_A)$$

$$W_{AB} = -mg\,(z_B - z_A) > 0$$

$$= \frac{m}{2}\left(v_B^2 - v_A^2\right) = E_k^B - E_k^A$$

$$\downarrow$$

$$E_k^B + U(z_B) = E_k^A + U(z_A)$$

- CAMPO di FORZE CONSERVATIVO

$$\overline{F}(\overline{r}) \longrightarrow U(\overline{r}) = E_p(\overline{r})$$

$$W_{AB} = \int_{A\gamma}^{B} \overline{F}(\overline{r}) \cdot d\overline{r} = -\left(E_p(\overline{r}_B) - E_p(\overline{r}_A)\right)$$

$$= -\left[E_p(\underbrace{x,y,z}_{\vec r}) + \frac{\partial E_p}{\partial x}dx + \frac{\partial E_p}{y}dy + \frac{\partial E_p}{\partial z}dz - E_p(\vec r)\right] = -\left(\frac{\partial E_p}{\partial x}dx + \frac{\partial E_p}{\partial y}dy + \frac{\partial E_p}{\partial z}dz\right)$$

$$= -\nabla E_p \cdot d\vec r$$

- ROTORE

$$\vec A = \mathrm{rot}\,\vec F = 0$$

$$\downarrow$$

$$A_x = \partial_y F_z - \partial_z F_y = 0$$

$$A_y = \partial_z F_x - \partial_x F_z = 0$$

$$A_z = \partial_x F_y - \partial_y F_x = 0$$

- TEOREMA di CONSERVAZIONE dell'ENERGIA

1) $$W_{AB} = \int_{A\gamma}^{B} \vec F \cdot d\vec r = \underbrace{\frac{m}{2}v_B^2}_{E_k^B} - \underbrace{\frac{m}{2}v_A^2}_{E_k^A}$$

2) se $\vec F$ = conservativo

$$W_{AB} = \int_{A\gamma}^{B} \vec F \cdot d\vec r = -\left(E_p(\vec r_B) - E_p(\vec r_A)\right)$$

$$\downarrow$$

$$W_{AB} = E_k^B - E_k^A = -\left(\underbrace{E_p(\vec r_B)}_{E_p^B} - \underbrace{E_p(\vec r_A)}_{E_p^A}\right) \longrightarrow E_k^B + E_p^B = E_k^A + E_p^A$$

$$= -K\left(\hat{u}_x\left(\frac{-x}{r^3}\right)+\hat{u}_y\left(\frac{-y}{r^3}\right)+\hat{u}_z\left(\frac{-z}{r^3}\right)\right) = +\frac{K}{r^3}\bar{r} \longrightarrow \bar{F}(\bar{r}) = -\nabla E_p$$

Se le forze sono conservative:

$$E^B_{TOT} = E^A_{TOT} \quad , \quad E^C_{TOT} = E^C_K + E^C_p \qquad C = A,B$$

$$W_{AB} = \int_{A\gamma}^{B} (\underbrace{\bar{F}}_{\text{CONSERVATIVA}} + \underbrace{\bar{F}_\alpha}_{\text{NON CONSERVATIVA}}) \cdot d\bar{r} = E^B_K - E^A_K$$ applicazione del teorema dell'energia cinetica

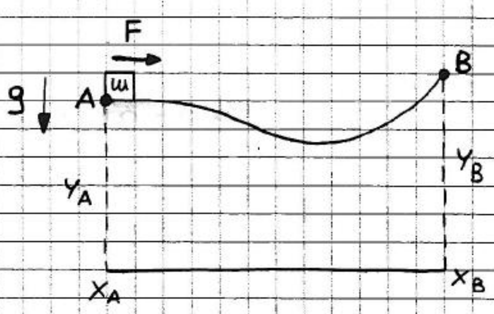
$$\downarrow$$

$$= \int_{A\gamma}^{B} \bar{F}\cdot d\bar{r} + \int_{A\gamma}^{B} \bar{F}_\alpha \cdot d\bar{r} = -\left(E^B_p - E^A_p\right) + W'_{AB}$$

$$\downarrow$$

$$E^B_K - E^A_K = -\left(E^B_p - E^A_p\right) + W'_{AB} \longrightarrow \underbrace{E^B_K + E^B_p}_{E^B_{TOT}} = \underbrace{E^A_K + E^A_p}_{E^A_{TOT}} + W'_{AB}$$

es. ALENIA



Hp: $X_A = 0$ $\quad m = 5\,kg$

$X_B = 2\,m$

$Y_A = 0{,}5\,m$

$Y_B = 0{,}8\,m$

$F = 20\,N$

Th: W_{AB}, V_B

perché le Y crescono verso l'alto

$$W_{AB} = \int_{A}^{B} \vec{F}\cdot d\vec{s} = \int_{X_A}^{X_B} F\cdot dx \ominus \int_{Y_A}^{Y_B} mg\cdot dy = 25{,}35\,J$$

$$W = \frac{1}{2} m V_B^2 \qquad V_B = \sqrt{\frac{2W}{m}} = 3{,}18\ m/s$$

- TEOREMA del MOMENTO ANGOLARE per una MASSA PUNTIFORME

$$\frac{d\overline{L}}{dt} = \overline{M}$$

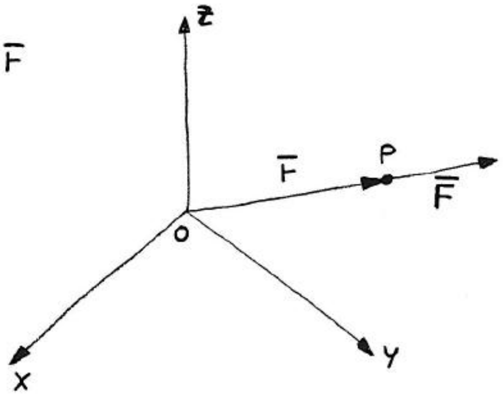
Dim. $\frac{d\overline{L}}{dt} = \frac{d}{dt}\left(m\overline{r} \wedge \overline{v}\right) = m \underbrace{\frac{d\overline{r}}{dt}}_{\overline{v}} \wedge \overline{v} + m\overline{r} \wedge \frac{d\overline{v}}{dt} = m\overline{r} \wedge \overline{a} = \overline{r} \wedge \overline{F} = \overline{M}$

$\downarrow$

$$\overline{M} = 0 \longrightarrow \frac{d\overline{L}}{dt} = 0 \longrightarrow \overline{L} = \text{costante in } t = m\underbrace{\overline{r} \wedge \overline{v}}_{\text{dipendono da } t}$$

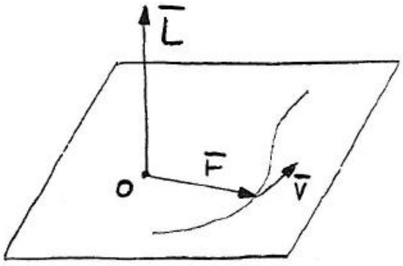
- TEOREMA del MOMENTO ANGOLARE a FORZE CENTRALI

$\overline{F} \parallel \overline{r}$



F è orientata nella stessa direzione di r

$$\frac{d\overline{L}}{dt} = \overline{M} = \overline{r} \wedge \overline{F} = 0 \qquad \overline{L} = \text{costante}$$



$t: \ \overline{r} = \overline{r}(t), \ \overline{v} = \overline{v}(t)$

$t + \varepsilon = t': \ r' = \overline{r}(t+\varepsilon) = \overline{r}(t) + \varepsilon v(t)$

$\qquad v' = \overline{v}(t+\varepsilon) = \overline{v}(t) + \varepsilon \overline{a}(t)$

perchè di ordine superiore

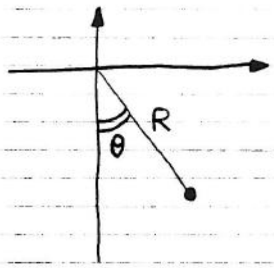
$$L' = m\overline{r}' \wedge \overline{v}' = m(\overline{r} + \varepsilon\overline{v}) \wedge (\overline{v} + \varepsilon\overline{a}) = m\overline{r} \wedge \overline{v} + \varepsilon \underbrace{m\overline{r} \wedge \overline{a}}_{m\overline{a} = \overline{F}} + m\varepsilon^2 \cancel{\overline{v} \wedge \overline{a}} \longrightarrow$$

$$L' = m\overline{r} \wedge \overline{v} = \overline{L}$$

$$\rightarrow \frac{d\overline{L}}{dt} = \overline{u} \quad \rightarrow m R^2 \ddot{\theta}\,\hat{u}_z = -R m g \,\mathrm{sen}\,\theta\, \hat{u}_z$$

$$\ddot{\theta} R = -m g\,\mathrm{sen}\theta \quad \rightarrow \quad \ddot{\theta} + \frac{g}{R}\,\mathrm{sen}\theta = 0$$

