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A P P U N T I

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MATERIA : Gasdinamica
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GASDINAMICA

· TEORIA CINETICA E TERMODINAMICA

• **LIBERO CAMMINO MEDIO**: distanza media percorsa da una particella tra due urti

$\ell = \frac{1}{d^2 \eta \pi \sqrt{2}}$ dove d = diametro particelle, η = numero di particelle; $\eta = \frac{Na}{Volume}$

• **VELOCITÀ** $V = \bar{V} + V'$ la velocità di un fluido è somma della sua velocità media d'insieme $\bar{V}$ (velocità macroscopica) e da una fluttuazione locale dovuta all'agitazione termica V' ($o\ V_{at}$)

• **VELOCITÀ DI AGITAZIONE TERMICA**

$V_{at} = \sqrt{2 E_t \frac{1}{m}}$ dove E_t = energia traslazionale media, m = massa di gas coinvolto

• **ENERGIA TRASLAZIONALE MEDIA**

$E_t = \frac{1}{2} m (\overline{uu} + \overline{vv} + \overline{ww})$ $\qquad E_{t\,caotico} = \frac{1}{2} m (\overline{u'u'} + \overline{v'v'} + \overline{w'w'})$

• legame con la temperatura: $E_t = \frac{3}{2} K_b \cdot T$ dove $K_b = 1{,}38 \cdot 10^{-23}\ J/K$ costante di boltzmann.

• **VELOCITÀ DEL SUONO**

$a = \sqrt{\gamma \frac{R}{M} T}$ $\qquad R = K_b \cdot Na = 8314\ J/KmolK$ = COSTANTE DEI GAS, $M = m \cdot Na$ = MASSA MOLARE, $\gamma = C_p/C_v$

• **IPOTESI DI FLUIDO CONTINUO**

$Kn = \frac{\ell}{L}$ n° di Knudsen; esprime il confronto tra il libero cammino medio e la lunghezza caratteristica del corpo

$Kn < 0{,}01$ FLUIDO CONTINUO
$Kn < 0{,}1$ SLIP FLOW
$Kn < 10$ FLUIDO MODERATAMENTE RAREFATTO
$Kn > 10$ FLUIDO ALTAMENTE RAREFATTO

• **EQUIPARTIZIONE DELL'ENERGIA**
data una molecola con L gradi di libertà l'energia complessiva $\bar{E}_L$ è data dalla somma delle energie spettanti a ogni singolo gdl

$\bar{\varepsilon} = \frac{1}{2} K_b T$ $\qquad \bar{E}_L = \Sigma \bar{\varepsilon} = L \cdot \frac{1}{2} K_b \cdot T$

• **CALORI SPECIFICI**

$$c_v = \left(\frac{dq}{dT}\right)_{V\,cost} = \left(\frac{de}{dT}\right) = \frac{L}{2} \cdot \frac{R}{M} \qquad c_v = \frac{\gamma}{\gamma - 1} \cdot \frac{R}{M}$$

$$c_p = \left(\frac{dq}{dT}\right)_{P\,cost} = \left(\frac{dh}{dT}\right) = \frac{L+2}{2} \frac{R}{M} \qquad c_p = \frac{1}{\gamma - 1} \cdot \frac{R}{M}$$

dove $\gamma = \frac{c_p}{c_v} = \frac{L+2}{2}$

$R/M = c_p - c_v$

• **LEGGE DI BERNOULLI**

$dp = -\rho V dV$ (forma differenziale) $\longrightarrow$ $P_0 = P \cdot \frac{1}{2} \rho V^2$ (caso incomprimibile)

$\frac{dp}{d\rho} = -\rho v \frac{dv}{d\rho} = a^2$ dove a = velocità del suono e $M = \frac{V}{a}$ = n° di MACH e $a^2 = \left(\frac{\partial p}{\partial \rho}\right)$

$\frac{d\rho}{\rho} = -M^2 \frac{dV}{V}$

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FLUSSO DI ENERGIA

- LEGGE DI FOURIER: $q_x = -\lambda \frac{\partial T}{\partial x}$ si ha un flusso di energia in presenza di un gradiente termico
- NUMERO DI PRANDTL

$$Pr = \frac{\mu c_p}{\lambda} = \frac{\nu}{K} \quad \text{dove} \quad K = \frac{\lambda}{\rho c_p}$$ è detta diffusività termica.

$Pr = \frac{\text{DIFFUSIVITÀ VISCOSA}}{\text{DIFFUSIVITÀ TERMICA}}$, Pr indica anche il rapporto delle estensioni degli strati limite viscoso δ e termico δ_T

- FORMULA DI EUCKEN $Pr = \frac{2L+4}{2L+9} = \frac{4\gamma}{9\gamma-5}$

FLUSSO DI MASSA

$J_x = -D \frac{ds}{dx}$ → LEGGE DI FICK

il trasporto di massa avviene seguendo il gradiente di concentrazione di una certa specie S. (ds/dx = gradiente lungo x)

TERMODINAMICA

- Eq. di STATO → $p \cdot V = n \cdot R/M \; T$ → $p \cdot V = NRT$

1° PRINCIPIO

$$de = \delta q + \delta L$$

- Sia il sistema considerato in equilibrio, se si fornisce calore o si compie lavoro, se ne varia l'energia interna.

da cui: $(\delta L)_{rev} = -p\,dv$; $de = \delta q - p\,dv$

in forma entalpica $dh = \delta q + V\,dp$

2° PRINCIPIO

s = entropia per unità di massa (FUNZIONE DI STATO)

$$ds = \frac{dq}{T} + ds_{irrev.} \quad \text{dove} \quad ds_{irrev} \geq 0$$

A causa dei fenomeni dissipativi, tutti i processi avvengono a entropia crescente.

I + II
$$T\,ds = de + p\,dv$$
$$T\,ds = dh + v\,dp$$

da un punto di vista lagrangiano: $T \frac{Ds}{Dt} = \frac{De}{Dt} + p\frac{Dv}{Dt}$

- CALCOLO DELL'ENTROPIA

$$ds = c_p \cdot \frac{dT}{T} - \frac{R}{M} \cdot \frac{dp}{p} \longrightarrow s_2 - s_1 = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - \frac{R}{M}\ln\left(\frac{p_2}{p_1}\right)$$

$$ds = c_v \cdot \frac{dT}{T} + \frac{R}{M} \cdot \frac{dv}{v} \longrightarrow s_2 - s_1 = c_v \cdot \ln\left(\frac{T_2}{T_1}\right) + \frac{R}{M}\ln\left(\frac{v_2}{v_1}\right)$$

EQ. DELL'ENTROPIA

- PROCESSI ISOENTROPICI

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/\gamma-1} ; \quad \frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{1/\gamma-1} ; \quad \frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} ; \quad \left(\frac{p}{\rho}\right)^{\gamma} = \text{cost.}$$

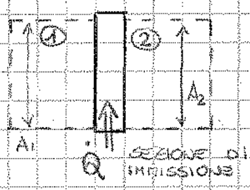
RIPASSO SU OPERATORI

- $\nabla = \frac{\partial}{\partial x}\bar{i} + \frac{\partial}{\partial y}\bar{j} + \frac{\partial}{\partial z}\bar{k}$
- Prodotto scalare: $\bar{a}\cdot\bar{b} = a\cdot b\cos\alpha = a_x b_x + a_y b_y + a_z b_z$
  $\bar{a}\cdot\bar{a} = |a^2|$
- Prodotto vettorale: $\bar{a}\times\bar{b} = a\cdot b\cdot\sin\alpha = (a_y b_z - a_z b_y)\bar{i} + \ldots\bar{j} + \ldots\bar{k}$
- GRADIENTE $\nabla s = \frac{\partial s}{\partial x}\bar{i} + \frac{\partial s}{\partial y}\bar{j} + \frac{\partial s}{\partial z}\bar{k}$ il gradiente di uno scalare è un vettore che indica la direzione in cui è MASSIMA LA VARIAZIONE DELLO SCALARE
- DIVERGENZA $\nabla\bar{v} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$ la divergenza di un vettore fornisce uno scalare che indica la tendenza del flusso a entrare/uscire
- ROTORE $\nabla\times\bar{V} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\bar{i} + \left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right)\bar{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\bar{k}$

  il rotore è legato alla velocità angolare $\nabla\times\bar{V} = 2\bar{\omega} = 2\omega_x\bar{i} + 2\omega_y\bar{j} + 2\omega_z\bar{k}$

FLUSSI UNIDIMENSIONALI 1D

SI CONSIDERA UN VOLUME DI CONTROLLO ARBITRARIO FISSO, INDEFORMABILE ED EULERIANO.
CONSIDERIAMO UN FLUSSO:
- STAZIONARIO ($\partial/\partial t = 0$)
- SENZA FORZE DI MASSA ($\bar{F} = 0$)
- NON VISCOSO ($\bar{V} = u\bar{i}$)
- UNIFORME (PROPRIETÀ COSTANTI LUNGO LA SEZ. TRASVERSALE



PER LE IPOTESI FATTE POSSO RISCRIVERE LE EQ. DI BILANCIO

- MASSA $\rho_1 u_1 = \rho_2 u_2$
- QDM $p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$
- ENERGIA $q + h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$

PROPAGAZIONE DI PICCOLE PERTURBAZIONI

LA PROPAGAZIONE DEL SUONO E' UN PROCESSO ADIABATICO e REVERSIBILE → ISOENTROPICO.

- dalla continuità ricavo $a = -\rho\cdot\frac{da}{d\rho}$
- dalla Q.d.M $\frac{da}{d\rho} = \frac{dp/d\rho + a^2}{-2a\rho}$

da cui ottengo: $a^2 = \left(\frac{dp}{d\rho}\right)_{S=cost} \Rightarrow \sqrt{\gamma\frac{p}{\rho}}$

OSSIA LE PICCOLE PERTURBAZIONI ISOENTROPICHE SI PROPAGANO ALLA VELOCITÀ DEL SUONO

URTO RETTO

dato un pistone in un fluido con condizioni iniziali T_i, p_i, a_i uguali sia in I che in II. Supponiamo di imprimere al pistone un impulso infinitesimo du.

I: $p_I + dp$, $T_I + dT$, $V_I = a_i$
II: $p_{II} - dp$, $T_{II} - dT$, $V_{II} = a_i$

incrementando ancora la velocità si TROVA CHE LE ONDE DI PERTURBAZIONE GENERATE AVRANNO VELOCITÀ:

$V_{I_2} = \sqrt{\gamma\cdot R/M\,(T_i + dT)} + du > a_i$

$V_{II_2} = \sqrt{\gamma\, R/M\,(T_i + dT)} - du < a_i$

LE ONDE DI ESPANSIONE SUCCESSIVE (II) SARANNO SEMPRE PIÙ LENTE E TENDERANNO A DISTANZIARSI, MENTRE LE ONDE DI COMPRESSIONE (I) RISULTANO PIÙ VELOCI E TENDONO AD IMPACCHETTARSI IN UN URTO.

FLUSSO DI RAYLEIGH:

CAMPO 1D
$q \neq 0$ → scambio di calore
$\tau_v = 0$ → No attriti.

EQUAZIONI:
① - CONTINUITÀ $\rho_1 u_1 = \rho_2 u_2$
② - QdM $P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2$
③ - ENERGIA $q + H_1 = H_2$
④ - $\frac{P}{\rho} = \frac{R}{M} T$

• Si assegnano le condizioni nel campo I, si fornisce calore q. Vogliamo valutare le condizioni nel campo II.

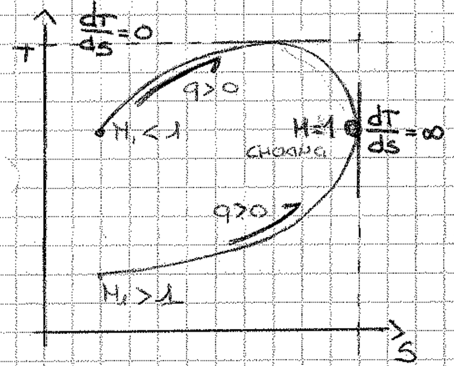
da ①, $H^0 = c_p \cdot T^0 \Rightarrow H_1^0 + q = H_2^0 \rightarrow T_2^0 = T_1^0 + \frac{q}{c_p}$

da ②, $\rho u^2 = \rho a^2 M^2 = \gamma P M^2 \rightarrow P_1 + \gamma P_1 M_1^2 = P_2 + \gamma P_2 M_2^2 \rightarrow \frac{P_2}{P_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}$

si ricavano

$$\frac{P_2}{P_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \; ; \quad \frac{\rho_2}{\rho_1} = \frac{1 + \frac{\gamma - 1}{2} M_1^2}{1 + \frac{\gamma - 1}{2} M_2^2} \cdot \left(\frac{M_1}{M_2}\right)^2 \; ; \quad \frac{T_2}{T_1} = \left(\frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}\right) \cdot \left(\frac{M_1}{M_2}\right)^2$$

$$S_2 - S_1 = c_p \ln\left[\frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \cdot \left(\frac{M_1}{M_2}\right)^2\right] - \frac{R}{M} \ln\left[\frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}\right]$$



$$\frac{dT}{ds} = c_v \cdot T \cdot \frac{(\gamma M^2 - 1)}{(M^2 - 1)\gamma}$$

$$\frac{du}{ds} = -\frac{c_v u}{\gamma (M^2 - 1)}$$

• INGRESSO SUBSONICO $M_1^2 < 1$,

$\frac{dT}{ds} > 0$ se $\gamma M_1^2 - 1 < 0 \Rightarrow M < 1/\sqrt{\gamma} = 0,85$

$\frac{dT}{ds} < 0$ se $\gamma M_1^2 - 1 > 0 \Rightarrow M_1 > 1/\sqrt{\gamma} = 0,85$

$\frac{du}{ds} > 0$ FORNENDO CALORE LA VELOCITÀ AUMENTA (u crescente)

• INGRESSO SUPERSONICO $M^2 > 1$

$\frac{dT}{ds} > 0 \quad \frac{du}{ds} < 0$ FORNENDO CALORE IL FLUSSO RALLENTA PORTANDOSI A $M = 1$

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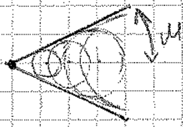
ONDE OBLIQUE IN FLUSSI SUPERSONICI

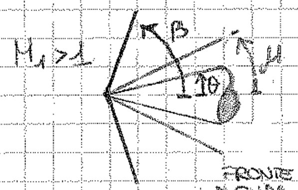
STUDIAMO UNA SORGENTE SONORA IN MOVIMENTO CON $V_s > a$



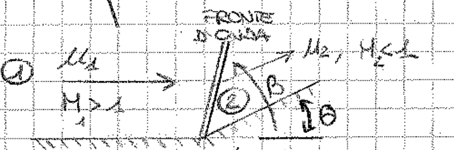
LE LINEE INDIVIDUATE DAI FRONTI D'ONDA PRENDONO IL NOME DI LINEE DI MACH, E FORMANO IL CONO DI MACH
μ = SEMIAPERTURA DEL CONO DI MACH