PENDOLO con l'ENERGIA



$$E_p(z) = mgz + \underbrace{c}_{mgR} = mg(-R\cos\theta) + mgR = mgR(1-\cos\theta)$$

$$E = E_k + E_p = \text{cost}$$

$$E = \frac{m}{2} R^2 \dot{\theta}^2 + mgR(1-\cos\theta) \rightarrow \underbrace{\frac{dE}{dt}}_{0} = \frac{m}{2} R^2 \frac{d\dot{\theta}^2}{dt} + mgR\frac{d}{dt}(\cancel{1}-\cos\theta)$$

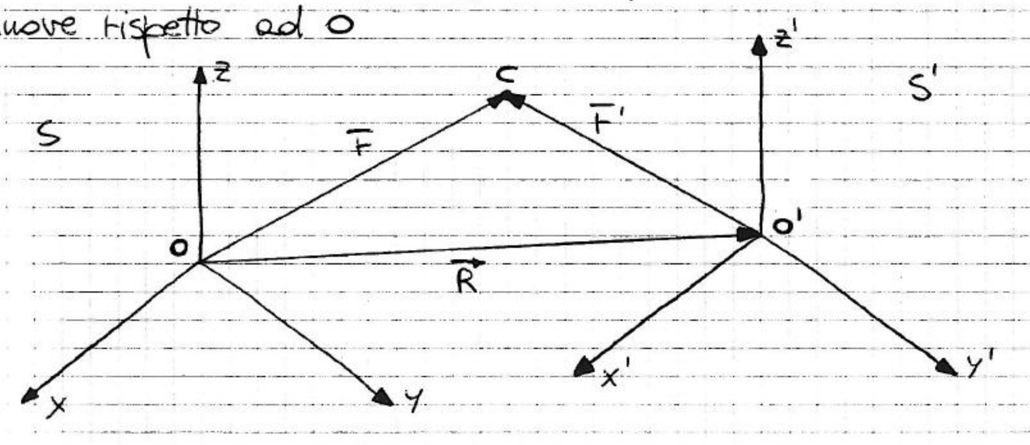
$$\rightarrow 0 = \frac{m}{2} R^2\, 2\dot{\theta}\ddot{\theta} - mgR(-\dot{\theta}\,\mathrm{sen}\theta) \rightarrow 0 = m\dot{\theta}R^2\left(\ddot{\theta} + \frac{g}{R}\,\mathrm{sen}\theta\right)$$

$$\rightarrow \ddot{\theta} + \frac{g}{R}\,\mathrm{sen}\theta = 0$$

- MOTI RELATIVI

La scelta del sistema di riferimento in cui descrivere un fenomeno NON è, in generale, univoca. Scelte diverse nascono dalla necessità di semplificare la descrizione di certi aspetti o proprietà fisiche di un fenomeno rispetto alle altre scelte possibili.

S ed S' hanno sempre gli assi paralleli ($x /\!/ x'$, $y /\!/ y'$, $z /\!/ z'$), O' si muove rispetto ad O



$\overline{OO'} = \overline{R}$

$\overline{r} = \overline{R} + \overline{r}'$

$\overline{r}' = \overline{r} - \overline{R}$

• TEOREMA delle VELOCITÀ RELATIVE

$$\bar{v} = \bar{v}' + \bar{v}_{o'} + \underbrace{\bar{\omega} \wedge \bar{r}'}_{\text{effetto della rotazione}}$$

se $\bar{\omega} = 0$ $\quad \bar{v} = \bar{v}' + \bar{v}_{o'}$

(se i sistemi non hanno movimenti relativi)

Dim.

$$\bar{r} = \bar{r}' + \bar{R} \longrightarrow \frac{d\bar{r}}{dt} = \frac{d\bar{r}'}{dt} + \frac{d\bar{R}}{dt} \longrightarrow \bar{v} = \bar{v}_{o'} + \frac{d\bar{r}'}{dt}$$

FORMULA di POISSON

$$\frac{d\hat{u}}{dt} = \dot{\theta}\, \hat{u}_\perp \wedge \hat{u} = \bar{\omega} \wedge \hat{u} \qquad \bar{\omega} = \dot{\theta}\, \hat{u}_\perp$$

$\downarrow$

$$\frac{d\bar{r}'}{dt} = \frac{d}{dt}\left(x'\hat{u}'_x + y'\hat{u}'_y + z'\hat{u}'_z\right) = \underbrace{\frac{dx'}{dt}}_{v'_x}\hat{u}'_x + x'\frac{d\hat{u}'_x}{dt} + \underbrace{\frac{dy'}{dt}}_{v'_y}\hat{u}'_y + y'\frac{d\hat{u}'_y}{dt} + \underbrace{\frac{dz'}{dt}}_{v'_z}\hat{u}'_z + z'\frac{d\hat{u}'_z}{dt} =$$

$$= v'_x\hat{u}'_x + v'_y\hat{u}'_y + v'_z\hat{u}'_z + x'\bar{\omega} \wedge \hat{u}'_x + y'\bar{\omega} \wedge \hat{u}'_y + z'\bar{\omega} \wedge \hat{u}'_z =$$

$$= \bar{v}' + \bar{\omega} \wedge \left(x'\hat{u}'_x + y'\hat{u}'_y + z'\hat{u}'_z\right) = \bar{v}' + \bar{\omega} \wedge \bar{r}' \longrightarrow = \bar{v}' + \underbrace{\bar{v}_{o'} + \bar{\omega} \wedge \bar{r}'}_{\text{velocità di trascinamento}}$$

• TEOREMA delle ACCELERAZIONI RELATIVE

$$\bar{a} = \bar{a}' + \bar{a}_{o'} + \bar{\omega} \wedge \left(\bar{\omega} \wedge \bar{r}\right) + \frac{d\bar{\omega}}{dt} \wedge \bar{r}' + 2\bar{\omega} \wedge \bar{v}'$$

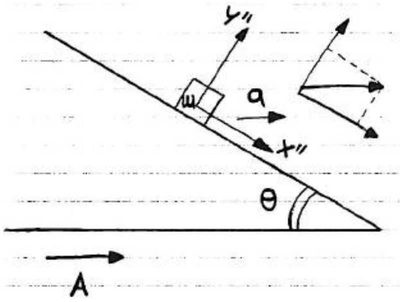
Dim. $\qquad$ se $\bar{\omega} = 0$ $\quad \bar{a} = \bar{a}' + \bar{a}_{o'}$

$$\frac{d\bar{v}}{dt} = \frac{d\bar{v}'}{dt} + \frac{d\bar{v}_{o'}}{dt} + \frac{d}{dt}\left(\bar{\omega} \wedge \bar{r}\right)$$

$$\bar{a} = \underbrace{\frac{d\bar{v}_{o'}}{dt}}_{\bar{a}_{o'}} + \frac{d}{dt}\left(v'_x\hat{u}'_x + v'_y\hat{u}'_y + v'_z\hat{u}'_z\right) + \frac{d\bar{\omega}}{dt} \wedge \bar{r}' + \bar{\omega} \wedge \frac{d\bar{r}'}{dt} = \longrightarrow$$

$\overline{T} = m\overline{a} \longrightarrow T = m\omega_0^2 r \longrightarrow m = \frac{T}{\omega_0^2 r} = 0,1\ Kg$

es. 2



$\overline{a}' = \overline{a} - \overline{A}$

$m\overline{a} = m\overline{g} + \overline{N} + \overline{f}_{attrito}$

$m\overline{a}' = m\overline{g} + \overline{N} + \overline{f} - m\overline{A} \longrightarrow \begin{cases} x'': & ma' = mg\,sen\theta - mA\cos\theta + f \\ y'': & 0 = N - mA\,sen\theta - mg\cos\theta \end{cases}$

$\downarrow$

$N = mA\,sen\theta + mg\cos\theta$

$\downarrow$

$g\,sen\theta > A\cos\theta \longrightarrow f < 0$

$g\,sen\theta < A\cos\theta \longrightarrow f > 0$

$\downarrow$

Il sistema è in equilibrio statico (m non si muove), se $\longrightarrow 0 < f \leq \mu_s N$

$\downarrow$

$0 = mg\,sen\theta - mA\cos\theta + f$

$0 \leq mg\,sen\theta - mA\cos\theta + \mu_s N$ **sotto** questo valore c'è movimento

$\downarrow$

$0 \leq mg\,sen\theta - mA\cos\theta + \mu_s m(A\,sen\theta + g\cos\theta)$

$\downarrow$

$0 \leq mg\,tg\theta - mA + \mu_s m(A\,tg\theta + g) \longrightarrow A \leq \frac{tg\theta + \mu_s}{1 - \mu_s tg\theta}\,g \ \vee\ A \geq \frac{tg\theta + \mu_s}{1 + \mu_s tg\theta}\,g$

• PRINCIPIO di RELATIVITÀ CLASSICA

Le leggi della fisica sono le stesse per tutti gli osservatori che descrivono i fenomeni da sistemi di riferimento inerziali

$$\rightarrow \begin{cases} \overline{V} = \overline{V}' + \overline{\omega} \wedge \overline{r}' \\ \overline{a} = \overline{a}' - \omega^2 \overline{r}' + 2\overline{\omega} \wedge \overline{V}' \end{cases}$$

1) Osservatore sulla giostra che osserva un corpo fermo a terra

$$\overline{r} = x\hat{u}_x + y\hat{u}_y = \overline{r}' = x'\hat{u}'_x + y'\hat{u}'_y$$

$$\underbrace{\overline{V}}_{0} = \overline{V}' + \overline{\omega} \wedge \overline{r}' \longrightarrow V' = -\overline{\omega} \wedge \overline{r}'$$

$$\downarrow$$

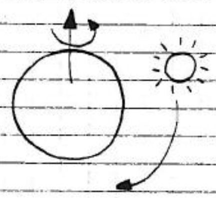
$$\underbrace{\overline{a}}_{0} = \overline{a}' - \omega^2 \overline{r}' + 2\overline{\omega} \wedge \overline{V}' \longrightarrow \overline{a}' = \omega^2 \overline{r}' - 2\overline{\omega} \wedge \overline{V}' = \omega^2 \overline{r}' - 2\overline{\omega} \wedge (-\overline{\omega} \wedge \overline{r}') = \omega^2 \overline{r}' + \underbrace{2\overline{\omega} \wedge (\overline{\omega} \wedge \overline{r}')}_{-\omega^2 \overline{r}'}$$

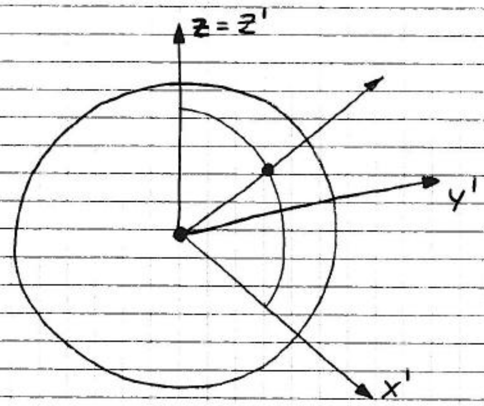
$$\downarrow$$

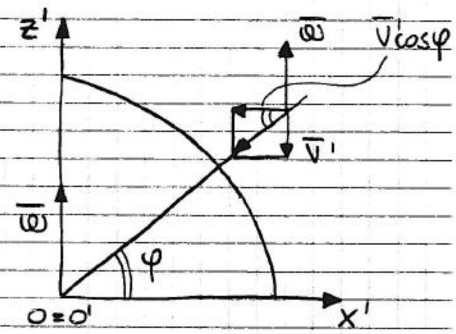
$$\overline{a}' = -\omega^2 \overline{r}'$$

• CADUTA dei GRAVI VERSO EST

$$m\overline{a}' = \overline{F} - m\overline{a}_{0'} - m\overline{\omega} \wedge (\overline{\omega} \wedge \overline{r}') - 2m\overline{\omega} \wedge \overline{V}' \qquad \overline{\omega} = cost$$







$$\downarrow$$

$$2\overline{\omega} \wedge \overline{V}' = 2\overline{\omega} \wedge (-\overline{V}'\cos\varphi\, \hat{u}_{x'} + \ldots) = -2\omega V' \cos\varphi\, \hat{u}_{z'} \wedge \hat{u}_{x'} = -2\omega V' \cos\varphi\, \hat{u}_{y'}$$

$$\downarrow$$

$$m\overline{a}' = \overline{F} - \overline{\omega} \wedge (\overline{\omega} \wedge \overline{r}') - 2m(\overline{\omega} \wedge \overline{V}')$$

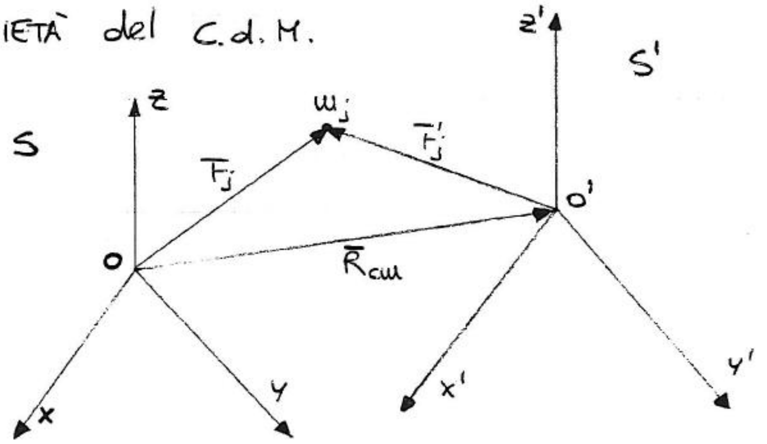
$$\downarrow$$

$$m\overline{a}' = \overline{F} \underbrace{- m\overline{\omega} \wedge (\overline{\omega} \wedge \overline{r}')}_{\text{ora calcolo questo}} + 2m\omega V' \cos\varphi\, \hat{u}_{y'} \longrightarrow$$

- POSIZIONE MEDIA delle PARTICELLE (m costante)

$$\frac{1}{N}\sum_j \bar{r}_j$$

- PROPRIETÀ del C.d.M.



$$\bar{r}'_j = \bar{r}_j - \bar{R}_{cm}$$

$$\bar{v}'_j = \bar{v}_j - \bar{V}_{cm}$$

$$M = \sum_j m_j$$

1) $$\bar{R}'_{cm} = \frac{1}{M}\sum_j m_j \bar{r}'_j = \frac{1}{M}\sum_j m_j (\bar{r}_j - \bar{R}_{cm}) = \frac{\sum_j m_j \bar{r}_j}{M} - \frac{\sum_j m_j \bar{R}_{cm}}{M} \longrightarrow$$

$$\bar{R}'_{cm} = \bar{R}_{cm} - \bar{R}_{cm}\overbrace{\frac{\sum_j m_j}{M}}^{1} = 0$$

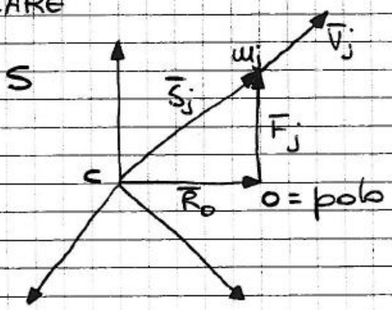
2) $$\bar{V}_{cm} = \frac{d\bar{R}_{cm}}{dt} = \frac{d}{dt}\frac{1}{M}\sum_j m_j \bar{r}_j = \frac{1}{M}\sum_j m_j \frac{d\bar{r}_j}{dt} \longrightarrow \bar{V}_{cm} = \frac{\sum_j m_j \bar{v}_j}{M}$$

$\downarrow$

$$\bar{V}'_{cm} = \frac{\sum_j m_j \bar{v}_j - \sum_j m_j \bar{V}_{cm}}{M} = \bar{V}_{cm} - \bar{V}_{cm}\cdot\overbrace{\frac{\sum_j m_j}{M}}^{1} = 0$$