PER LE PICCOLE PERTURBAZIONI IL PROCESSO È ISOENTROPICO $\Rightarrow T_1^0 = T_2^0,\ p_1^0 = p_2^0$

GRANDI PERTURBAZIONI

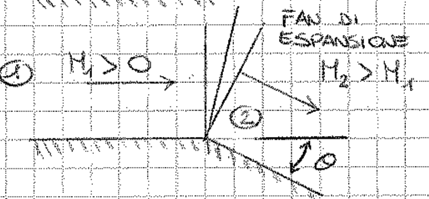


dove θ = Semiapertura del cono
μ = " " del cono di Mach
β = " " del cono d'urto



CAMPO ①
- uniforme
- parallelo alla parete

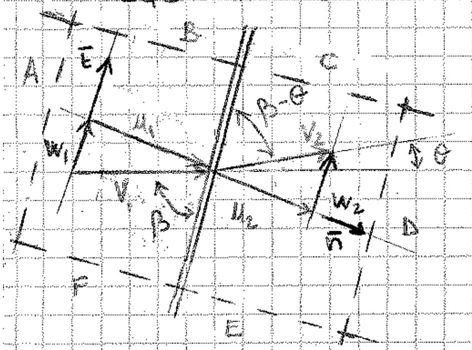
CAMPO ②
- uniforme
- deflesso di θ
- PARALLELO ALLA PARETE



- occorre rispettare la condizione di tangenza

COMPRESSIONE $\rightarrow M_2 < M_1,\ \ p_2 > p_1,\ \ \rho_2 > \rho_1,\ T_2 > T_1$
ESPANSIONE $\rightarrow M_2 > M_1,\ \ p_2 < p_1,\ \ \rho_2 < \rho_1,\ T_2 < T_1$

- EQUAZIONI DELL'URTO OBLIQUO



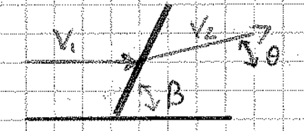
$$\begin{cases} \vec{V_1} = \vec{u_1} \cdot \vec{W_1} \\ \vec{V_2} = \vec{u_2} \cdot \vec{W_2} \end{cases}$$

- FLUSSO 2D $(\vec{V} = u\vec{n} + W\vec{t})$
- " " STAZIONARIO
- " " INVISCIDO
- " " ADIABATICO

ADOTTO IL VOLUME DI CONTROLLO $\overline{ABCDEF}$

- CONTINUITÀ $\int_S \rho(\vec{V} \cdot \vec{n})\, dS \Rightarrow \rho_1 u_1 = \rho_2 u_2$
- Q.D.M.: direzione NORMALE (N) $\Rightarrow p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$
  direzione TANGENZIALE (T) $\Rightarrow \rho_1 u_1 W_1 = \rho_2 u_2 W_2 \Rightarrow W_1 = W_2$
- ENERGIA: $h_1 + u_1^2/2 = h_2 + u_2^2/2$

- URTO OBLIQUO



θ = deflessione della corrente
β = inclinazione dell'urto

$$M_{1N} = \frac{u_1}{a_1} = \frac{V_1 \sin\beta}{a_1} = M_1 \sin\beta$$

considero solo la direz. N poiché per T, $W_1 = W_2$.

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma - 1}\left[M_1^2 \sin^2\beta - 1\right] \quad e \quad \frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{(\gamma+1) M_1^2 \sin^2\beta}{2 + (\gamma-1) M_1^2 \sin^2\beta} \quad e \quad \frac{T_2}{T_1} = \frac{p_2}{p_1} \cdot \frac{\rho_1}{\rho_2}$$

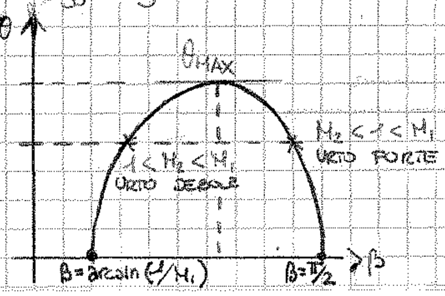
$$M_2 < M_1, \quad M_2 = \frac{1}{\sin^2(\beta-\theta)} \cdot \frac{2 + (\gamma-1) M_1^2 \sin^2\beta}{2\gamma M_1^2 \sin^2\beta - (\gamma-1)}$$

- A MONTE $\ \mathrm{tg}(\beta) = u_1/w_1$
- A VALLE $\ \mathrm{tg}(\beta-\theta) = u_2/w_2$

$$\Rightarrow \frac{\mathrm{tg}(\beta-\theta)}{\mathrm{tg}(\beta)} = \frac{u_2}{u_1} = \frac{\rho_1}{\rho_2} = \frac{2 + (\gamma-1) M_1^2 \sin^2\beta}{(\gamma+1) M_1^2 \sin^2\beta}$$

ricaviamo θ con alcuni passaggi trigonometrici

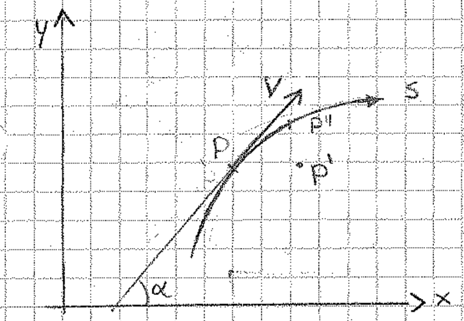
$$\mathrm{tg}\,\theta = \frac{2}{\mathrm{tg}\beta}\left[\frac{M_1^2 \sin^2\beta - 1}{M_1^2(\gamma + \cos 2\beta) + 2}\right]$$

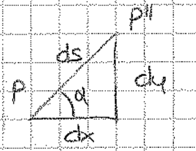


dato un qualunque valore $\theta < \theta_{max}$ sono possibili 2 casi. La scelta tra urto forte e urto debole è funzione della pressione a valle.

PER $\theta > \theta_{MAX}$ HO UN URTO CURVO, FORTE ALLA BASE CHE DIMINUISCE VERSO L'ESTERNO.

• **LINEE CARATTERISTICHE**





$\frac{dy}{dx} = m = tg\,\alpha$ pendenza di S nota.

V velocità punti di S $(u\hat{i} + v\hat{j}) = \bar{V}$

supponendo il flusso irrotazionale: $\frac{\partial u}{\partial x} = \frac{\partial^2 \Phi}{\partial x^2}$; $\frac{\partial v}{\partial y} = \frac{\partial^2 \Phi}{\partial y^2}$; $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} = \frac{\partial \Phi^2}{\partial x \partial y}$

$u(P') = u(P) + \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy$ $\qquad \frac{dx}{ds} = \cos\alpha$; $\frac{dy}{ds} = \sin\alpha$

$v(P') = v(P) + \frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy$

$A_x = \frac{du}{ds} = \frac{\partial u}{\partial x}\frac{dx}{ds} + \frac{\partial u}{\partial y}\frac{dy}{ds}$;

$A_y = \frac{dv}{ds} = \frac{\partial v}{\partial x}\frac{dx}{ds} + \frac{\partial v}{\partial y}\frac{dy}{ds}$;

$$\begin{cases} B_x = \frac{A_x}{\cos\alpha} = \frac{\partial^2 \Phi}{\partial x^2} + m \frac{\partial^2 \Phi}{\partial x \partial y} \\ B_y = \frac{A_y}{\cos\alpha} = \frac{\partial^2 \Phi}{\partial x \partial y} + m \frac{\partial^2 \Phi}{\partial y^2} \\ H \frac{\partial^2 \Phi}{\partial x^2} + 2K \frac{\partial^2 \Phi}{\partial x \partial y} + L \frac{\partial^2 \Phi}{\partial y^2} \end{cases}$$

EQ. DEL POTENZIALE PER VELOCITÀ

dove: $H = \left(\frac{u^2}{a^2} - 1\right)$, $L = \left(\frac{v^2}{a^2} - 1\right)$ $\qquad 2K = \frac{2uv}{a^2}$

• risolvendo il sistema trovo la velocità in ogni punto dell'intorno di $\bar{S}$
IL SISTEMA NON HA SOLUZIONE SE: $L - 2Km - Hm^2 = 0$
ossia quando la tangente al punto P ha tangente $\boxed{m}$, ossia
è DIREZIONE CARATTERISTICA.
SE LA TANGENTE E' SEMPRE DIREZIONE CARATTERISTICA, LA LINEA S SI DICE **LINEA CARATTERISTICA**.

$\Delta = M^2 - 1$ → discriminante dell'equazione

• Se la corrente è supersonica $(M^2 > 1)$, $\Delta > 0$, l'equazione ammette due soluzioni, pertanto vi saranno DUE DIREZIONI CARATTERISTICHE. le cui tangenti sono inclinate di un angolo $\pm\mu = \arcsin(1/M)$

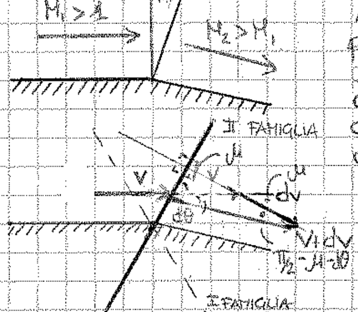
• In caso di corrente sonica $(M = 1)$ ho un unica soluzione, con $\mu = 90°$

• Per correnti subsoniche $(M^2 < 1)$ l'equazione non ammette soluzioni e conseguentemente nemmeno direzioni caratteristiche.

RELAZIONE FONDAMENTALE:

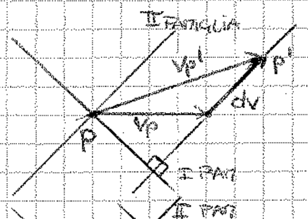
• ESPANSIONE ISOENTROPICA

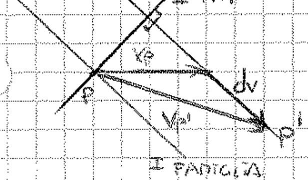
$d\theta = \pm\sqrt{M^2 - 1} \cdot \frac{dV}{V}$



Nel caso dell'espansione per piccole deflessioni, si ha che la variazione dv è ortogonale alla direzione caratteristica della II famiglia.

Se la deviazione fosse opposta sarebbe ortogonale alla direzione della I° FAMIGLIA





Voglio $\frac{dV}{V} f\left(\frac{dM}{M}\right)$ → $\frac{dV}{V} = \frac{dM}{M} \cdot \frac{da}{a}$

$\frac{da}{a}$: $\left(\frac{a^0}{a}\right)^2 = \frac{T^0}{T} = \left(1 + \frac{\gamma - 1}{2} M_\infty^2\right)$ → $a = a_0 \cdot \left(1 + \frac{\gamma-1}{2} M_\infty^2\right)^{-1/2}$

$da = a_0 \left[-\frac{1}{2}\left(1 + \frac{\gamma-1}{2} M_\infty^2\right)^{-\frac{3}{2}} \cdot (\gamma - 1) M dM\right]$

↳ $\frac{da}{a} = -\left(\frac{\gamma-1}{2}\right)\left(1 + \frac{\gamma-1}{2}\right)^{-1} M dM$

$\frac{dV}{V} = \frac{dM}{M} \cdot \left(1 / 1 + \frac{\gamma - 1}{2} M^2\right)$ → $d\theta = \pm \frac{\sqrt{M^2 - 1}}{\left(1 + \frac{\gamma-1}{2} M^2\right)} \cdot \frac{dM}{M}$

INTEGRANDO OTTENGO LA DEFLESSIONE $\nu(M)$

$$\nu(M) = \theta + cost = \left[\sqrt{\frac{\gamma+1}{\gamma-1}} \, arctg\left(\sqrt{\frac{\gamma-1}{\gamma+1}} \sqrt{M^2 - 1}\right) - arctg\left(\sqrt{M^2 - 1}\right)\right]$$

CHE RAPPRESENTA LA FUNZIONE PER UN EPICICLOIDE

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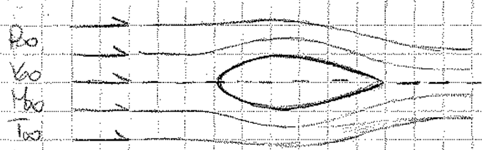
TEORIA DELLE PICCOLE PERTURBAZIONI

È UNA TEORIA LINEARIZZATA PER FLUSSI SUPERSONICI O SUBSONICI COMPRESSIBILI. L'ipotesi di piccole perturbazioni permette di linearizzare e quindi semplificare il problema



$$\bar{V} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k}$$

A MONTE $\begin{cases} V_x = V_\infty \\ V_y = 0 \\ V_z = 0 \end{cases}$ A VALLE $\begin{cases} V_x = V_\infty + u' \\ V_y = v' \\ V_z = w' \end{cases}$

LA VELOCITÀ È SOGGETTA A DEFLESSIONE E VARIAZIONE DEL MODULO.