- TEOREMA del MOMENTO ANGOLARE

Per ogni massa



$C \neq O$ in generale

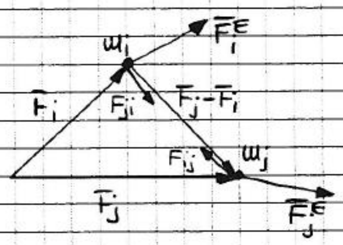
$$m_j \begin{cases} \overline{L}_j = m_j \overline{r}_j \wedge \overline{v}_j \\ \overline{M}_j = \overline{r}_j \wedge \overline{F}_j \end{cases}$$

rispetto a C vedo:

$$\begin{cases} \text{posizione} \quad \overline{s}_j = \overline{R}_o + \overline{r}_j \\ \text{velocità} \quad \overline{v}_j = \overline{v}_o + \dfrac{d\overline{r}_j}{dt} \end{cases}$$

rispetto al polo O vedo:

$\overline{L} = \sum_j \overline{L}_j = \sum_j m_j \overline{r}_j \wedge \overline{v}_j$ MOMENTO ANGOLARE TOTALE rispetto ad O



$$\frac{d\overline{L}}{dt} = \sum_j \frac{dL_j}{dt} = \sum_j m_j \frac{d}{dt}\left(\overline{r}_j \wedge \overline{v}_j\right) = \sum_j \left(m_j \frac{d\overline{r}_j}{dt} \wedge \overline{v}_j + m_j \overline{r}_j \wedge \frac{d\overline{v}_j}{dt} \right)$$

$$\frac{d\overline{r}_j}{dt} = \overline{v}_j - \overline{v}_o \quad = \sum_j m_j (\cancel{\overline{v}_j} - \overline{v}_o) \wedge \overline{v}_j + \sum_j m_j \overline{r}_j \cdot \overline{a}_j = -\overline{v}_o \wedge \sum_j m_j \overline{v}_j + \sum_j \overline{r}_j \wedge \overline{R}_j \longrightarrow$$

$$\longrightarrow = -\overline{v}_o \wedge M \cdot \overline{v}_{cm} + \sum_j \overline{r}_j \wedge \left(\overline{F}_j^E + \overline{F}_j^I\right) = M \cdot \overline{v}_{cm} \wedge \overline{v}_o + \sum_j \overline{r}_j \wedge \overline{F}_j^E + \sum_j \overline{r}_j \wedge \overline{F}_j^I = \longrightarrow$$

$$\longrightarrow = \sum_j \overline{r}_j \wedge \overline{F}_j^I = \sum_j \overline{r}_j \wedge \sum_{i \neq j} \overline{F}_{ij} = \sum_j \sum_{i \neq j} \overline{r}_j \wedge \overline{F}_{ij} = 0$$

$\forall$ coppia i,j trovo sempre:

$$\overline{r}_j \wedge \overline{F}_{ij} + \overline{r}_i \wedge \overline{F}_{ji} = \overline{r}_j \wedge \overline{F}_{ij} - \overline{r}_i \wedge \overline{F}_{ij} = \left(\overline{r}_j - \overline{r}_i\right) \wedge \overline{F}_{ij}$$

Quindi $\sum_j \sum_{i \neq j} \overline{r}_j \wedge \overline{F}_{ij} = 0 \longrightarrow \dfrac{d\overline{L}}{dt} = M \overline{v}_{cm} \wedge \overline{v}_o + \overline{M}^E \longrightarrow$

$\frac{d\overline{L}}{dt} = \overline{M}^E$ derivo L o M^E con la nuova notazione

$\downarrow$

$\overline{L} = \sum_J m_J \overline{r}'_J \wedge \overline{v}_J$ (visto da S)

$\overline{M}^E = \sum_J \overline{r}'_J \wedge \overline{F}_J^E$

considero L:

$$\overline{L} = \sum_J m_J \overline{r}'_J \wedge \overline{v}_J = \sum_J m_J \overline{r}'_J \wedge (\overline{v}'_J + \overline{V}_{cm}) = \sum_J m_J \overline{r}'_J \wedge \overline{v}'_J + \sum_J (m_J \overline{r}'_J \wedge \overline{V}_{cm}) =$$

$$= \sum_J m_J \overline{r}'_J \wedge \overline{v}'_J + \underbrace{\left(\sum_J m_J \overline{r}'_J\right)}_{M \cdot \frac{1}{M} \sum_J m_J \overline{r}'_J = \overline{R}'_{cm} = 0} \wedge \overline{V}_{cm}$$

$\downarrow$

$$\overline{L} = \sum_J m_J \overline{r}'_J \wedge \overline{v}'_J = \overline{L}'$$

considero M^E:

$$\overline{M}^E = \sum_J \overline{r}'_J \wedge \overline{F}^E \longrightarrow \overline{M}'^E = \sum_J \overline{r}'_J \wedge \overline{F}'_J \quad \text{ma} \quad m_J \overline{a}'_J = \overline{F}'_J = \overline{F}_J^E + \overline{F}_J^I - m_J \overline{a}_{cm}$$

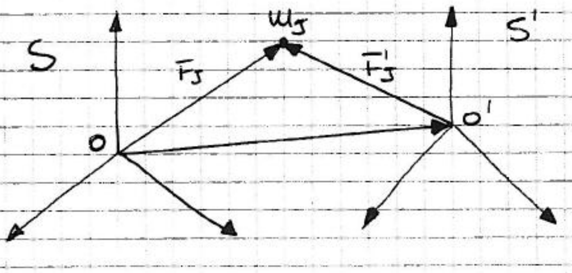
$$\longrightarrow \overline{M}'^E = \sum_J \overline{r}'_J \wedge (\overline{F}_J^E + \overline{F}_J^I - m_J \overline{a}_{cm}) = \sum_J \overline{r}'_J \wedge \overline{F}_J^E + \underbrace{\sum_J \overline{r}'_J \wedge \overline{F}_J^I}_{0} - \underbrace{\left(\sum_J m_J \overline{r}'_J\right)}_{0} \wedge \overline{a}_{cm} = \overline{M}^E$$

$\downarrow$ $\overline{M}'^E = \sum_J \overline{r}'_J \wedge \overline{F}_J'^E = \overline{M}^E$

$$\frac{d\overline{L}}{dt} = \overline{M}^{(E)} \longrightarrow \frac{d\overline{L}'}{dt} = \overline{M}'^{(E)}$$

nel sistema S — nel sistema del CdM

Come l'energia cinetica totale e il momento angolare totale cambiano passando da un sistema di riferimento inerziale S al sistema del CdM?



$O' = CdM$

$\overline{r}_J = \overline{r}'_J + \overline{R}_{cm}$

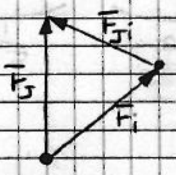
$\overline{V}_J = \overline{v}'_J + \overline{V}_{cm}$

$$W_{AB} = \sum_j W_{AB}^{(j)} = \sum_j \int_{A\gamma_j}^{B} \bar R_j \cdot d\bar r_j = \sum_j \int_{A\gamma_j}^{B} (\bar F_j^E + \underbrace{\bar F_j^I}_{\sum_{i\neq j} \bar F_{ij}}) \cdot d\bar r_j = -\sum_j (U_j^{EB} - U_j^{EA}) + \sum_j \int_{A\gamma_j}^{B} \bar F_j^I \cdot d\bar r_j =$$

$$= W_{AB}^E + \sum_j \sum_{i\neq j} \int_{A\gamma_j}^{B} \bar F_{ij} \cdot d\bar r_j \qquad \text{(tutte le possibili coppie)}$$

$\downarrow$

coppie (i,j) $\quad \bar F_{ij} \cdot d\bar r_j + \underbrace{\bar F_{ji}}_{-\bar F_{ij}} \cdot d\bar r_i = \bar F_{ij} \cdot (d\bar r_j - d\bar r_i) = \bar F_{ij} \cdot d\bar r_{ji}$



$\downarrow$

$$W_{AB} = W_{AB}^E + \frac{1}{2} \sum_{\substack{(ij)\\ i\neq j}} \int_A^B \bar F_{ij} \cdot d\bar r_{ij}$$ LAVORO per N PARTICELLE

FORZA GRAVITAZIONALE

$$\bar F(\bar r) = -\frac{\gamma m_1 m_2}{r^3} \bar r \longrightarrow \bar F(\bar r_2 - \bar r_1) = -\frac{\gamma m_1 m_2}{|\bar r_2 - \bar r_1|^3} (\bar r_2 - \bar r_1) \qquad \bar r = \bar r_2 - \bar r_1 = \bar r_{21}$$