• IPOTESI:
- PROFILO SOTTILE (Valida per 2D e corpi assialsimmetrici snelli e 3D)
- BASSA INCIDENZA
- FLUSSO ROTAZIONALE E INVISCIDO

ϕ = FUNZIONE POTENZIALE DI PERTURBAZIONE. $\quad u' = \frac{\partial\phi}{\partial x} \quad v' = \frac{\partial\phi}{\partial y} \quad w' = \frac{\partial\phi}{\partial z}$

$\bar{\Phi}$ = POTENZIALE TOTALE

$$\begin{cases} V_x = V_\infty + u' = \frac{\partial\bar\Phi}{\partial x} = V_\infty + \frac{\partial\phi}{\partial x} \\ V_y = \frac{\partial\bar\Phi}{\partial y} = \frac{\partial\phi}{\partial y} = v' \\ V_z = \frac{\partial\bar\Phi}{\partial z} = \frac{\partial\phi}{\partial z} = w' \end{cases}$$

$$\left(1-\frac{\bar\Phi_x^2}{a^2}\right)\bar\Phi_{xx} + \left(1-\frac{\bar\Phi_y^2}{a^2}\right)\bar\Phi_{yy} + \left(1-\frac{\bar\Phi_z^2}{a^2}\right)\bar\Phi_{zz} - 2\frac{\bar\Phi_x\bar\Phi_y}{a^2}\bar\Phi_{xy} - 2\frac{\bar\Phi_x\bar\Phi_z}{a^2}\bar\Phi_{xz} - 2\frac{\bar\Phi_y\bar\Phi_z}{a^2}\bar\Phi_{yz} = 0$$

SOSTITUENDO:

$$\left[a^2-\left(V_\infty+\frac{\partial\phi}{\partial x}\right)^2\right]\phi_{xx} + \left[a^2-\left(\frac{\partial\phi}{\partial y}\right)^2\right]\phi_{yy} + \left[a^2-\left(\frac{\partial\phi}{\partial z}\right)^2\right]\phi_{zz} - 2\left(V_\infty+\frac{\partial\phi}{\partial x}\right)\frac{\partial\phi}{\partial y}\frac{\partial^2\phi}{\partial x\partial y} - 2\left(V_\infty+\frac{\partial\phi}{\partial x}\right)\frac{\partial\phi}{\partial z}\frac{\partial^2\phi}{\partial x\partial z} - 2\left(\frac{\partial\phi}{\partial y}\cdot\frac{\partial\phi}{\partial z}\right)\frac{\partial\phi^2}{\partial y\partial z} = 0$$

DETTA FUNZIONE POTENZIALE DI PERTURBAZIONE

riscrivo come:

$$[a^2-(V_\infty+u')^2]\frac{\partial u'}{\partial x} + [a^2-v'^2]\frac{\partial v'}{\partial y} + [a^2-w'^2]\frac{\partial w'}{\partial z} - 2(V_\infty+u')v'\cdot\frac{\partial u'}{\partial y} - 2(V_\infty+u')\cdot w'\cdot\frac{\partial u'}{\partial z} - 2v'w'\cdot\frac{\partial v'}{\partial z} = 0$$

ESPLICITO a^2 IN FUNZIONE DELLE VELOCITÀ DI PERTURBAZIONE $\qquad h_\infty + \frac{V_\infty^2}{2} = h + \frac{V^2}{2}$

$$\frac{a_\infty^2}{\gamma-1} + \frac{V_\infty^2}{2} = \frac{a^2}{\gamma-1} + \frac{(V_\infty+u')^2+v'^2+w'^2}{2} \Rightarrow a^2 = a_\infty^2 - \frac{\gamma-1}{2}\cdot(2u'V_\infty + u'^2 + v'^2 + w'^2)$$

E SOSTITUISCO NELL'EQ. DI POTENZIALE:

$$\begin{aligned}(1-M_\infty^2)\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = {} & \underbrace{M_\infty^2\left[(\gamma+1)\frac{u'}{V_\infty} + \frac{\gamma+1}{2}\frac{u'^2}{V_\infty^2} + \frac{\gamma+1}{2}\left(\frac{v'^2+w'^2}{V_\infty^2}\right)\right]\frac{\partial u'}{\partial x}}_{D_1} + \\ & + \underbrace{M_\infty^2\left[(\gamma-1)\frac{u'}{V_\infty} + \frac{\gamma+1}{2}\frac{v'^2}{V_\infty^2} + \frac{\gamma+1}{2}\left(\frac{w'^2+u'^2}{V_\infty^2}\right)\right]\frac{\partial v'}{\partial y}}_{D_2} + \\ & + \underbrace{M_\infty^2\left[(\gamma-1)\frac{u'}{V_\infty} + \frac{\gamma+1}{2}\frac{w'^2}{V_\infty^2} + \frac{\gamma+1}{2}\left(\frac{u'^2+v'^2}{V_\infty^2}\right)\right]\frac{\partial w'}{\partial z}}_{D_3} + \\ & + \underbrace{M_\infty^2\left[\frac{v'}{V_\infty}\left(1+\frac{u'}{V_\infty}\right)\left(\frac{\partial u'}{\partial y}+\frac{\partial v'}{\partial x}\right) + \frac{w'}{V_\infty}\left(1+\frac{u'}{V_\infty}\right)\left(\frac{\partial u'}{\partial z}+\frac{\partial w'}{\partial x}\right) + \frac{v'w'}{V_\infty}\left(\frac{\partial v'}{\partial z}+\frac{\partial w'}{\partial y}\right)\right]}_{D_4}\end{aligned}$$

L'EQUAZIONE È ANCORA ESATTA (E NON-LINEARE). PUÒ ESSERE APPLICATA SIA A FLUSSI INCOMPRESSIBILI CHE COMPRESSIBILI.

• ASSUNZIONI PER PICCOLE PERTURBAZIONI: $\quad \frac{u'}{V_\infty}, \frac{v'}{V_\infty}, \frac{w'}{V_\infty} \lll 1, \quad \frac{u'^2}{V_\infty^2}, \frac{v'^2}{V_\infty^2} + \frac{w'^2}{V_\infty^2} \lll 1$

PER $0 < M_\infty \leq 0{,}8 \wedge 1{,}2 < M_\infty < 5$

$D_1 \ll (1-M_\infty^2)$

$D_2, D_3, D_4 \lll 1$

EQ. POTENZIALE DELLE VELOCITÀ $\quad (1-M_\infty^2)\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0$

la linearizzazione non è più accettabile in regime ipersonico, mentre non è possibile a causa degli urti in regimi transonici.

• Cp per flusso $\qquad C_p = \frac{p-p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2} = \frac{2}{\gamma M_\infty^2}\left(\frac{p}{p_\infty}-1\right)$

cerco p/p_∞

$$T - T_\infty = \frac{V_\infty^2 - V^2}{2c_p} \quad \text{con} \quad c_p = \frac{\gamma}{\gamma-1}\cdot\frac{R}{M} \Rightarrow \frac{T}{T_\infty} = 1 - \frac{\gamma-1}{2a_\infty^2}(2u'V_\infty + u'^2 + v'^2 + w'^2) \Rightarrow \frac{p}{p_\infty} = \left(\frac{T}{T_\infty}\right)^{\gamma/\gamma-1} = 1 - (\ldots)^{\gamma/\gamma-1}$$

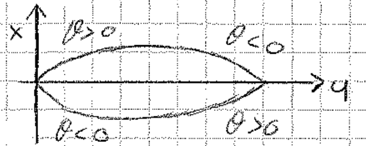
espando p/p_∞ in serie binomiale $(1-\varepsilon)^\alpha = 1 - \alpha\varepsilon$

$$\frac{p}{p_\infty} = 1 - \frac{\gamma}{2}M_\infty^2\left(\frac{2u'}{V_\infty} + \frac{u'^2+v'^2+w'^2}{V_\infty}\right) \Rightarrow \boxed{C_{p_{LIN}} = -\frac{2u'}{V_\infty}} - \underbrace{\cancel{\frac{u'^2+v'^2+w'^2}{V_\infty^2}}}_{=0 \text{ PER PICCOLE PERT.}}$$

TEORIA DELLE PICCOLE PERTURBAZIONI IN SUPERSONICO

tiene conto di compressioni, espansioni e URTI.

Ricordando la TEORIA LINEARIZZATA: • $C_{P_C} = C_{P_I} / \sqrt{1 - M_\infty^2}$



θ_{LE} = LEADING EDGE

• Subsonico: $\beta^2 \cdot \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$; $\beta = \sqrt{1 - M_\infty^2}$

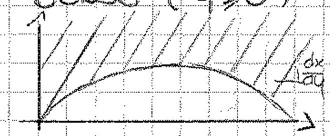
• Supersonico: $\lambda^2 \cdot \frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial y^2} = 0$; $\lambda = \sqrt{M_\infty^2 - 1}$

Cerco la funzione potenziale di perturbazione, ASSEGNATA LA GEOMETRIA

In generale: $\lambda^2 \frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial y^2} = 0$ è l'eq di un'onda, con soluzione $\phi = f(x - \lambda y) + g(x + \lambda y)$

trovo quindi le soluzioni per $y>0$ e $y<0$

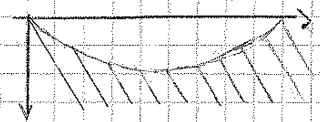
• DORSO ($y \geq 0$) -> $\phi = f(x - \lambda y)$



le linee a ϕ = costante individuano LINEE CARATTERISTICHE

$$\frac{dx}{dy} = tg(\mu_\infty) = \frac{1}{\sqrt{M_\infty^2 - 1}} = \frac{1}{\lambda}$$

• VENTRE ($y < 0$) -> $\phi = g(x + \lambda y)$



anche in questo caso le linee a ϕ = cost. sono LINEE CARATTERISTICHE

$$\frac{dx}{dy} = tg(-\mu_\infty) = -\frac{1}{\sqrt{M_\infty^2 - 1}} = -\frac{1}{\lambda}$$

• CONDIZIONE DI TANGENZA (LINEARIZZATA)

occorre imporre la condizione di tangenza del flusso attorno al profilo

$tg(\theta) = \frac{v'}{V_\infty + u'}$ -> DEVO ESPRIMERLA IN FUNZIONE DEL POTENZIALE SIA PER IL DORSO CHE PER IL VENTRE

- DORSO

$$u' = \frac{\partial \phi(x - \lambda y)}{\partial (x - \lambda y)} \cdot \frac{\partial (x - \lambda y)}{\partial x} = f'$$

$$v' = \frac{\partial \phi(x - \lambda y)}{\partial (x - \lambda y)} \cdot \frac{\partial (x - \lambda y)}{\partial y} = f' \cdot (-\lambda)$$

$f' = u'$, $f' = -\frac{v'}{\lambda}$ -> $u' = -\frac{v'}{\lambda}$

$$tg\theta = \frac{v'}{V_\infty + u'} \xrightarrow[u' \ll V_\infty,\ tg\theta \approx \theta]{} v' = V_\infty \theta$$

da cui: $u' = \frac{V_\infty \cdot \theta}{\lambda}(-1)$

$$C_p = \frac{2\theta}{\lambda}$$

- VENTRE

$$u' = \frac{\partial \phi(x + \lambda y)}{\partial (x + \lambda y)} \cdot \frac{\partial (x + \lambda y)}{\partial x} = g'$$

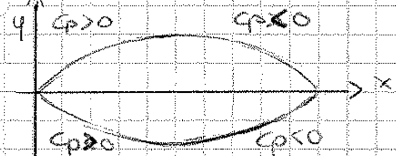
$$v' = \frac{\partial \phi(x + \lambda y)}{\partial (x + \lambda y)} \cdot \frac{\partial (x + \lambda y)}{\partial y} = g' (\lambda)$$

$g' = u'$, $g' = \frac{v'}{\lambda}$ $u' = \frac{v'}{\lambda}$

$\longrightarrow v' = V_\infty \theta$

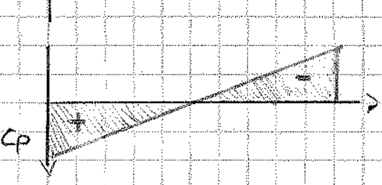
da cui $u' = \frac{V_\infty \theta}{\lambda}$

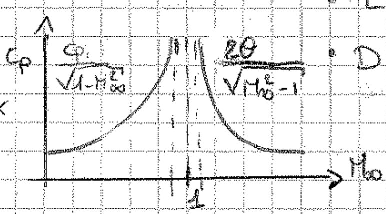
$$C_p = -\frac{2\theta}{\lambda}$$



$$C_p = \frac{p - p_\infty}{\frac{1}{2} \rho_\infty V_\infty^2}$$

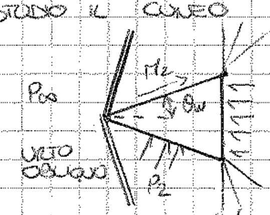
• deflessione nota data dalla geometria
• fluido NON VISCOSO
• $\alpha = 0$
• $L = 0$ se il profilo è simmetrico i contributi sono uguali
• $D \neq 0$ -> RESISTENZA D'ONDA



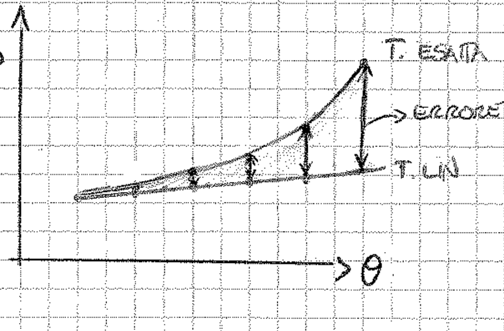


• CHE ERRORE SI COMMETTE CON LA TEORIA LINEARIZZATA?

STUDIO IL CUNEO

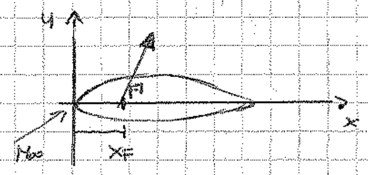


TEORIA ESATTA



TEORIA LINEARIZZATA

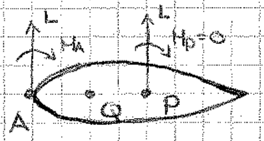
PROPRIETÀ FOCALI



IL FUOCO DI UN PROFILO E' UN PUNTO CARATTERISTICO PER CUI IL MOMENTO M_F E' COSTANTE

• PER UN FLUIDO INCOMPRESSIBILE $x_F = 0{,}25 \cdot c$

$$C_{MA} = -\frac{2\alpha}{\sqrt{M_\infty^2-1}} \cdot \frac{2}{\sqrt{M_\infty^2-1}} \cdot \frac{(S_u - S_L)}{c^2}$$



SIA P IL CENTRO DI PRESSIONE DEL PROFILO, CON x_{cp} NOTO,

$$\begin{cases} M_p = L \cdot (x_p - x_{cp}) = 0 \\ M_A = -L x_{cp} \\ M_Q = M_A + L x_Q = M_p - L(x_p - x_{cp}) \end{cases} \qquad \begin{aligned} M_p &= 0 \\ M_A &= -L x_{cp} \\ x_{cp} &= -M_A / L \end{aligned} \;\rightarrow\; \frac{x_{cp}}{c} = \frac{C_{MA}}{C_L}$$

CONSIDERIAMO IL PUNTO O IL FUOCO DELL'ALA, SE $P \equiv O$, $M_O = M_A + L x_O \rightarrow C_{M_O} = C_{MA} + C_L \cdot \frac{x_O}{c}$

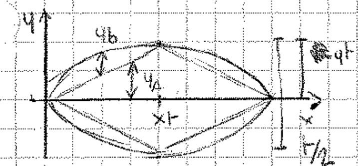
$$C_{m_O} = \left[-\frac{2\alpha}{\sqrt{M_\infty^2-1}} - \frac{2}{\sqrt{M_\infty^2-1}} \frac{[S_u + S_L]}{c^2}\right] + \frac{4\alpha}{\sqrt{M_\infty^2-1}} \cdot \frac{x_O}{c}$$; PER LA PROPRIETÀ FOCALE $\frac{\partial c_{m_O}}{\partial \alpha} = 0$

$$\frac{\partial c_{m_O}}{\partial \alpha} = -\frac{2}{\sqrt{\;}} + \frac{4}{\sqrt{\;}} \cdot \frac{x_O}{c} = 0 \quad \Rightarrow \quad \frac{x_F}{c} = \frac{1}{2}$$

NEL SUPERSONICO IL FUOCO E' POSIZIONATO A METÀ DEL PROFILO

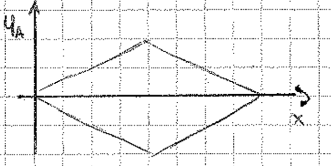
• DATI $\alpha = 0$, spessore t, CERCO LA FORMA CON MINIMA RESISTENZA.