$\downarrow$

$$\bar F(\bar r) = -\nabla_{\bar r} U(\bar r)$$

$$\bar F_{12} = \bar F(\bar r_2 - \bar r_1) = -\nabla_{\bar r_2} \underbrace{U(\bar r_2 - \bar r_1)}_{U_{12}}$$

$\downarrow$ LAVORO

$$W_{AB} = W_{AB}^E - \frac{1}{2} \sum_{(ij)} (U_{ij}^B - U_{ij}^A)$$

$\downarrow$

$$W_{AB} = \sum_j \int_{A\gamma_j}^{B} \bar R_j \cdot d\bar r_j = \sum_j \frac{m_j}{2} (v_{Bj}^2 - v_{Aj}^2) = E_K^B - E_K^A$$

$\downarrow$

$$E_K^B - E_K^A = W_{AB} = W_{AB}^E - \left(\frac{1}{2} \sum_{(ij)} U_{ij}^B - \frac{1}{2} \sum_{(ij)} U_{ij}^A \right) \longrightarrow$$

allora:

$$E_k^A + E_p^A = E_k^B + E_p^B \qquad N \text{ particelle} = 2$$

$$E_k^B - E_k^A = -\left(E_p^B - E_p^A\right) = -\left[U^{\epsilon B} + U_{12}^B - U^{\epsilon A} - U_{12}^A\right]$$

$$\underbrace{U^{\epsilon B} - U^{\epsilon A}}_{0} \qquad \underbrace{U_{12}^B - U_{12}^A}_{0}$$

$$\downarrow$$

$$E_k^A = E_k^B \qquad \text{URTO ELASTICO}$$

e quindi:

- $\overline{p}_1 + \overline{p}_2 = \overline{p}_1' + \overline{p}_2'$
- $E_k^A = E_k^B$
- $\frac{1}{2} m_1 \overline{v}_1^2 + \frac{1}{2} m_2 \overline{v}_2^2 = \frac{1}{2} m_1 \overline{v}_1'^2 + \frac{1}{2} m_2 \overline{v}_2'^2$

caso ANELASTICO:

$$E^B - E^A = \sum_J \int_{A\gamma_J}^{B} \overline{F}_{aJ} \cdot d\overline{r}_J = W'_{AB}$$

$$E_k^B - E_k^A + \underbrace{E_p^B - E_p^A}_{0} = W'_{AB} \qquad \text{URTO ANELASTICO}$$

e quindi:

- $\overline{p}_1 + \overline{p}_2 = \overline{p}_1' + \overline{p}_2'$
- $E_k^B - E_k^A = W'_{AB}$

2) sperimentale $\overline{V_L} = \dfrac{2\pi R_{TL}}{T_L}$ → 28 giorni

3) considero un corpo m sulla Terra

$$\gamma \frac{mM}{R_T^2} = mg$$

↓

$$\gamma \frac{\cancel{m_L} M}{R_{TL}^2} = \cancel{m_L}\frac{V_L^2}{R_{TL}}$$

$$\gamma \frac{\cancel{m} M}{R_T^2} = \cancel{m} g$$

$$\to \frac{\dfrac{\gamma M}{R_{TL}^2}}{\dfrac{\gamma M}{R_T^2}} = \frac{\dfrac{V_L^2}{R_{TL}}}{g} \to \frac{R_T^2}{R_{TL}^2} = \frac{V_L^2}{g R_{TL}}$$

dimostrazione legge gravitazione universale

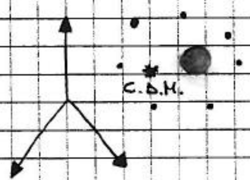
Prova della 2ª LEGGE di KEPLERO

$$\frac{d\overline{L}}{dt} = \overline{M} = \overline{r} \wedge \overline{F} = 0 \to \overline{L} = cost = m r^2 \theta' \hat{u}_z$$

se $\overline{F}$ è centrale
$\overline{r} /\!/ \overline{F}$

velocità areolare

$$\frac{dA}{dt} = \frac{r^2}{2}\frac{d\theta}{dt} = \frac{1}{2} r^2 \theta' \to \frac{dA}{dt} = \frac{1}{2m} L = cost$$



$$\overline{R}_{cm} = \sum_j m_j \overline{r}_j \qquad M\overline{A}_{cm} = M\frac{d^2\overline{R}_{cm}}{dt^2} = \overline{R}^E \simeq 0 \to \overline{A}_{cm} = 0$$

↓

$$\overline{V}_{cm} = cost$$

È quindi un sistema inerziale

$$\rightarrow \frac{1}{2}\frac{4\pi^2}{T^2}a^2b^2\left(\frac{1}{r_1^2}-\frac{1}{r_2^2}\right)=\gamma M\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

$$\downarrow$$

$$a^2b^2=\underbrace{r_1r_2}_{b^2}\,\underbrace{\frac{1}{4}(r_1+r_2)^2}_{a^2} \rightarrow \frac{2\pi^2}{T^2}\frac{1}{4}\cancel{r_1r_2}(r_1+r_2)^2\left(\frac{r_2^2-r_1^2}{\cancel{r_1^2r_2^2}}\right)=\gamma M\,\frac{r_2-r_1}{\cancel{r_1r_2}}$$

$$\rightarrow \frac{\pi^2}{2T^2}(r_1+r_2)^3=\gamma M \rightarrow T^2=\underbrace{\frac{4\pi^2}{\gamma M}}_{K}\underbrace{\left(\frac{r_1+r_2}{2}\right)^3}_{R^3}$$

- FORZA GRAVITAZIONALE con N MASSE

$$\overline{G}(\overline{r})=\sum_{j=1}^{N}\frac{-\gamma m_j}{|\overline{r}-\overline{r}_j|^2}\hat{u}_j \quad , \quad \hat{u}_j=\frac{\overline{r}-\overline{r}_j}{|\overline{r}-\overline{r}_j|}$$

$\downarrow$ con una massa di prova m

$$\overline{F}(\overline{r})=m\,\overline{G}(\overline{r})$$

$$\downarrow$$

$$\overline{F}(\overline{r})=m\sum_{j=1}\underbrace{\overline{G}_j(\overline{r})}_{\frac{-\gamma m_j}{|\overline{r}-\overline{r}_j|^2}\hat{u}_j}=\sum_j \overline{F}_j(\overline{r})$$

$$\downarrow$$

$$E_p(\overline{r})=\sum_j E_p^j(\overline{r})=\sum_j m\left(\frac{-\gamma m_j}{|\overline{r}-\overline{r}_j|}\right)+c$$

$$\overline{F}(\overline{r})=-\nabla_{\overline{r}}E_p(\overline{r})=-\nabla_{\overline{r}}\sum_j E_p^j(\overline{r})=-\sum_j\nabla E_p^j(\overline{r})=\sum_j\big(\underbrace{-\nabla E_p^j(\overline{r})}_{\overline{F}_j(\overline{r})}\big)=\overline{F}(\overline{r})$$

$$=\sum_j\nabla\frac{\gamma m m_j}{|\overline{r}-\overline{r}_j|}=\sum_j m\,\frac{-\gamma m_j(\overline{r}-\overline{r}_j)}{|\overline{r}-\overline{r}_j|^3}=\sum_j m\,\frac{-\gamma m_j}{|\overline{r}-\overline{r}_j|^2}\hat{u}_j=m\,\overline{G}(\overline{r})$$

- **ORBITE POSSIBILI in un SISTEMA a 2 CORPI** (Sole-Terra, ...)

$$E = \frac{m_2}{2}\underbrace{\bar{v}^2}_{\bar{v} = \frac{d\bar{r}}{dt} = r'\hat{u}_r + r\theta'\hat{u}_\theta} - \gamma\frac{m_2 \overset{\text{SOLE}}{\widehat{m_1}}}{r} = \frac{m_2}{2}\left(r'^2 + r^2\theta'^2\right) - \gamma\frac{m_1 m_2}{r} = \longrightarrow$$