- SCOMPONGO IL PROFILO



$$\begin{cases} y_A = \frac{y_t}{x_t} \cdot x, & \text{per } 0 < x < x_t \;; \quad \frac{dy_A}{dx} = \frac{y_t}{x_t} \\ y_A = \frac{y_t}{(c - x_t)} \cdot (c - x_t), & \text{per } x_t < x < c \;; \quad \frac{dy_A}{dx} = -\frac{y_t}{c - x_t} \end{cases}$$

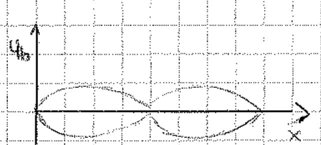
$y_b = 0$ in 0, x_t e c



$$\int_0^c \left(\frac{dy}{dx}\right)_u^2 dx = \int_0^{x_t} \left(\frac{y_t}{x_t} + \frac{dy_b}{dx}\right)^2 dx + \int_{x_t}^c \left(-\frac{y_t}{c - x_t} + \frac{dy_b}{dx}\right)^2 dx =$$

$$= \left[\frac{y_t^2}{x_t} + \int_0^{x_t} \left(\frac{dy_b}{dx}\right)^2 dx + \frac{y_t^2}{(c - x_t)} + \int_{x_t}^c \left(\frac{dy_b}{dx}\right)^2 dx\right]$$

ANALOGAMENTE PER LA FACCIA DI SOTTO. SOSTITUISCO NELL EQ.



$$C_D = \frac{2}{\sqrt{M_\infty^2-1}} \cdot \left[\int_0^c \left(\frac{dy}{dx}\right)_u^2 dx + \int_0^c \left(\frac{dy}{dx}\right)_L^2 dx\right]$$

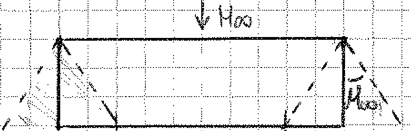
SI VERIFICA CHE IL MINIMO DI C_D SI HA PER $\frac{dy_b}{dx} = 0$

VALE A DIRE PER IL PROFILO A ROMBO.

• $C_{D\,MIN\,DORSO} = \frac{2}{\sqrt{M_\infty^2-1}} \cdot 4 \left(\frac{y_t}{c}\right)^2$ • $C_{D\,MIN\,VENTRE} = \frac{2}{\sqrt{M_\infty^2-1}} \cdot 4 \left(\frac{t - y_t}{c}\right)^2$, Per $x_t = \frac{c}{2}$; $y_t = \frac{t}{2}$

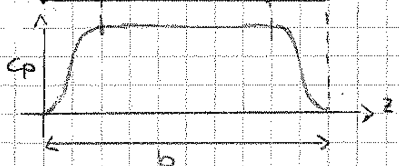
EFFETTI BIDIMENSIONALI

SUPPONIAMO UNA LASTRA PIANA DI DIMENSIONI FINITE



$\sin \mu_\infty = 1/M_\infty$

• PER L'ALA INFINITAMENTE LUNGA, $\begin{cases} C_{L_{2D}} = \frac{4\alpha}{\sqrt{M_\infty^2-1}} \\ C_{p_u} = -C_{pL} = -\frac{2\alpha}{\sqrt{M_\infty^2-1}} \end{cases}$



Indico con L_W gli effetti reali e con L_{2D} il caso ideale. NEL CASO REALE HO PERDITE DI PORTANZA.

$\Delta L = L_{2D} - L_W$; $\frac{C_{LW}}{C_{L_{2D}}} = 1 - \frac{1}{2A\sqrt{M_\infty^2-1}}$

$A = \frac{b}{c}$ ALLUNGAMENTO ALARE

A PARITÀ DI M_∞, SE $A \uparrow$, $\frac{C_{LW}}{C_{L_{2D}}} \rightarrow 1$, SI RIDUCE IL PESO DEGLI EFFETTI

A PARITÀ DI A, SE $M_\infty \uparrow$, $\frac{C_{LW}}{C_{L_{2D}}} \rightarrow 1$, si restringe il cono di MACH e i suoi effetti sul profilo

→ SE IL BORDO D'ATTACCO DELL'ALA E' FUORI DAL CONO DI MACH, ALLORA IL PROFILO E' SUPERSONICO.

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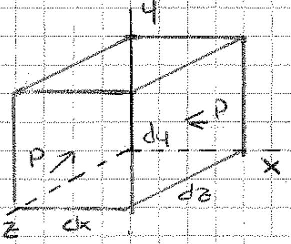
EFFETTI DELLA VISCOSITÀ E DELLA CONDUCIBILITÀ TERMICA

- VISCOSITÀ μ → GRADIENTI DI VELOCITÀ GENERANO SFORZI VISCOSI
- CONDUCIBILITÀ λ → " " TEMPERATURA " " FLUSSI DI ENERGIA

→ PUNTO DI VISTA EULERIANO (CONSERVATIVO) = VOLUME DI CONTROLLO FISSO
→ " " " LAGRANGIANO (NON CONSERVATIVO) = SEGUO LA PARTICELLA FLUIDA.

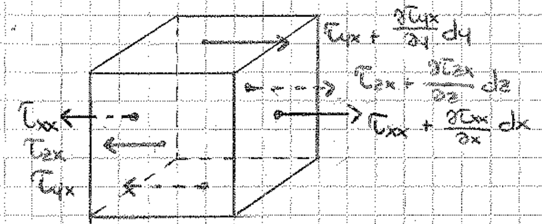


$$\mu \neq 0 \begin{cases} p_i \\ \tau_{ij} \end{cases}$$

→ EQ DI CONTINUITÀ (NON VARIA)

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0 \qquad \frac{D\rho}{Dt} = -\rho \overbrace{\nabla \cdot \vec{V}}^{\text{DIVERGENZA}}$$

FORMA EULERIANA — FORMA LAGRANGIANA



SFORZI AGENTI SOLO LUNGO X.

→ BILANCIO DELLA QDM

$$\left[(p - \tau_{xx})dydz + \left(\tau_{xx} + \frac{\partial \tau_{xx}}{\partial x}dx - p - \frac{\partial p}{\partial x}dx\right)dydz\right] +$$
$$+\left[-\tau_{yx}\,dxdz + \left(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y}dy\right)dxdz\right] +$$
$$+\left[-\tau_{zx}\,dxdy + \left(\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z}dz\right)dxdy\right] = m \cdot a_x$$

$$\text{dove } m = \rho\,dxdydz \quad e \quad a_x = \frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

Analogamente per Y e Z. riordino e divido per dxdydz.

$$x)\ \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}$$
$$y)\ \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}$$
$$z)\ \rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}$$

DIMENSIONALMENTE SI TRATTA DI FORZE PER UNITÀ DI VOLUME (N/m^3)

PER L'EQUILIBRIO ALLA ROTAZIONE SI OTTIENE:

$$\begin{cases} \tau_{xy} = \tau_{yx} \\ \tau_{xz} = \tau_{zx} \\ \tau_{yz} = \tau_{zy} \end{cases} \quad e \quad \tau_{ij} = \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \tau_{yy} & \tau_{zy} \\ \tau_{xz} & \tau_{zy} & \tau_{zz} \end{bmatrix}$$

TENSORE DEGLI SFORZI VISCOSI

GLI SFORZI VISCOSI DETERMINANO LE DEFORMAZIONI A VOLUME COSTANTE

$$\tau_{xx} = 2\mu\frac{\partial u}{\partial x} + \lambda_V \nabla\cdot\vec{V}, \quad \tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \tau_{yx}, \quad \lambda_V = -\frac{2}{3}\mu$$

COEFFICIENTE DI STOKES O SECONDO COEFF. DI VISCOSITÀ

$$\tau_{yy} = 2\mu\frac{\partial v}{\partial y} + \lambda_V \nabla\cdot\vec{V}, \quad \tau_{xz} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = \tau_{zx}$$
$$\tau_{zz} = 2\mu\frac{\partial w}{\partial z} + \lambda_V \nabla\cdot\vec{V}, \quad \tau_{yz} = \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = \tau_{zy}$$

IN FORMA COMPATTA:

$$\tau_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\overbrace{\frac{\partial u_k}{\partial x_k}}^{\text{DIVERGENZA}}\delta_{ij}\right) \quad \text{con} \quad \delta_{ij} = \begin{cases} 0 & \text{se } i \neq j \\ 1 & \text{se } i = j \end{cases}$$

SOSTITUENDO τ_{ij} NELLE EQ LUNGO I 3 ASSI OTTENGO LE EQUAZIONI DI NAVIER-STOKES (NON-LINEARI, CON SOLUZ. SOLO IN CASI SEMPLICI).
SE IL FLUSSO FOSSE INCOMPRESSIBILE (ρ = cost), DALLA CONTINUITÀ AVREI CHE LA DIVERGENZA È NULLA.

- PER RISOLVERE LE EQ DI NAVIER-STOKES SI IMPIEGANO SIMULAZIONI NUMERICHE DIRETTE.

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• IN FORMA COMPATTA:

$$\rho \cdot \frac{D(e+V^2/2)}{Dt} = \left[-\nabla\cdot(p\cdot\vec{v})\right] + \left[\bar{\bar{\tau}}\times\vec{v}\right] + \left[\rho\dot{q} - \frac{\partial q_x}{\partial x} - \frac{\partial q_y}{\partial y} - \frac{\partial q_z}{\partial z}\right]$$

PRESSIONE VISCOSITA' CALORE

SAPENDO CHE $\rho\cdot\frac{D(e+V^2/2)}{Dt} = \rho\,\frac{D(h+V^2/2)}{Dt} - \frac{\partial p}{\partial t} - \nabla(p\cdot\vec{v})$

E ASSUMENDO:
- FLUSSO STAZIONARIO $\partial H/\partial t = 0$, $\partial p/\partial t = 0$
- FLUSSI ESTERNI DI CALORE NULLI ($\dot{q}=0$, $\rho\dot{q}=0$)

OTTENIAMO

$$\rho\cdot u_j\frac{\partial H}{\partial x_j} = \frac{\partial p}{\partial t} + \frac{\partial(\tau_{ij}u_j - q_i)}{\partial x_j} \quad \text{EQ. ENERGIA}$$

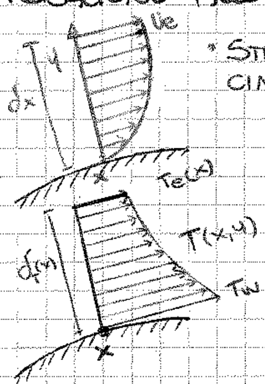
RIASSUMENDO LE EQUAZIONI

• CONTINUITA' $\Rightarrow$ $\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_j)}{\partial x_j} = 0$

• Q.D.M $\Rightarrow$ $\rho\frac{\partial u_i}{\partial t} + \rho u_j\frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial(\tau_{ij})}{\partial x_j}$

• ENERGIA $\Rightarrow$ $\rho\frac{\partial H}{\partial t} + \rho u_j\frac{\partial H}{\partial x_j} = \frac{\partial p}{\partial t} + \frac{\partial(\tau_{ij}u_j - q_i)}{\partial x_j}$

APPLICAZIONE ALLO STRATO LIMITE



• STRATO LIMITE CINEMATICO $\tau_{xy} = \mu\left(\frac{\partial u}{\partial y}\right)$

$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$ (A PARETE)

• STRATO LIMITE TERMICO $q_y = -\lambda\left(\frac{\partial T}{\partial y}\right)$

$q_w = -\lambda\left(\frac{\partial T}{\partial y}\right)_{y=0}$ (A PARETE)

$Re = \frac{V_\infty L \rho_\infty}{\mu}$

$Pr = \frac{\mu\cdot c_p}{\lambda}$

Per uno strato limite laminare, si ha che: $Pr \propto \sqrt{\frac{\delta(x)}{\delta_T(x)}}$

OSSIA IL PRANDTL E' PROPORZIONALE AL RAPPORTO DELLE ESTENSIONI DEGLI STRATI LIMITE.

$Pr = \frac{4\gamma}{9\gamma-5}$ (ARIA $Pr = 0{,}71$)

EQ. DELLO STRATO LIMITE 2D, STAZIONARIO, COMPRESSIBILE e TERMICO

2D implica $w=0$, $\frac{\partial}{\partial z}=0$
LE EQUAZIONI DUNQUE:

• CONTINUITA': $\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$

• Q.D.M

$$\begin{cases}\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial\tau_{xx}}{\partial x} + \frac{\partial\tau_{yx}}{\partial y}\\ \rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \frac{\partial\tau_{xy}}{\partial x} + \frac{\partial\tau_{yy}}{\partial y}\end{cases}$$

• ENERGIA: $\rho\left[u\frac{\partial H}{\partial x} + v\frac{\partial H}{\partial y}\right] = \frac{\partial(\tau_{xx}u + \tau_{xy}v - q_x)}{\partial x} + \frac{\partial(\tau_{yx}u + \tau_{yy}v - q_y)}{\partial y}$

ASSUNZIONI:

$Re \gg 1$, $\delta/L \ll 1$, $\tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \cancel{\frac{\partial v}{\partial x}}_{\text{TRASCURABILE}}\right)$, $T = \frac{h}{c_p}$

• dalla Q.D.M $\rightarrow \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{dp_e}{dx} + \frac{\partial\tau_{yx}}{\partial y}$

$\rightarrow \frac{\partial p}{\partial y} = 0$ LO STRATO LIMITE E' ISOBARO LUNGO y

• dall' ENERGIA

$$\rightarrow \rho\left[u\frac{\partial H}{\partial x} + v\frac{\partial H}{\partial y}\right] = \frac{\partial}{\partial y}\left[\mu\frac{\partial u^2/2}{\partial y} + \frac{\lambda}{c_p}\frac{\partial h}{\partial y}\right] = \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial(u^2/2)}{\partial y} + \frac{1}{Pr}\frac{\partial h}{\partial y}\right)\right]$$

Con $q_w \neq 0$, $dp_e/dx \neq 0$, $Pr \neq 1$ NON ESISTONO SOLUZIONI E DEVO STUDIARE FLUSSI SIMILI.

PROFILO DI TEMPERATURA: CASO ADIABATICO ($Pr \neq 1$)

$\frac{dp_e}{dx} = 0$, $Pr \neq 1$, $q_w = 0$

SIA IL PUNTO DI ARRESTO ISOENTROPICO

$$\begin{cases} H = H_e = H_w = \text{cost.} \\ h_\infty + \frac{U_\infty^2}{2} = h_e + \frac{u_e^2}{2} \end{cases}$$

$H_\infty = H_0$
$T_W = T_0$
$U = 0$

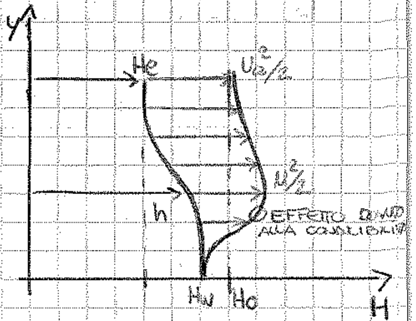
Se $Pr \neq 1$, $h + R\frac{u^2}{2} = \text{cost}$

$H_r = h_e + R\frac{u_e^2}{2} = H_W$ entalpia di recupero

$T_r = T_e + R\frac{u_e^2}{2c_p} = T_e\left(1 + \frac{\gamma-1}{2} R M_e^2\right)$

$R = Pr^{1/2}$ LAMINARE
$R = Pr^{1/3}$ TURBOLENTO
$R = Pr$ COUETTE

PROFILO DI TEMPERATURA $T = T_e + R\frac{U_e^2}{2c_p}(1 - \varphi^2)$

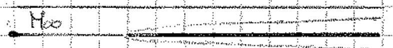


PROFILO DI TEMPERATURA: CASO NON ADIABATICO

$q_w = 0$, $Pr \neq 1$, $\frac{dp_e}{dx} \neq 0$, $\left(-\lambda\left(\frac{\partial T}{\partial y}\right)_{y=0}\right)$ NON HO SOLUZIONI.