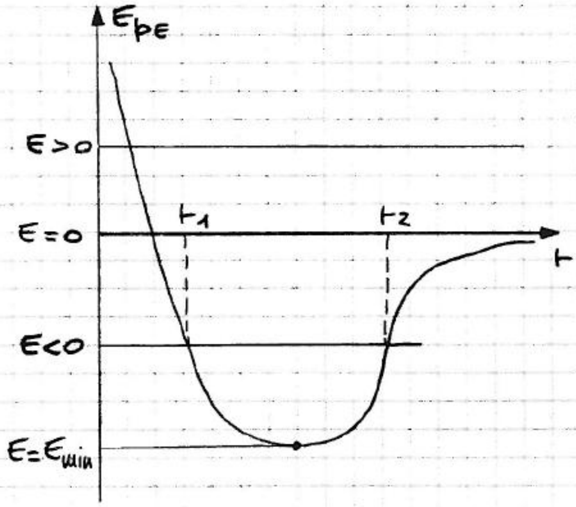
$$L = m_2 r^2 \theta' = cost$$

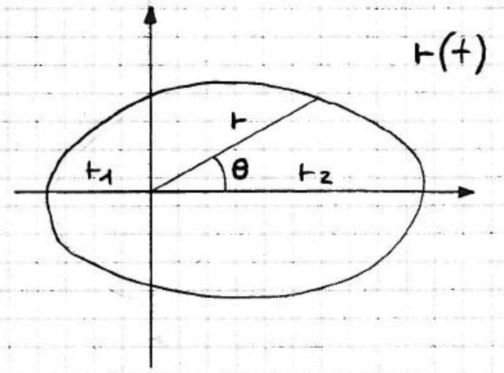
$$\theta' = \frac{L}{m_2 r^2}$$

$$\longrightarrow = \frac{m_2}{2}r'^2 + \underbrace{\frac{L^2}{2m_2r^2} - \frac{\gamma m_1 m_2}{r}}_{E_{pE}(r)}$$

$E_{pE}(r)$ ENERGIA POTENZIALE EFFICACE

$$E = \frac{m_2}{2}r'^2 + E_{pE}(r)$$





$$\frac{dE_{pE}(r)}{dr} = \frac{d}{dr}\left(\frac{L^2}{2m_2r^2} - \frac{\gamma m_1 m_2}{r}\right) = \frac{-2L^2}{2m_2r^3} + \frac{\gamma m_1 m_2}{r^2} = \frac{1}{r^3}\underbrace{\left(\gamma m_1 m_2 r - \frac{L^2}{m_2}\right)}_{0} \longrightarrow$$

$$r = r_m = \frac{L^2}{m_2^2\gamma m_1}$$ valore del raggio minimo, orbita circolare

$$L = m_2 r_m^2 \theta' \quad — \quad r_m = \frac{\cancel{m_2^2} r_m^4 \theta'^2}{\cancel{m_2^2} m_1 \gamma} \longrightarrow r_m^3 = \frac{\gamma m_1}{\theta'^2} \qquad \theta' = \frac{L}{m_2 r^2}$$

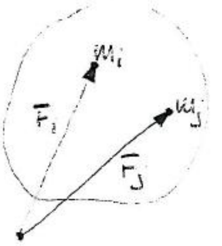
3ª legge di KEPLERO

$$\rightarrow v_1^2 = 2\gamma M \frac{r_2^2}{r_2^2 - r_1^2}\left(\frac{r_2 - r_1}{r_1 r_2}\right)$$

$$\downarrow$$

$$v_1 = \sqrt{\frac{2\gamma M r_2}{r_1(r_1 + r_2)}}$$

• DINAMICA ROTAZIONALE dei CORPI RIGIDI



$|\bar r_i - \bar r_j| =$ costante
$\forall$ i,j punti del corpo

Ricordiamo che:

$$\frac{d\bar L}{dt} = \bar M^E - M \bar v_o \wedge \bar v_{cm}$$

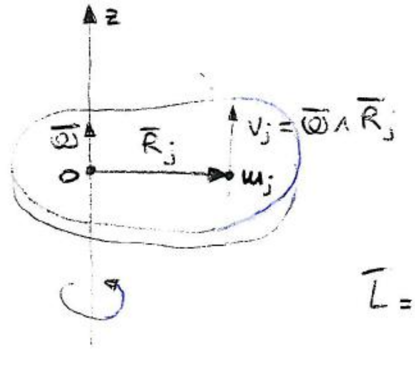
$$\bar L = \sum_j m_j \bar r_j \wedge \bar v_j$$

$$\bar M^E = \sum_j r_j \wedge \bar F_j^E$$

Con un asse fisso conviene prendere il polo O sull'asse e assumere O come origine

O = C dunque $\bar v_o = 0 \rightarrow \frac{d\bar L}{dt} = \bar M^E$

1)



$$\bar L = \sum_j m_j \bar R_j \wedge \bar v_j = \sum_j m_j R_j v_j \hat u_z = \sum_j m_j R_j^2 \omega \hat u_z$$

$$\downarrow$$

$$\bar L = \underbrace{\left(\sum_j m_j R_j^2\right)}_{I_z} \bar\omega = I_z \bar\omega \qquad \bar L \parallel \bar\omega$$

CORPO PIATTO

$$I_z = \iint \rho(x,y)(x^2 + y^2)\, dx\, dy$$

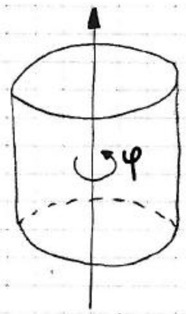
ASSI PRINCIPALI d'INERZIA

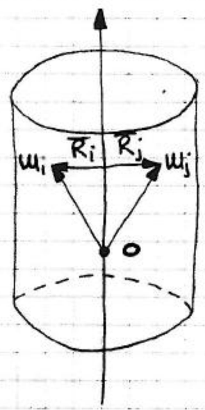
$\overline{L} = \overline{\omega} I_z$ cioè $\overline{L}_\perp = 0$

caso semplice:

CORPI OMOGENEI

In tal caso gli ASSI di SIMMETRIA del corpo coincidono con gli ASSI PRINCIPALI d'INERZIA





$\overline{L}_\perp = \Sigma_j \left(-m_j z_j \overline{R}_j\right) \omega$

$\overline{R}_i = -\overline{R}_j$

$z_i = z_j$

$m_i = m_j = m$

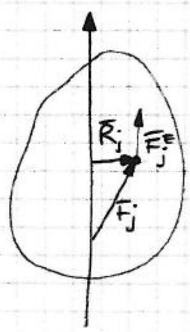
• EQUAZIONE del MOTO ROTAZIONALE del CORPO RIGIDO

$$\frac{d\overline{L}}{dt} = \overline{M}^E$$

1) se $\overline{L} /\!/ \overline{\omega}$ $\quad (\overline{L}_\perp = 0)$

$$\frac{d\overline{L}}{dt} = \frac{d}{dt}\left(I_z \overline{\omega}\right) = I_z \underbrace{\frac{d\omega}{dt}}_{\overline{\alpha}} \longrightarrow I_z \overline{\alpha} = \overline{M}^E$$

$$\overline{M}^E = \Sigma_j \, r_j \wedge \overline{F}_j^E$$



Analizziamo le forze lungo i vari assi:

$x: \; R_x = T\cos\frac{\pi}{6}$

$y: \; R_y = Mg - T\sin\frac{\pi}{6}$

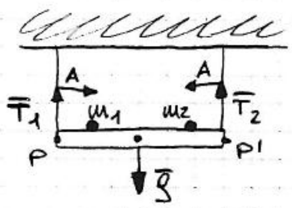
Ricordando che il momento è $M = Fr\sin\alpha$

$M = \underbrace{mg\frac{l}{2}\cos\frac{\pi}{6}}_{\text{momento in P}} + \underbrace{Tl\cos\frac{\pi}{6}}_{\text{momento in B}} = 0$ da qui trovo T

iona $T = \frac{1}{2}Mg$

$R_x = \frac{\sqrt{3}}{4}Mg \quad R_y = \frac{3}{4}Mg \longrightarrow R = \frac{\sqrt{3}}{2}Mg$

es. 2



Hp: $l = 3\,m$ Th: T_1, T_2

$A = 0{,}5\,m$

$M = 100\,Kg$

$m_1 = 50\,Kg$

$m_2 = 150\,Kg$

Il sistema è in equilibrio, quindi:

$M\bar{g}$ $\sum_j \bar{F}^E = M\bar{A}_{cm} = 0$ forze $= 0$

$\sum_j F_j \wedge \bar{F}_j^E = M^E = 0$ momenti $= 0$

$\downarrow$

) forze: $T_1 + T_2 - (m_1 + m_2 + M)g = 0$

, momenti (considero P come punto di rotazione):