- SEMPLIFICAZIONE: $Pr = 1$, $\frac{dp_e}{dx} = 0$, $\frac{\partial}{\partial x} = 0$

PLACCA PIANA A INCIDENZA NULLA



INTEGRALE DI CROCCO

$H = C_1 + C_2 u$ $\quad u \to$ Q.D.M, $H \to$ ENERGIA

per trovare C_1 e C_2 impongo le condizioni iniziali e al contorno

$\begin{cases} y = 0, \; u_w = 0 & \to \text{Aderenza} \\ H_W = c_p T_W & \to \text{Eq termico} \end{cases}$ A PARETE

$\begin{cases} y = \infty, \; u = u_e & \to \text{Aderenza} \\ H_e = H_\infty & \to \text{Eq} \end{cases}$ ESTERNO

$\to \begin{cases} y = 0 \\ u = 0 \\ H = H_W \end{cases} \to C_1 = H_W$ $\quad \begin{cases} y = \infty \\ H_e = H_W + C_2 u_e \end{cases} \to C_2 = (H_e - H_W) \cdot 1/u_e$

Pertanto $H = H_W + \frac{H_e - H_W}{u_e} \cdot u$

$\frac{u}{u_e} = \frac{H - H_W}{H_e - H_W} = \frac{h + u^2/2 - h_w}{h_e + u_e^2/2 - h_w}$ ANALOGIA TRA CAMPO DI VELOCITÀ ED ENERGIA

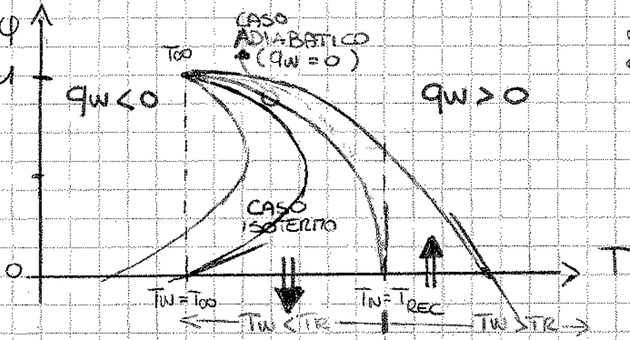
- PROFILI DI TEMPERATURA; $T = T(\varphi)$ e $\varphi = u/u_e$

$$T\left(1 + \frac{\gamma-1}{2}M^2\right) = \varphi\left[T_\infty\left(1 + \frac{\gamma-1}{2}M_\infty^2\right) - T_W\right] + T_W$$

$T = T_W(1-\varphi) + T_\infty \varphi^2 + T_0(\varphi - \varphi^2)$, con $T_0 = T_W\left(1 + \frac{\gamma-1}{2}M_\infty^2\right)$ se $Pr = 1$

se $q_w \neq 0$, $Pr \neq 1$ e $\frac{dp_e}{dx} \neq 0$, correggo il profilo con R

$$T = T_W(1-\varphi) + T_\infty\varphi^2 + T_{rec}(\varphi - \varphi^2)$$



- CASO ADIABATICO ($T_W = T_R$)
- CASO ISOTERMO ($T_W = T_\infty$)

- PER $T_W < T_R$ la parete può essere più calda della corrente ($T_W > T_\infty$), TUTTAVIA PER EFFETTO DELL'ATTRITO IL FLUSSO DI CALORE È DIRETTO DALLA CORRENTE ALLA PARETE
- PER $T_W > T_R$ LA PARETE È SUFFICIENTEMENTE CALDA DA TRASMETTERE CALORE AL FLUSSO

Se $Pr = 1$, $T_W = T^0$
Se $Pr \neq 1$, $T_W = T_{REC}$

METODO DELLA SEZIONE RAPPRESENTATIVA

siano $\mu(T)$ e $\rho(T)$, scelto $\varphi = 0{,}5$

- $T = T_W(1-\varphi) + T_\infty \varphi^2 + T_{REC}(\varphi - \varphi^2)$
  $T^* = 0{,}5\, T_W + 0{,}25\, T_\infty + 0{,}25\, T_{REC}$

$C_{D_{COMP}}(Re, M_\infty, T_W/T_\infty) = C_{D_{INCOMP}} \cdot \chi\left(M_\infty, \frac{T_W}{T_\infty}\right)$ dove χ è un fattore correttivo.

$Re_\infty = \dfrac{V_\infty L \rho_\infty}{\mu_\infty}$; se $Re_l < Re_{CR} = \underset{\text{LASTRA PIANA}}{5 \cdot 10^5}$ → LAMINARE; se $Re > Re_{CR}$ → TURBOLENTO

- LAMINARE, $C_D = \dfrac{1{,}328}{\sqrt{Re_\infty}}$; $Re < 5 \cdot 10^5$
- TURBOLENTO (BASSO), $C_D = \dfrac{0{,}074}{Re_\infty^{1/5}}$; $5 \cdot 10^5 < Re < 10^7$
- TURBOLENTO (ALTO), $C_D = \dfrac{0{,}0295}{Re_\infty^{1/7}}$; $Re > 10^7$

$\tau_W = \frac{1}{2} \rho_\infty V_\infty^2 \cdot C_D$

$\chi = \dfrac{C_{DC}}{C_{DI}} = \dfrac{\tau_{WC}}{\tau_{WI}}$

$\tau_{W_{INC}} \propto (\rho_\infty ; \mu_\infty)^{1/2}$

$\tau_{W_{COMPR}} (\rho_\infty, \mu_\infty)^{1/2}$

$\tau_{W_{COMP}} \propto (\rho_\infty^*, \mu_\infty^*)$ è detta sezione rappresentativa; $\chi = \left(\dfrac{\rho_\infty^* \mu_\infty^*}{\rho_\infty \mu_\infty}\right)^{1/2}$

Poiché $\dfrac{\mu^*}{\mu_\infty} = \left(\dfrac{T^*}{T_\infty}\right)^\omega$ e $\dfrac{\rho^*}{\rho_\infty} = \left(\dfrac{T^*}{T_\infty}\right)^{-1}$, con $\omega = \omega(T)$, per $T \approx 300K$, $\omega = 0{,}75$

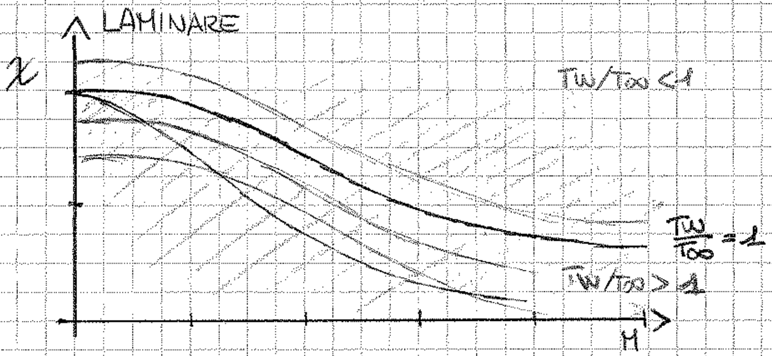
quindi $\chi_{LAM} = \left(\dfrac{T^*}{T_\infty}\right)^{\frac{\omega-1}{2}} = \left[\dfrac{T_W + T_\infty}{2T_\infty} + R\,\dfrac{\gamma-1}{2} \cdot M_\infty^2 \cdot \dfrac{1}{4}\right]^{\frac{\omega-1}{2}}$

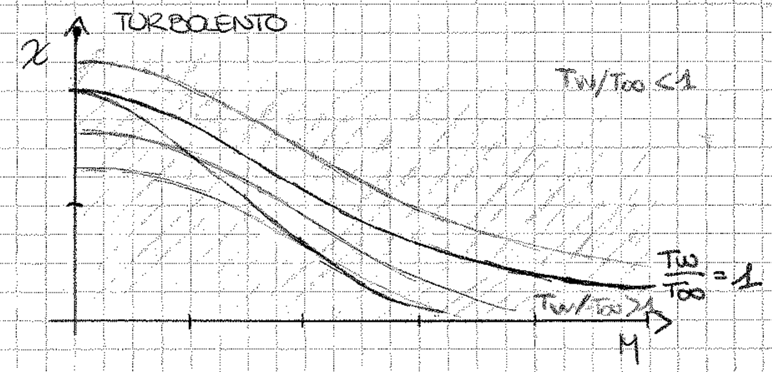
$\dfrac{\omega-1}{2} = 0{,}125$ $(T \sim 300K)$

per il turbolento cambiano solo gli esponenti

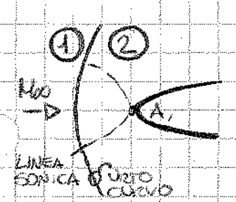
$\chi_{TURB} = \left[\dfrac{T_W + T_\infty}{2T_\infty} + R \cdot \dfrac{\gamma-1}{2} \cdot M_\infty^2 \cdot \dfrac{1}{4}\right]^{\frac{\omega-4}{5}}$

$\dfrac{\omega-4}{5} = -0{,}65$ $(T \sim 300K)$





FLUSSO DI CALORE NEL PUNTO DI ARRESTO

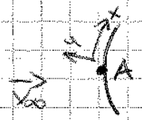


$P_A = P_2^0$ $\quad$ $u_A = 0$

$T_A = T_2^0 = T_1^0 = T_\infty^0$ $\quad$ $T_2/T_1 = f(M)$

se $M_\infty > 1$, l'urto è curvo; $(3 < M_\infty < 10)$ $\quad$ in A, $\begin{cases} \tau_w = 0 \\ q_w = max \end{cases}$

• FLUSSO NELL'INTORNO DEL PUNTO DI ARRESTO INCOMPRESSIBILE



$U_e(x) = K \cdot x$

$U_e(A) = 0 \quad (x_A = 0)$

$\left(\frac{dU_e}{dx}\right)_{x=0} = K$; $\quad \rho U_e \frac{dU_e}{dx} = \frac{dP_e}{dx}$ GRADIENTE DI PRESSIONE

METODI DI RISOLUZIONE INTEGRALI:

• si perdono soluzioni locali a beneficio di una soluzione GLOBALE

δ^* = SPESSORE DI SPOSTAMENTO

θ = SPESSORE DI QUANTITÀ DI MOTO

$x \rightarrow \begin{cases} \delta^*(x) \\ \theta(x) \\ c_f(x) \rightarrow \tau_w(x) \end{cases}$

$H = \frac{\delta^*}{\theta}$ = PARAMETRO DI FORMA

$\delta^* = \int_0^\infty \left(1 - \frac{u}{U_e}\right) dy$; $\quad \theta = \int_0^\infty \frac{u}{U_e}\left(1 - \frac{u}{U_e}\right) dy$

$H = \begin{cases} 2{,}5 \rightarrow \text{STRATO LIMITE LAMINARE} \\ 1{,}3 \rightarrow \text{STRATO LIMITE TURBOLENTO} \end{cases}$

• METODO DI THWAITES (x IL FLUSSO LAMINARE)

NOTO IL FLUSSO ESTERNO FORNISCE FACILMENTE $\theta^2(x) = \frac{0{,}45\,\nu}{U_e^6(x)} \int_0^x U_e^5(x)\, dx$

NELL'INTORNO DEL PUNTO DI ARRESTO (dove $K = (dU_e/dx)_{x=0}$)

$$\theta^2(x)_{ARRESTO} = \frac{0{,}45\,\nu}{k^6 x^6} \int_0^x (kx)^5 dx = \frac{0{,}45\,\nu}{6K}$$

OSSIA È GIÀ PRESENTE LO STRATO LIMITE E θ È COSTANTE

CONSIDERIAMO: $\lambda = \frac{\theta^2}{\nu}\left(\frac{dU_e}{dx}\right)$ e $P = \left(\frac{du}{dy}\right)_{y=0} \cdot \frac{\theta}{U_e(x)} = P(\lambda)$

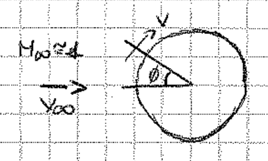
PER FLUSSI ACCELERATI: $P = 0{,}22 + 1{,}57\lambda - 1{,}8\lambda^2 \quad (\lambda > 0)$

PER IL PT. DI ARRESTO: $\lambda_{ARR} = \theta_{ARR} \cdot \frac{K}{\nu} = 0{,}075$

$P_{ARR} = P(\lambda_{ARR}) = 0{,}328$

da cui, sostituendo: $\tau_w = 0{,}328 \cdot x/B = f(x, \mu, P, K)$ e $K = f\begin{pmatrix} \text{FORMA} \\ \text{GEOMETRIA} \\ V_\infty \end{pmatrix}$

- APPLICAZIONE (SFERA - CILINDRO)



$V(\phi) = 2V_\infty \sin\phi$, con $\phi = x/R$ $\rightarrow$ FLUSSO POTENZIALE

$V(x) = 2V_\infty \cdot \sin(x/R)$ $\rightarrow$ Approssimo $U_e(x) = V(x)$

$\frac{dV}{dx} = 2V_\infty \cos(x/R) = K\Big|_{x=0} \Rightarrow$ $\quad$ SFERA $= 4V_\infty/D \rightarrow 4 = KD/V_\infty$

$\qquad$ CILINDRO $= 3V_\infty/D \rightarrow 3 = KD/V_\infty$

per flussi supersonici incomprimibili:

· CILINDRO: $\tau_w(x) = 1{,}232 \cdot \sqrt{\rho_\infty \mu_\infty U_e^2 \,(dU_e/dx)}$

$q_w(x) = 0{,}57\ Pr^{-0{,}6} (\rho_e \mu_e k)^{1/2} \left(\frac{\rho_w \mu_w}{\rho_e \mu_e}\right)^{0{,}1} \cdot (H_e - H_w)$

w = parete
e = esterno

SFERA: $\tau_w(x) = 1{,}312 \sqrt{\rho_\infty \mu_\infty U_e^2 \,(dU_e/dx)}$

$q_w(x) = 0{,}763\ Pr^{-0{,}6} (\rho_e \mu_e k)^{1/2} \left(\frac{\rho_w \mu_w}{\rho_e \mu_e}\right)^{0{,}1} \cdot (H_e - H_w)$