$LT_2 - Mg\frac{l}{2} - m_1 g A - m_2 g(L-A) = 0$ da qui trovo T_2, sostituisco sopra e trovo T_1

g,

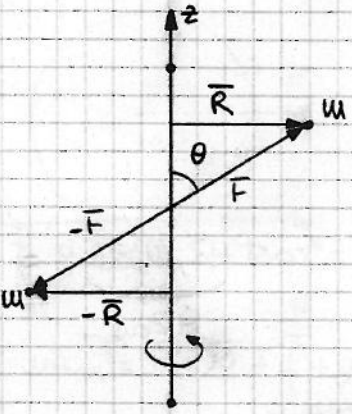
e

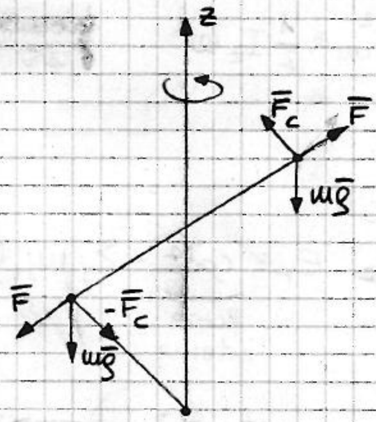
$$\overline{L} = m\bar{r}\wedge\bar{v} + m(-\bar{r})\wedge(-\bar{v}) = 2m\bar{r}\wedge\bar{v} = 2mr\underbrace{v}_{\omega r}\,\hat{u}_z = 2mr^2\omega\,\hat{u}_z = I_z\,\bar{\omega}$$

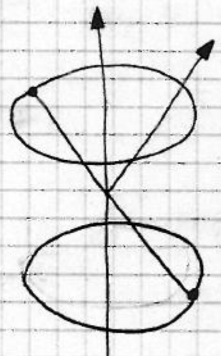
$$\overline{M} = \bar{r}\wedge(\bar{F} + m\bar{g}) + (-\bar{r})\wedge(-\bar{F} + m\bar{g}) = 2\bar{r}\wedge F = 2rF\,\hat{u}_z$$

$\downarrow$

$$\frac{d\overline{L}}{dt} = \overline{M}^E \longrightarrow I_z\,\omega' = 2rF$$







$$\overline{L} = 2m\bar{r}\wedge\bar{v} \qquad \bar{v} = \bar{\omega}\wedge\bar{R}$$

$$= 2m(\bar{R} + z\hat{u}_z)\wedge\bar{v} = 2mRv\,\hat{u}_z + 2mz\,\hat{u}_z\wedge\bar{v} = \underbrace{2mRv\,\hat{u}_z}_{I_z\bar{\omega}} - 2mz\omega\bar{R}$$

$\downarrow$

$$\overline{L} = I_z\bar{\omega} + \overline{L}_\perp \quad , \quad \overline{L}_\perp = -2mz\omega\bar{R}$$

$$\overline{M} = \bar{r}\wedge(\cancel{m\bar{g}} + \bar{F}_c + \bar{F}) + (-\bar{r})\wedge(\cancel{m\bar{g}} - \bar{F}_c - \bar{F}) = 2\bar{r}\wedge(\underbrace{\bar{F}}_{\overline{M}^E} + \underbrace{\bar{F}_c}_{\overline{M}_{VIN}})$$

$\downarrow$

$$\frac{d\overline{L}}{dt} = \overline{M}^E_{TOT} \longrightarrow I_z\frac{d\bar{\omega}}{dt} + \frac{d\overline{L}_\perp}{dt} = \overline{M}^E + \overline{M}_{VIN}$$

$\downarrow$

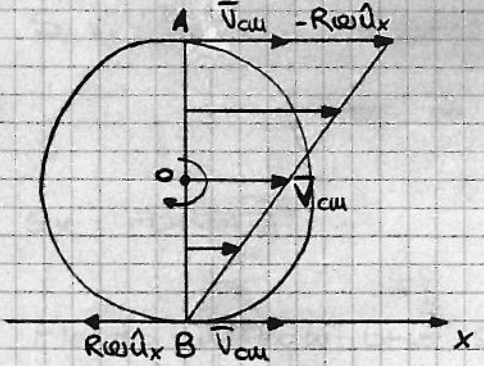
$\omega = cost$

$\overline{M}^E = 0$

$$\frac{d\overline{L}_\perp}{dt} = \overline{M}_{VIN} \longrightarrow \frac{d}{dt}\left(-2m\omega\bar{R}\right) = 2z\,\hat{u}_z\wedge\bar{F}_c$$

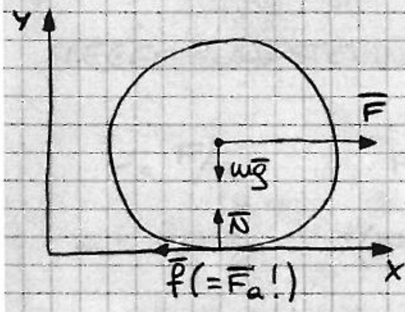
$$-\cancel{2}m\cancel{z}\omega\frac{d\bar{R}}{dt} = \cancel{2}\cancel{z}\,\hat{u}_z\wedge\bar{F}_c \qquad -m\omega(\bar{\omega}\wedge\bar{R}) = \hat{u}_z\wedge\bar{F}_c$$

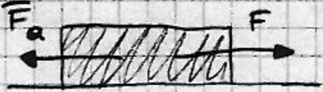
$$-m\bar{\omega}^2R\,(\hat{u}_z\wedge\hat{u}_R) = F_c\,\hat{u}_z\wedge\hat{u}_R \longrightarrow F_c = -m\omega^2R$$



$$A)\ \overline{V}_{cm} - R\omega\hat{u}_x = \underbrace{-R\omega\hat{u}_x}_{\overline{V}_{cm}} - R\omega\hat{u}_x = -2R\omega\hat{u}_x = 2R|\omega|\hat{u}_x$$

$$B)\ \overline{V}_{cm} + R\omega\hat{u}_x = -R\omega\hat{u}_x + R\omega\hat{u}_x = 0$$





$$\overline{F}_a = F < \mu_s N$$

$$N = mg$$

$$\begin{cases} m\overline{a}_{cm} = \overline{F} + \cancel{\overline{N}} + \overline{f} + \cancel{mg} \\ \\ \dfrac{d\overline{L}}{dt} = \overline{M}^E \longrightarrow \overline{F} \wedge \overline{f} \end{cases}$$

$$\overline{F} = F\hat{u}_x \quad , \quad \overline{f} = -f\hat{u}_x \quad , \quad \overline{a}_{cm} = a_{cm}\hat{u}_x$$

$$\overline{L} = I_{cm}\overline{\omega} \quad , \quad \overline{\omega} = \omega\hat{u}_z$$

$$\downarrow$$

$$\begin{cases} ma_{cm} = F - f \\ \\ I_{cm}\dfrac{d\omega}{dt}\hat{u}_z = -r + \hat{u}_z \end{cases} = \begin{cases} ma_{cm} = F - f \\ \\ I_{cm}\dfrac{d\omega}{dt} = -rf \end{cases}$$

$$V_{cm} = -R\omega$$

$$a_{cm} = -R\alpha$$

$$\alpha = -\frac{a_{cm}}{R}$$

$$R = r$$

$$\downarrow$$

$$\begin{cases} ma_{cm} = F - f \\ \\ I_{cm}\dfrac{a_{cm}}{r} = rf \end{cases} \longrightarrow \begin{matrix} ma_{cm} = F - I_{cm}\dfrac{a_{cm}}{r^2} \\ \\ \left(m + \dfrac{I_{cm}}{r^2}\right)a_{cm} = F \end{matrix} \longrightarrow \begin{cases} a_{cm} = \dfrac{r^2 F}{I_{cm} + mr^2} \\ \\ f = \dfrac{I_{cm}a_{cm}}{r^2} = \dfrac{I_{cm}F}{I_{cm} + mr^2} \end{cases}$$

$\longrightarrow$

$S_A v_A = S_B v_B$ LEGGE di LEONARDO

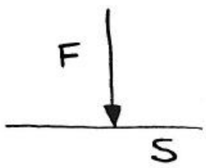
eq. di CONTINUITÀ

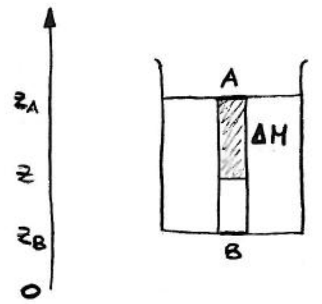
Sv = PORTATA

Fluido in equilibrio statico = fluido in quiete, cioè tale che ogni elemento ha velocità e accelerazione nulla