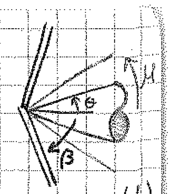
FORMULARIO

$$T^o = T \cdot \left(1 + \frac{\gamma-1}{2} M^2\right)$$

$$p^o = p \cdot \left(1 + \frac{\gamma-1}{2} M^2\right)^{\gamma/\gamma-1}$$

$$\rho^o = \rho \cdot \left(1 + \frac{\gamma-1}{2} M^2\right)^{1/\gamma-1}$$

$$a = \sqrt{\gamma R T} = \sqrt{\gamma\, P/\rho}$$



$$\mu = a\sin\left(\frac{1}{M}\right)$$

- **URTI**

$T_{01} = T_{02}$

$p_1 < p_2$ $V_1 > V_2$

$T_1 < T_2$ $\rho_{01} > \rho_{02}$

$\rho_1 < \rho_2$ $p_{01} > p_{02}$

$S_1 < S_2$ $M_{01} > M_{02}$

- **RETTO**

$$V_1 \cdot V_2 = V_{cr}^2 = a^{*2}$$

$$1 < M_2 < 0{,}45 \quad (\gamma = 1{,}4,\ M_1 < 5)$$

$$M_2^2 = \frac{2 + (\gamma-1) M_1^2}{2\gamma M_1^2 - (\gamma-1)} \ , \qquad M_1^2 = \frac{2 + (\gamma-1) M_2^2}{2\gamma M_2^2 - (\gamma-1)}$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma-1} \cdot (M_1^2 - 1) \ ; \qquad \frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{(\gamma+1) M^2}{2 + (\gamma-1) M_1^2} \ ; \qquad \frac{T_2}{T_1} = 1 + \frac{2(\gamma-1)}{(\gamma+1)^2} \cdot \frac{\gamma M_1^2 + 1}{M_1^2} (M_1^2 - 1)$$

$$* \quad \frac{p_{02}}{p_{01}} = \frac{\rho_{02}}{\rho_{01}} = \left[1 + \frac{2\gamma}{\gamma+1} (M_1^2 - 1)\right]^{-1/\gamma-1} \cdot \left[\frac{(\gamma+1) M_1^2}{2 + (\gamma-1) M_1^2}\right]^{\gamma/\gamma-1}$$

$$\frac{S_2 - S_1}{R^*} = -\ln \frac{p_{02}}{p_{01}} = \frac{\gamma}{\gamma-1} \cdot \ln \frac{T_2}{T_1} - \ln \frac{p_2}{p_1}$$

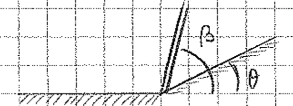
$$R^* = c_p - c_v = 287$$
$$c_p = 1005$$

$$\frac{a^2}{\gamma-1} + \frac{V^2}{2} = \text{cost} = \frac{a_0^2}{\gamma-1} = \frac{a^{*2}}{2} \cdot \left(\frac{\gamma+1}{\gamma-1}\right) \ , \qquad c_p T^o = c_p T_1 + V^2/2$$

$$M^{*2} = \left(\frac{V}{a^*}\right)^2 = \frac{M^2 (\gamma-1)}{2 + M^2 (\gamma-1)} \ \to \ M^2 = \frac{2}{(\gamma+1)/M^{*2} - (\gamma-1)} \ , \qquad S_2 - S_1 = c_v \ln\left\{\left[1 + \frac{2\gamma}{(\gamma+1)} (M_1^2 - 1)\right]\left[\frac{2 + (\gamma-1) M_1^2}{(\gamma+1) M_1^2}\right]^{\gamma}\right\}$$

- **OBLIQUO**

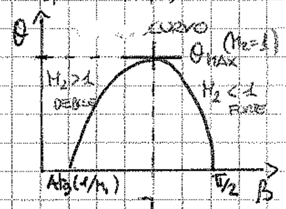
$$V_{1N} \cdot V_{2N} = V_{cr}^2 - \frac{(\gamma-1)}{(\gamma+1)} V_t^2$$



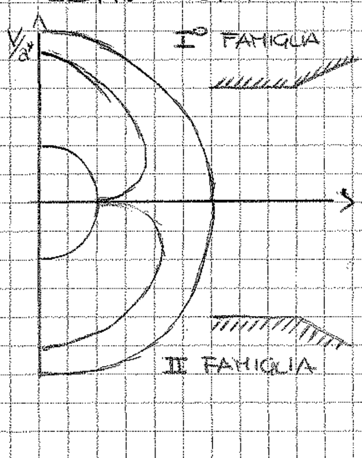
$$\frac{1}{tg(\theta)} = \left(\frac{\gamma+1}{2} \cdot \frac{M_1^2}{M^2 \cdot \sin^2(\beta) - 1}\right) \cdot tg(\beta)$$

$*$ VALGONO, CON $M_1 \to M_{1N} = M_1 \cdot \sin(\beta)$

$$F_1 \quad [M_2 \cdot \sin(\beta - \theta)]^2 = \frac{2 + (\gamma-1)(M_1 \sin\beta)^2}{2\gamma (M_1 \sin\beta)^2 - (\gamma-1)} \ ; \qquad tg(\theta) = \frac{2}{tg(\beta)} \left[\frac{M_1^2 \sin^2\beta - 1}{M_1^2 (\gamma + \cos 2\beta) + 2}\right]$$



- **ESPANSIONI**



DATI M_1, θ_W

$$\nu(M_1) = \left[\sqrt{\frac{\gamma+1}{\gamma-1}} \cdot \text{arctg}\left(\sqrt{\frac{\gamma-1}{\gamma+1}} \sqrt{M_1^2 - 1}\right) - \text{arctg}\sqrt{M^2 - 1}\right] + \text{cost}$$

$$\theta_0 = \nu(M_1) \qquad \begin{cases} \text{I° FAM} \to \theta_W - \nu(M) = \text{cost} = \theta_W + \theta_0 \\ \text{II° FAM} \to \theta_W + \nu(M) = \text{COST} = \theta_W + \theta_0 \end{cases}$$

$$\theta_0 + \theta_W = \theta_2 = \nu(M_2) \Rightarrow M_2 \quad (\text{iterativo})$$

L'ESP E' ISOENTROPICA $\begin{cases} T_1^o = T_2^o \\ p_1^o = p_2^o \\ \rho_1^o = \rho_2^o \end{cases}$

• STRATO LIMITE TERMICO

ARIA STANDARD

$\mu = 1{,}781 \cdot 10^{-5}$ Kg/ms
$\nu = 1{,}454 \cdot 10^{-5}$ m²/s
$T = 288$ K
$c_p = 1005$
$R^* = 287$
$\gamma = 1{,}4$
$P_r = 0{,}71$
$\rho = 1{,}225$ Kg/m³

FOURIER: $q = -\lambda \cdot \text{grad}\, T \rightarrow q_y = -\lambda \dfrac{\partial T}{\partial y}$

PRANDTL: $P_r = \dfrac{\mu c_p}{\lambda} = \dfrac{\nu}{K} = \dfrac{4\gamma}{9\gamma - 5} = \dfrac{2L+4}{2L+9}$, $P_r \propto \left(\dfrac{\delta}{\delta_T}\right)^2$

• CASO ADIABATICO ($q_w = 0$)

$PR = 1$

BUSEMANN: $H = h + \dfrac{u^2}{2} = h_\infty + \dfrac{V_\infty^2}{2} = h_e + \dfrac{U_e^2}{2} = h_w = H^0 =$ COSTANTE

$$T = T_e + \frac{U_e^2}{2c_p}(1 - \varphi^2) \qquad \text{con} \quad \varphi = \frac{u}{U_e}$$

$$T_w(\varphi = 0) = T_e + \frac{U_e^2}{2c_p} = T_e \left(1 + \frac{\gamma - 1}{2} M_e^2\right)$$

$PR \neq 1$

FATTORE DI RECUPERO $R \longrightarrow$ $\begin{cases} P_r^{1} \rightarrow \text{COUETTE} \\ P_r^{1/2} \rightarrow \text{LAMINARE} \\ P_r^{1/3} \rightarrow \text{TURBOLENTO} \end{cases}$

$$H_{REC} = h + R \cdot \frac{u^2}{2} = h_e + R\frac{U_e^2}{2} = h_w = \text{COST.}$$

$$T = T_e + R \cdot \frac{U_e^2}{2c_p}(1 - \varphi^2)$$

$$T_w = T_{REC} = T_e + \frac{U_e^2}{2c_p} = T_e\left(1 + \frac{\gamma - 1}{2} R M_e^2\right)$$

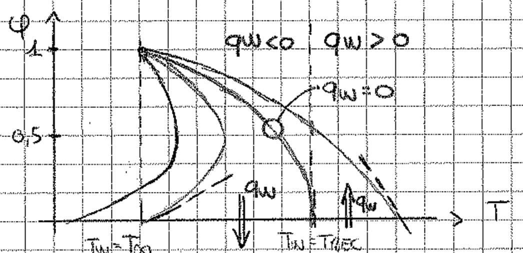
• CASO NON ADIABATICO ($q_w \neq 0$)

CROCCO $\Rightarrow H = C_1 + C_2 u$ $\begin{cases} \rightarrow C_1 = H_w \\ \rightarrow C_2 = (H_e - H_w)/U_e \end{cases}$

$$H = H_w + \frac{H_e - H_w}{U_e} \cdot u$$

$T = T_w(1 - \varphi) + T_\infty \varphi^2 + T_0(\varphi - \varphi^2)$ per $P_r = 1$ $\qquad T_0 = T_w\left(1 + \frac{\gamma - 1}{2} M_\infty^2\right)$

$T = T_w(1 - \varphi) + T_\infty \varphi^2 + T_{REC}(\varphi - \varphi^2)$ per $P_r \neq 1$



$T_\infty < T_w < T_R \rightarrow T_w$ causano $q_w \Downarrow$ (oppure $T_w > T_\infty$)
$T_w > T_R \rightarrow q_w \Uparrow$
$T_w < T_\infty \rightarrow q_w \Downarrow$

ANALOGIA DEI CAMPI $\longrightarrow \dfrac{u}{U_e} = \dfrac{H - H_p}{H_e - H_p} = \dfrac{h + \frac{1}{2}u - h_p}{h_e + \frac{1}{2}U_e - h_p}$ (solo se $P_r = 1$)

$T = T_\infty \varphi^2 + T_{REC}(1 - \varphi^2)$ CASO ADIABATICO ($T_w = T_{REC}$)
$T = T_\infty + (T_{REC} - T_\infty)(\varphi - \varphi^2)$ " ISOTERMO ($T_w = T_\infty$)

ANALOGIA DI REYNOLDS $\longrightarrow \dfrac{\tau_w(x)}{U_e} = \dfrac{-q_w(x)}{h_\infty + \frac{1}{2}U_\infty^2 - h_w}$ se $P_r = 1$ $\qquad \begin{cases} A = P_r^{a} \\ R = P_r^{r} \end{cases}$

$\begin{cases} a = 1, & r = 1 & \text{COUETTE} \\ a = 2/3, & r = 1/2 & \text{LAMINAR} \\ a = 0{,}6, & r = 1/3 & \text{TURBOLENTO} \end{cases}$ $\qquad \dfrac{\tau_w(x)}{U_e} = \dfrac{-A q_w(x)}{c_p(T_{REC} - T_w)} = \dfrac{-A q_w(x)}{h_\infty + R \cdot \frac{1}{2}U_\infty^2 - h_w}$ se $P_r \neq 1$

ANALOGIA DEGLI SCAMBI INTEGRALI $\longrightarrow \dfrac{\overline{\tau_w}}{U_e} = \dfrac{-A\overline{q_w}}{c_p(T_{rec} - T_w)}$

CONVEZ. FORZATA

$\overline{q_w} = -\overline{K}\Delta T = -\dfrac{\overline{\tau_w}}{V_\infty A} c_p(T_{rec} - T_w)$, con $\overline{K} = \rho_\infty c_p U_\infty C_D / 2A$

• SEZIONE RAPPRESENTATIVA ($\varphi = 0{,}5 \rightarrow T^*, Re^*$)

$Re = \dfrac{V_\infty L \rho_\infty}{\mu_\infty} < 5 \cdot 10^5$ LAMINARE P.P.

$$T^* = 0{,}5\, T_w + 0{,}25\, T_\infty + 0{,}25\, T_{rec}$$

χ = FATTORE CORRETTIVO: $\chi_L = \left(\dfrac{T_w + T_\infty}{2T_\infty} + R\dfrac{\gamma - 1}{2} \cdot \dfrac{M_\infty^2}{4}\right)^{-0{,}125}$

$\chi_T = \left(\dfrac{T_w + T_\infty}{2T_\infty} + R\dfrac{\gamma - 1}{2} \cdot \dfrac{M_\infty^2}{4}\right)^{-0{,}66}$

dove

$Cf_{COMP} = Cf_{INCOMP} \cdot \chi$; $\tau_{w_C} = \chi \cdot \tau_{w_{INC}}$

$C_{D_{COMP}} = C_{D_{INCOMP}} \cdot \chi$; $\overline{\tau_{w_C}} = \chi\, \overline{\tau_{w_{INC}}}$

• FLUSSO DI CALORE SU P.P.