LEGGE di STEVINO in un fluido con $\rho = cost$

$P = F/S$





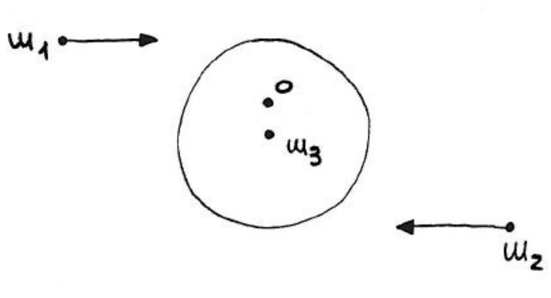
$$P_B = P_A + \frac{Mg}{\Delta S_B} = P_A + \frac{g}{\cancel{\Delta S_B}}\left[\rho \cancel{\Delta S_B}(z_A - z_B)\right]$$

$$P_B = P_A + \rho g (z_A - z_B)$$

$$P(z) = P_A + \frac{\Delta M(z) g}{\Delta S} = P_A + \frac{g}{\Delta S}\left[\rho \Delta S (z_A - z)\right]$$

$$P(z) = P_A + \rho g (z_A - z)$$

es. ALENIA



Hp: $m_3 = 2,5$ kg
$R = 0,3$ m
$m_1 = 2$ kg
$m_2 = 0,3$ kg
$v = 4$ m/s

$$V_{cm} = \frac{m_1 v_1 - m_2 v_2}{m_1 + m_2 + m_3}$$

$$L = m_1 v_1 R_1 + m_2 v_2 R_2$$

$$L = I_z \omega$$

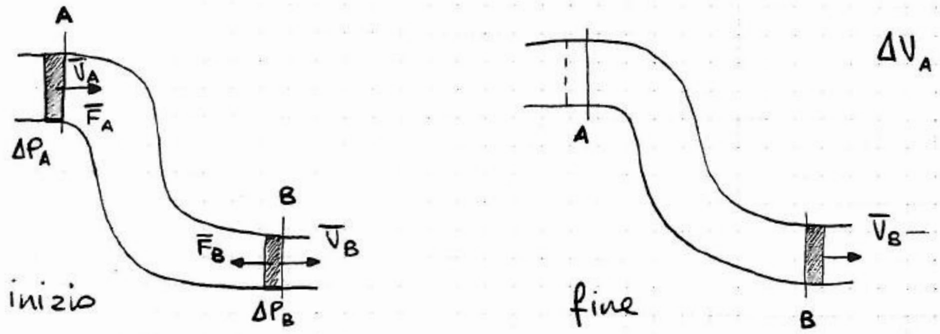
UNITÀ di MISURA

1 Pascal (Pa) = 1 N / 1 m²

1 bar = 10^5 Pa

1 atm = 1,01325 bar

• TEOREMA di BERNULLI



$\Delta V_A = \Delta V_B$

$\Delta M_A = \rho \Delta V_A = \rho S_A \Delta P_A$

$\Delta M_B = \rho \Delta V_B = \rho S_B \Delta P_B$

$\Delta W' = \Delta E_p + \Delta E_K = (E_p^{FIN} - E_p^{IN}) \longrightarrow$

$\longrightarrow = \left(\Delta M_B\, g z_B - \Delta M_A\, g z_A\right) + \left(\frac{1}{2}\Delta M_B V_B^2 - \frac{1}{2}\Delta M_A V_A^2\right)$

$\Delta W' = \overline{F}_A \cdot \Delta\overline{l}_A + \overline{F}_B \cdot \Delta\overline{l}_B = P_A S_A \Delta l_A - P_B S_B \Delta l_B$

$\downarrow$

$g(z_B - z_A)\Delta M_A + \frac{1}{2}(V_B^2 - V_A^2)\Delta M_A = (P_A - P_B)\Delta V_A$ $\qquad \Delta M_A = \rho \Delta V_A$

$\downarrow$

$P_A - P_B = \rho g (z_B - z_A) + \frac{\rho}{2}(V_B^2 - V_A^2) \longrightarrow P_A + \rho g z_A + \frac{\rho}{2}V_A^2 = P_B + \rho g z_B + \frac{\rho}{2}V_B^2$

$\downarrow$

$\forall$ punto sulla linea di corrente

$$P + \rho g z + \frac{1}{2}\rho V^2 = cost$$

7) Un sistema è in equilibrio termodinamico se valgono le 3 condizioni

- eq. MECCANICO
- eq. TERMICO
- eq. CHIMICO

8) EQUAZIONE di STATO : per tutti i sistemi termodinamici in equilibrio esiste un'eq. di stato che vincola le variabili di stato termodinamiche

GAS :
- $P = P(V,T)$
- $V = V(P,T)$
- $T = T(P,V)$.

- TEMPERATURA EMPIRICA e ASSOLUTA

Empirica, in quanto basata su un fenomeno fisico

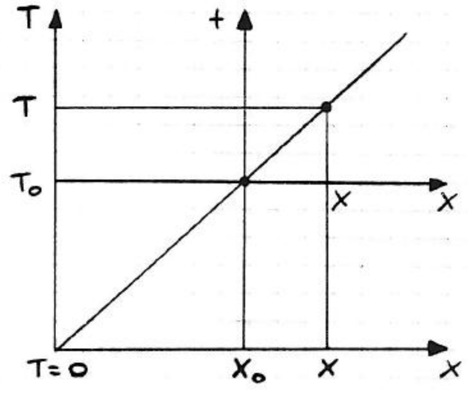
TERMOMETRO:

- LIQUIDO misura la dilatazione termica
- RESISTENZA misura la variazione di resistenza in un conduttore
- GAS misura la variazione di pressione

In generale ho uno strumento che misura $t = \theta(x)$ tramite la misura di x (caratteristica termometrica)

$\theta(x)$ funzione TERMOMETRICA

TEMPERATURA ASSOLUTA (gradi Kelvin)



$$\frac{x}{T} = \frac{x_0}{T_0} \longrightarrow T = T_0 \frac{x}{x_0}$$

si assume $T_0 = 273,16\ K$

$$\frac{P_A V_A}{T_A} = \frac{P_C V_C}{T_C} = C \longrightarrow \frac{PV}{T} = C \longrightarrow \begin{cases} PV = TC \\ V = \left(\frac{C}{P}\right)T \\ P = \left(\frac{C}{V}\right)T \end{cases}$$

$PV = CT \qquad C = nR \longrightarrow PV = nRT$

$R = 8{,}314 \frac{J}{K \cdot mol}$

$$\longrightarrow PV = \underbrace{nN_A}_{N} \underbrace{\frac{R}{N_A}}_{k} T \longrightarrow PV = NKT$$

$N_A = 6{,}022 \cdot 10^{23}$

$K = 1{,}38 \cdot 10^{-23} \frac{J}{K}$ COSTANTE di BOLTZMANN

↓ come si determina R

$n = 1\,mol \qquad T = T_0 = 273{,}16\,K \qquad P = 1\,atm = 1{,}01325 \cdot 10^5\,Pa \qquad V = 22{,}414\,l =$
$= 0{,}022414\,m^3$

$$R = \frac{PV}{nT_0} = 8{,}3144$$

- TEORIA CINETICA dei GAS

1) Trovare la relazione tra T e l'energia cinetica media

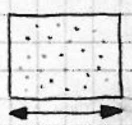
2) Definire l'energia interna a V

① $E_k^{TOT} = \sum_j \frac{m_j}{2} \overline{v}_j^2 \qquad \langle v^2 \rangle$ velocità quadratica media

$$\overline{E}_k = \langle E_k \rangle = \frac{E_k^{TOT}}{N} = \frac{1}{2} m \underbrace{\left(\frac{1}{N} \sum_j \overline{v}_j^2\right)}_{\langle v^2 \rangle} \qquad \langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle = \frac{1}{3} \langle v^2 \rangle$$

↓

$$\langle v^2 \rangle = \frac{1}{N} \sum_j v_{jx}^2 + \frac{1}{N} \sum_j v_{jy}^2 + \frac{1}{N} \sum_j v_{jz}^2 = \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle$$



L

$\Delta t = \frac{2L}{v_{jx}} \longrightarrow$ forza media esercitata nel tempo da una particella

$\Delta P_j = 2 m v_{jx}$

$$f_{jx} = \frac{\Delta P_j}{\Delta t} = \frac{2 m v_{jx}}{\frac{2L}{v_{jx}}} = \frac{m v_{jx}^2}{L} \longrightarrow$$