FORMULA DI ROMIG: $q_w = 0{,}0407 \cdot M_\infty^{3,1} \sqrt{\dfrac{\rho_\infty}{Re}}$ $\left[W/m^2\right]$

- **MISCELE DI GAS**

$x_i = N_i / N_t \; ; \quad c_i = m_i / m_t \; ; \quad M_t = m_t / N_t = \sum x_i M_i = 1 / \sum c_i M_i$

DALTON: $P_t = \sum P_i \; , \quad P_i / P_t = N_i / N_t = V_i / V_t$

SUTHERLAND: $\mu = S \cdot \frac{T^{3/2}}{\chi + T} \; , \quad \chi = T_{RIF} \; , \quad S = \mu|_{T=\chi}$

WILKE: $\mu_{MISC} = \sum_i \mu_i \cdot \left(1 + \sum_k G_{ik} \cdot x_k / x_i\right)^{-1}$

dove $G_{ik} = \frac{\left[1 + (\mu_i/\mu_k)^{1/2} (M_i/M_k)^{1/4}\right]^2}{2^{3/2} \cdot \left[1 + (M_i/M_k)\right]^{1/2}}$

$c_v = \frac{L}{2} \cdot \frac{R}{M} = \frac{1}{\gamma - 1} \cdot \frac{R}{M}$

$c_p = \frac{L+2}{2} \cdot \frac{R}{M} = \frac{\gamma}{\gamma - 1} \cdot \frac{R}{M}$

$\gamma = \frac{L+2}{2} = \frac{c_p}{c_v} \; ; \quad R = c_p - c_v$

$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/\gamma-1} , \quad \frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{1/\gamma-1}$

$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} \quad \left(\frac{p}{\rho}\right)^{\gamma} = cost$

EQ. ENTROPIA: $S_2 - S_1 = c_p \cdot \ln(T_2/T_1) - R^* \cdot \ln(P_2/P_1)$

$S_2 - S_1 = c_v \cdot \ln(T_2/T_1) + R^* \cdot \ln(\rho_1/\rho_2)$

- **FORZE E MOMENTI**

$C_p = \frac{P - P_\infty}{1/2 \, \rho_\infty V_\infty} \; ; \quad$ se $C_p > 0$ $\downarrow \Delta P$; $C_p < 0$ $\uparrow \Delta P$

$C_L = \frac{L}{1/2 \, \rho_\infty V_\infty S} \; , \quad C_D = \frac{D}{1/2 \, \rho_\infty V_\infty S} \; , \quad C_M = \frac{M}{1/2 \, \rho_\infty V_\infty S C}$

- **CROCCO**

$\underbrace{T \nabla s}_{\text{ENTROPIA}} = \underbrace{\nabla H}_{\text{ENTALPIA}} - \bar{V} \times \nabla \times \bar{V} + \underbrace{\frac{\partial \bar{V}}{\partial t}}_{\text{VELOCITA' NON STAZ.}}$

- **VEL. POTENZIALE**

$\bar{V} = \nabla \phi \rightarrow u\bar{i} + v\bar{j} + w\bar{k} = \frac{\partial \phi}{\partial x} \bar{i} + \frac{\partial \phi}{\partial y} \bar{j} + \frac{\partial \phi}{\partial z} \bar{k}$

- **MECCANICA STATISTICA**

$K_b = 1{,}38 \cdot 10^{-6}$

$\left.\begin{array}{l} v_{AT} = \sqrt{2 \varepsilon_q / m} \\ \varepsilon_q = \frac{3}{2} K_b T \end{array}\right\} \; V_{AT} = \sqrt{\frac{3RT}{M}}$

KNUDSENT $K_N = \frac{l}{L} \; , \quad l = 1/d^2 n \cdot 1/\sqrt{2}\,\pi$

$\mathcal{E} = \frac{L}{2} RT$ (en molare) ; $H = \frac{L+2}{2} RT$

- **EQUAZIONI FLUSSI 1D**

- C $\rho u = cos$
- QDM $P + \rho u^2 = cost$
- E $H_1 + q = H_2$

$\theta = \nu(M)$

$\theta = f(M)$

M	θ	M	θ	M	θ	M	θ	M	θ	M	θ	M	θ	M	θ	M	θ	M	θ
1,00	0,0000	1,50	11,9052	2,00	26,3798	2,50	39,1236	3,00	49,7573	3,50	58,5298	4,00	65,7848	4,50	71,8317	5,00	76,9202	5,50	81,2448
1,01	0,0447	1,51	12,1999	2,01	26,6550	2,51	39,3565	3,01	49,9499	3,51	58,6886	4,01	65,9167	4,51	71,9422	5,01	77,0136	5,51	81,3246
1,02	0,1257	1,52	12,4949	2,02	26,9295	2,52	39,5886	3,02	50,1417	3,52	58,8469	4,02	66,0480	4,52	72,0522	5,02	77,1067	5,52	81,4041
1,03	0,2294	1,53	12,7901	2,03	27,2033	2,53	39,8199	3,03	50,3328	3,53	59,0045	4,03	66,1789	4,53	72,1619	5,03	77,1995	5,53	81,4834
1,04	0,3510	1,54	13,0856	2,04	27,4762	2,54	40,0503	3,04	50,5231	3,54	59,1616	4,04	66,3093	4,54	72,2712	5,04	77,2921	5,54	81,5625
1,05	0,4874	1,55	13,3812	2,05	27,7484	2,55	40,2798	3,05	50,7127	3,55	59,3180	4,05	66,4393	4,55	72,3801	5,05	77,3843	5,55	81,6413
1,06	0,6367	1,56	13,6770	2,06	28,0197	2,56	40,5085	3,06	50,9016	3,56	59,4739	4,06	66,5688	4,56	72,4887	5,06	77,4762	5,56	81,7199
1,07	0,7973	1,57	13,9728	2,07	28,2903	2,57	40,7363	3,07	51,0897	3,57	59,6291	4,07	66,6978	4,57	72,5968	5,07	77,5678	5,57	81,7983
1,08	0,9680	1,58	14,2686	2,08	28,5600	2,58	40,9633	3,08	51,2771	3,58	59,7838	4,08	66,8263	4,58	72,7046	5,08	77,6591	5,58	81,8764
1,09	1,1479	1,59	14,5645	2,09	28,8290	2,59	41,1894	3,09	51,4638	3,59	59,9379	4,09	66,9544	4,59	72,8121	5,09	77,7501	5,59	81,9543
1,10	1,3362	1,60	14,8604	2,10	29,0971	2,60	41,4147	3,10	51,6497	3,60	60,0915	4,10	67,0820	4,60	72,9192	5,10	77,8409	5,60	82,0319
1,11	1,5321	1,61	15,1561	2,11	29,3644	2,61	41,6392	3,11	51,8350	3,61	60,2444	4,11	67,2092	4,61	73,0259	5,11	77,9313	5,61	82,1093
1,12	1,7350	1,62	15,4518	2,12	29,6308	2,62	41,8628	3,12	52,0195	3,62	60,3968	4,12	67,3359	4,62	73,1322	5,12	78,0215	5,62	82,1865
1,13	1,9445	1,63	15,7473	2,13	29,8965	2,63	42,0855	3,13	52,2033	3,63	60,5486	4,13	67,4621	4,63	73,2382	5,13	78,1113	5,63	82,2634
1,14	2,1600	1,64	16,0427	2,14	30,1613	2,64	42,3074	3,14	52,3864	3,64	60,6998	4,14	67,5879	4,64	73,3438	5,14	78,2009	5,64	82,3401
1,15	2,3810	1,65	16,3379	2,15	30,4253	2,65	42,5285	3,15	52,5688	3,65	60,8504	4,15	67,7132	4,65	73,4491	5,15	78,2902	5,65	82,4166
1,16	2,6073	1,66	16,6328	2,16	30,6884	2,66	42,7488	3,16	52,7505	3,66	61,0005	4,16	67,8381	4,66	73,5540	5,16	78,3792	5,66	82,4929
1,17	2,8385	1,67	16,9276	2,17	30,9507	2,67	42,9682	3,17	52,9315	3,67	61,1501	4,17	67,9626	4,67	73,6586	5,17	78,4679	5,67	82,5689
1,18	3,0743	1,68	17,2220	2,18	31,2121	2,68	43,1868	3,18	53,1118	3,68	61,2990	4,18	68,0866	4,68	73,7628	5,18	78,5564	5,68	82,6447
1,19	3,3142	1,69	17,5161	2,19	31,4727	2,69	43,4045	3,19	53,2914	3,69	61,4474	4,19	68,2101	4,69	73,8666	5,19	78,6445	5,69	82,7202
1,20	3,5582	1,70	17,8099	2,20	31,7325	2,70	43,6215	3,20	53,4703	3,70	61,5953	4,20	68,3332	4,70	73,9701	5,20	78,7324	5,70	82,7956
1,21	3,8060	1,71	18,1034	2,21	31,9914	2,71	43,8376	3,21	53,6486	3,71	61,7426	4,21	68,4559	4,71	74,0733	5,21	78,8200	5,71	82,8707
1,22	4,0572	1,72	18,3964	2,22	32,2494	2,72	44,0529	3,22	53,8261	3,72	61,8893	4,22	68,5782	4,72	74,1761	5,22	78,9074	5,72	82,9456
1,23	4,3117	1,73	18,6891	2,23	32,5066	2,73	44,2673	3,23	54,0029	3,73	62,0355	4,23	68,7000	4,73	74,2786	5,23	78,9944	5,73	83,0203
1,24	4,5694	1,74	18,9814	2,24	32,7629	2,74	44,4810	3,24	54,1791	3,74	62,1812	4,24	68,8213	4,74	74,3807	5,24	79,0812	5,74	83,0947
1,25	4,8299	1,75	19,2732	2,25	33,0184	2,75	44,6938	3,25	54,3546	3,75	62,3263	4,25	68,9423	4,75	74,4824	5,25	79,1677	5,75	83,1689
1,26	5,0931	1,76	19,5646	2,26	33,2730	2,76	44,9059	3,26	54,5294	3,76	62,4709	4,26	69,0628	4,76	74,5839	5,26	79,2539	5,76	83,2430
1,27	5,3590	1,77	19,8554	2,27	33,5268	2,77	45,1171	3,27	54,7036	3,77	62,6149	4,27	69,1829	4,77	74,6850	5,27	79,3399	5,77	83,3167
1,28	5,6272	1,78	20,1458	2,28	33,7796	2,78	45,3275	3,28	54,8770	3,78	62,7584	4,28	69,3026	4,78	74,7858	5,28	79,4256	5,78	83,3903
1,29	5,8977	1,79	20,4357	2,29	34,0316	2,79	45,5371	3,29	55,0498	3,79	62,9014	4,29	69,4218	4,79	74,8862	5,29	79,5110	5,79	83,4637
1,30	6,1703	1,80	20,7251	2,30	34,2828	2,80	45,7459	3,30	55,2220	3,80	63,0438	4,30	69,5406	4,80	74,9863	5,30	79,5962	5,80	83,5368
1,31	6,4449	1,81	21,0139	2,31	34,5331	2,81	45,9539	3,31	55,3935	3,81	63,1857	4,31	69,6590	4,81	75,0860	5,31	79,6810	5,81	83,6097
1,32	6,7213	1,82	21,3021	2,32	34,7825	2,82	46,1611	3,32	55,5643	3,82	63,3271	4,32	69,7770	4,82	75,1855	5,32	79,7657	5,82	83,6824
1,33	6,9995	1,83	21,5898	2,33	35,0310	2,83	46,3675	3,33	55,7344	3,83	63,4679	4,33	69,8946	4,83	75,2846	5,33	79,8500	5,83	83,7549
1,34	7,2794	1,84	21,8768	2,34	35,2787	2,84	46,5731	3,34	55,9040	3,84	63,6083	4,34	70,0118	4,84	75,3833	5,34	79,9341	5,84	83,8272
1,35	7,5607	1,85	22,1633	2,35	35,5255	2,85	46,7779	3,35	56,0728	3,85	63,7481	4,35	70,1285	4,85	75,4818	5,35	80,0180	5,85	83,8993
1,36	7,8435	1,86	22,4492	2,36	35,7715	2,86	46,9820	3,36	56,2411	3,86	63,8874	4,36	70,2449	4,86	75,5799	5,36	80,1015	5,86	83,9711
1,37	8,1276	1,87	22,7344	2,37	36,0165	2,87	47,1852	3,37	56,4086	3,87	64,0262	4,37	70,3608	4,87	75,6777	5,37	80,1848	5,87	84,0428
1,38	8,4130	1,88	23,0190	2,38	36,2607	2,88	47,3877	3,38	56,5756	3,88	64,1645	4,38	70,4763	4,88	75,7752	5,38	80,2679	5,88	84,1142
1,39	8,6995	1,89	23,3029	2,39	36,5041	2,89	47,5894	3,39	56,7419	3,89	64,3023	4,39	70,5914	4,89	75,8723	5,39	80,3507	5,89	84,1855
1,40	8,9870	1,90	23,5861	2,40	36,7465	2,90	47,7903	3,40	56,9075	3,90	64,4395	4,40	70,7062	4,90	75,9691	5,40	80,4332	5,90	84,2565
1,41	9,2756	1,91	23,8687	2,41	36,9881	2,91	47,9905	3,41	57,0725	3,91	64,5763	4,41	70,8205	4,91	76,0657	5,41	80,5155	5,91	84,3273
1,42	9,5650	1,92	24,1506	2,42	37,2289	2,92	48,1898	3,42	57,2369	3,92	64,7125	4,42	70,9344	4,92	76,1619	5,42	80,5976	5,92	84,3979
1,43	9,8553	1,93	24,4318	2,43	37,4687	2,93	48,3884	3,43	57,4007	3,93	64,8483	4,43	71,0479	4,93	76,2577	5,43	80,6793	5,93	84,4683
1,44	10,1464	1,94	24,7123	2,44	37,7077	2,94	48,5863	3,44	57,5639	3,94	64,9836	4,44	71,1611	4,94	76,3533	5,44	80,7609	5,94	84,5385
1,45	10,4381	1,95	24,9920	2,45	37,9459	2,95	48,7833	3,45	57,7264	3,95	65,1183	4,45	71,2738	4,95	76,4486	5,45	80,8421	5,95	84,6085
1,46	10,7305	1,96	25,2711	2,46	38,1831	2,96	48,9796	3,46	57,8883	3,96	65,2526	4,46	71,3862	4,96	76,5435	5,46	80,9232	5,96	84,6783
1,47	11,0235	1,97	25,5494	2,47	38,4195	2,97	49,1752	3,47	58,0496	3,97	65,3864	4,47	71,4982	4,97	76,6382	5,47	81,0039	5,97	84,7479
1,48	11,3169	1,98	25,8269	2,48	38,6551	2,98	49,3700	3,48	58,2102	3,98	65,5197	4,48	71,6097	4,98	76,7325	5,48	81,0845	5,98	84,8173
1,49	11,6109	1,99	26,1037	2,49	38,8897	2,99	49,5640	3,49	58,3703	3,99	65,6525	4,49	71,7209	4,99	76,8265	5,49	81,1648	5,99	84,8865

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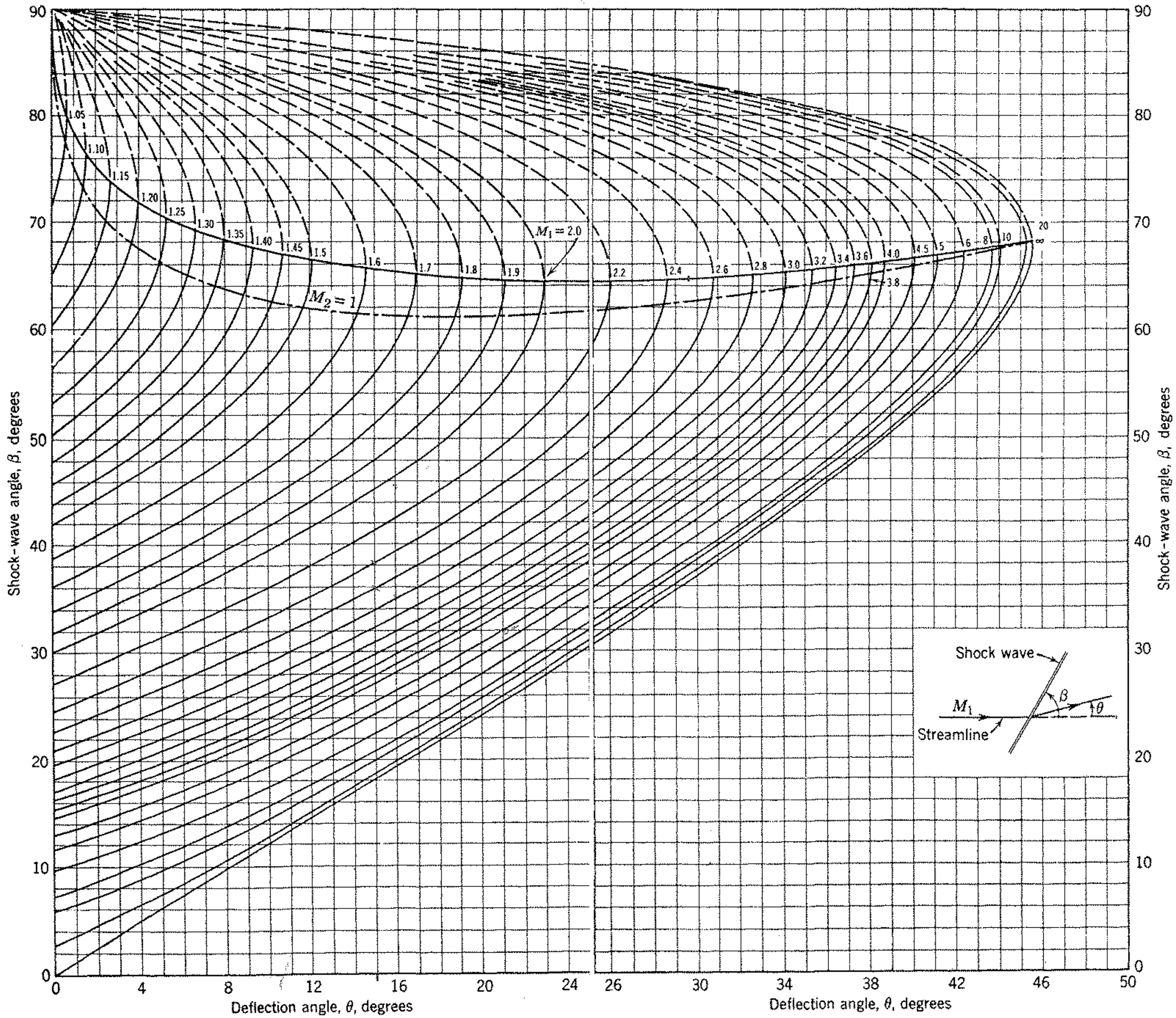
37°
83°



Oblique Shock Chart 1 Variation of shock-wave angle with flow-deflection angle for various upstream Mach numbers. Perfect gas, $\gamma = 1.40$. (From *NACA Report* 1135.)

Chart 1 (continued)

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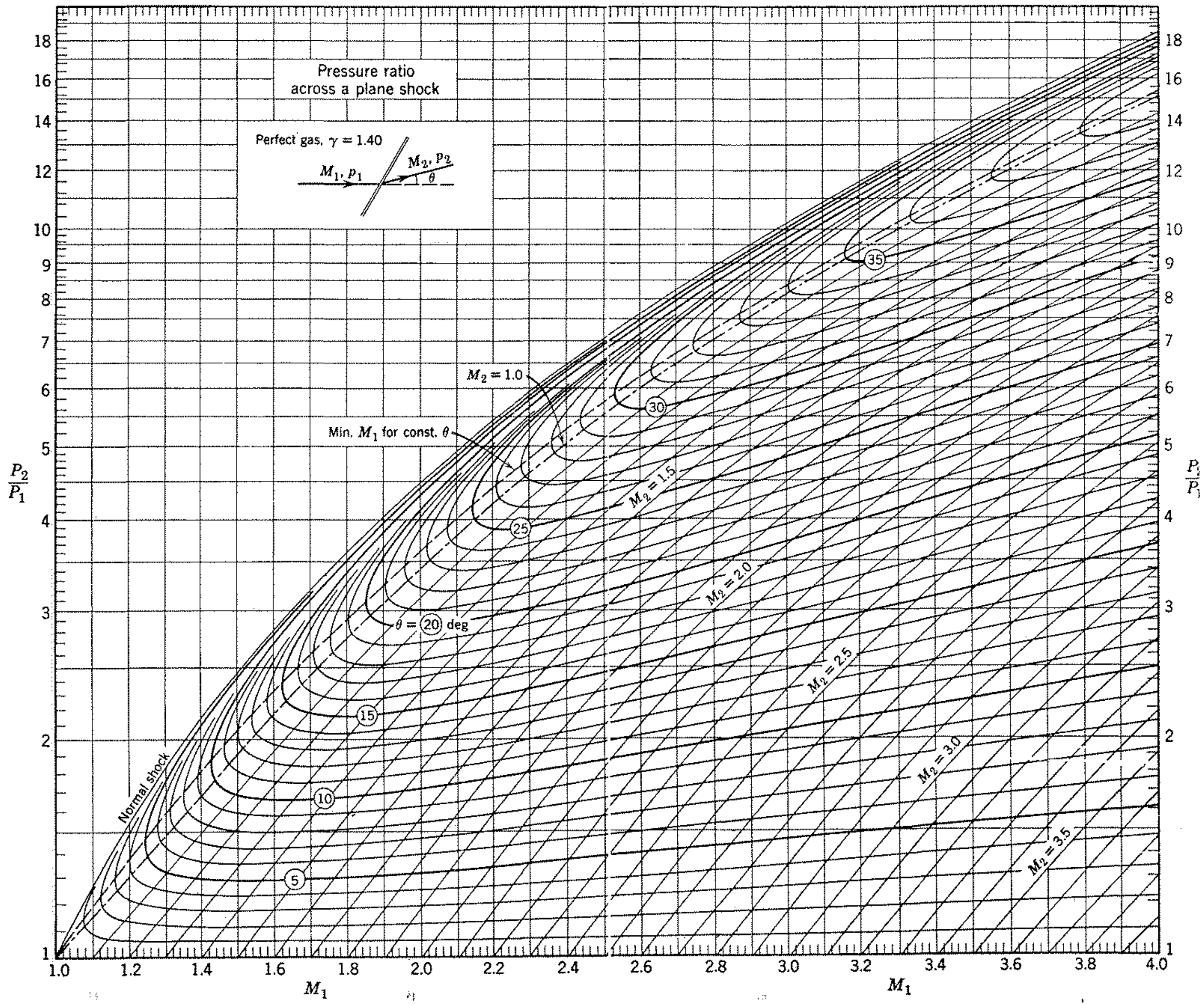

Oblique Shock Chart 2 Variation of pressure ratio and downstream Mach number with flow-deflection angle and upstream Mach number. (Data from C. L. Dailey and F. C. Wood, *Computation Curves for Compressible Flow Problems*, Wiley, 1949.)

Chart 2 (continued)

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PARAMETRI DI FLUSSO SUBSONICO

TABLE II

FLOW PARAMETERS VERSUS M FOR SUBSONIC FLOW

M	p/p_0	ρ/ρ_0	T/T_0	a/a_0	A^*/A
.00	1.0000	1.0000	1.0000	1.0000	.00000
.01	.9999	1.0000	1.0000	1.0000	.01728
.02	.9997	.9998	.9999	1.0000	.03455
.03	.9994	.9996	.9998	.9999	.05181
.04	.9989	.9992	.9997	.9998	.06905
.05	.9983	.9988	.9995	.9998	.08627
.06	.9975	.9982	.9993	.9996	.1035
.07	.9966	.9976	.9990	.9995	.1206
.08	.9955	.9968	.9987	.9994	.1377
.09	.9944	.9960	.9984	.9992	.1548
.10	.9930	.9950	.9980	.9990	.1718
.11	.9916	.9940	.9976	.9988	.1887
.12	.9900	.9928	.9971	.9986	.2056
.13	.9883	.9916	.9966	.9983	.2224
.14	.9864	.9903	.9961	.9980	.2391
.15	.9844	.9888	.9955	.9978	.2557
.16	.9823	.9873	.9949	.9974	.2723
.17	.9800	.9857	.9943	.9971	.2887
.18	.9776	.9840	.9936	.9968	.3051
.19	.9751	.9822	.9928	.9964	.3213
.20	.9725	.9803	.9921	.9960	.3374
.21	.9697	.9783	.9913	.9956	.3534
.22	.9668	.9762	.9904	.9952	.3693
.23	.9638	.9740	.9895	.9948	.3851
.24	.9607	.9718	.9886	.9943	.4007
.25	.9575	.9694	.9877	.9938	.4162
.26	.9541	.9670	.9867	.9933	.4315
.27	.9506	.9645	.9856	.9928	.4467
.28	.9470	.9619	.9846	.9923	.4618
.29	.9433	.9592	.9835	.9917	.4767
.30	.9395	.9564	.9823	.9911	.4914
.31	.9355	.9535	.9811	.9905	.5059
.32	.9315	.9506	.9799	.9899	.5203
.33	.9274	.9476	.9787	.9893	.5345
.34	.9231	.9445	.9774	.9886	.5486

TABLE II (*Continued*)

FLOW PARAMETERS VERSUS M FOR SUBSONIC FLOW

M	p/p_0	ρ/ρ_0	T/T_0	a/a_0	A^*/A
.35	.9188	.9413	.9761	.9880	.5624
.36	.9143	.9380	.9747	.9873	.5761
.37	.9098	.9347	.9733	.9866	.5896
.38	.9052	.9313	.9719	.9859	.6029
.39	.9004	.9278	.9705	.9851	.6160
.40	.8956	.9243	.9690	.9844	.6289
.41	.8907	.9207	.9675	.9836	.6416
.42	.8857	.9170	.9659	.9828	.6541
.43	.8807	.9132	.9643	.9820	.6663
.44	.8755	.9094	.9627	.9812	.6784
.45	.8703	.9055	.9611	.9803	.6903
.46	.8650	.9016	.9594	.9795	.7019
.47	.8596	.8976	.9577	.9786	.7134
.48	.8541	.8935	.9560	.9777	.7246
.49	.8486	.8894	.9542	.9768	.7356
.50	.8430	.8852	.9524	.9759	.7464
.51	.8374	.8809	.9506	.9750	.7569
.52	.8317	.8766	.9487	.9740	.7672
.53	.8259	.8723	.9468	.9730	.7773
.54	.8201	.8679	.9449	.9721	.7872
.55	.8142	.8634	.9430	.9711	.7968
.56	.8082	.8589	.9410	.9701	.8063
.57	.8022	.8544	.9390	.9690	.8155
.58	.7962	.8498	.9370	.9680	.8244
.59	.7901	.8451	.9349	.9669	.8331
.60	.7840	.8405	.9328	.9658	.8416
.61	.7778	.8357	.9307	.9647	.8499
.62	.7716	.8310	.9286	.9636	.8579
.63	.7654	.8262	.9265	.9625	.8657
.64	.7591	.8213	.9243	.9614	.8732
.65	.7528	.8164	.9221	.9603	.8806
.66	.7465	.8115	.9199	.9591	.8877
.67	.7401	.8066	.9176	.9579	.8945
.68	.7338	.8016	.9153	.9567	.9012
.69	.7274	.7966	.9131	.9555	.9076

TABLE II (*Continued*)

FLOW PARAMETERS VERSUS M FOR SUBSONIC FLOW

M	p/p_0	ρ/ρ_0	T/T_0	a/a_0	A^*/A
.70	.7209	.7916	.9107	.9543	.9138
.71	.7145	.7865	.9084	.9531	.9197
.72	.7080	.7814	.9061	.9519	.9254
.73	.7016	.7763	.9037	.9506	.9309
.74	.0951	.7712	.9013	.9494	.9362
.75	.6886	.7660	.8989	.9481	.9412
.76	.6821	.7609	.8964	.9468	.9461
.77	.6756	.7557	.8940	.9455	.9507
.78	.6690	.7505	.8915	.9442	.9551
.79	.6625	.7452	.8890	.9429	.9592
.80	.6560	.7400	.8865	.9416	.9632
.81	.6495	.7347	.8840	.9402	.9669
.82	.6430	.7295	.8815	.9389	.9704
.83	.6365	.7242	.8789	.9375	.9737
.84	.6300	.7189	.8763	.9361	.9769
.85	.6235	.7136	.8737	.9347	.9797
.86	.6170	.7083	.8711	.9333	.9824
.87	.6106	.7030	.8685	.9319	.9849
.88	.6041	.6977	.8659	.9305	.9872
.89	.5977	.6924	.8632	.9291	.9893
.90	.5913	.6870	.8606	.9277	.9912
.91	.5849	.6817	.8579	.9262	.9929
.92	.5785	.6764	.8552	.9248	.9944
.93	.5721	.6711	.8525	.9233	.9958
.94	.5658	.6658	.8498	.9218	.9969
.95	.5595	.6604	.8471	.9204	.9979
.96	.5532	.6551	.8444	.9189	.9986
.97	.5469	.6498	.8416	.9174	.9992
.98	.5407	.6445	.8389	.9159	.9997
.99	.5345	.6392	.8361	.9144	.9999
1.00	.5283	.6339	.8333	.9129	1.0000

Numerical values taken from NACA TN 1428, courtesy of the National Advisory Committee for Aeronautics. Setup from A. M. Kuethe and J. D. Schetzer, *Foundations of Aerodynamics*, John Wiley & Sons, New York, 1950.

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PARAMETRI DI FLUSSO SUPERSONICO

TABLE III

Flow Parameters versus M for Supersonic Flow

M	$\frac{p}{p_0}$	$\frac{\rho}{\rho_0}$	$\frac{T}{T_0}$	$\frac{a}{a_0}$	$\frac{A^*}{A}$	$\frac{\frac{\rho}{2}V^2}{p_0}$	θ
1.00	.5283	.6339	.8333	.9129	1.0000	.3698	0
1.01	.5221	.6287	.8306	.9113	.9999	.3728	.04473
1.02	.5160	.6234	.8278	.9098	.9997	.3758	.1257
1.03	.5099	.6181	.8250	.9083	.9993	.3787	.2294
1.04	.5039	.6129	.8222	.9067	.9987	.3815	.3510
1.05	.4979	.6077	.8193	.9052	.9980	.3842	.4874
1.06	.4919	.6024	.8165	.9036	.9971	.3869	.6367
1.07	.4860	.5972	.8137	.9020	.9961	.3895	.7973
1.08	.4800	.5920	.8108	.9005	.9949	.3919	.9680
1.09	.4742	.5869	.8080	.8989	.9936	.3944	1.148
1.10	.4684	.5817	.8052	.8973	.9921	.3967	1.336
1.11	.4626	.5766	.8023	.8957	.9905	.3990	1.532
1.12	.4568	.5714	.7994	.8941	.9888	.4011	1.735
1.13	.4511	.5663	.7966	.8925	.9870	.4032	1.944
1.14	.4455	.5612	.7937	.8909	.9850	.4052	2.160
1.15	.4398	.5562	.7908	.8893	.9828	.4072	2.381
1.16	.4343	.5511	.7879	.8877	.9806	.4090	2.607
1.17	.4287	.5461	.7851	.8860	.9782	.4108	2.839
1.18	.4232	.5411	.7822	.8844	.9758	.4125	3.074
1.19	.4178	.5361	.7793	.8828	.9732	.4141	3.314
1.20	.4124	.5311	.7764	.8811	.9705	.4157	3.558
1.21	.4070	.5262	.7735	.8795	.9676	.4171	3.806
1.22	.4017	.5213	.7706	.8778	.9647	.4185	4.057
1.23	.3964	.5164	.7677	.8762	.9617	.4198	4.312
1.24	.3912	.5115	.7648	.8745	.9586	.4211	4.569
1.25	.3861	.5067	.7619	.8729	.9553	.4223	4.830
1.26	.3809	.5019	.7590	.8712	.9520	.4233	5.093
1.27	.3759	.4971	.7561	.8695	.9486	.4244	5.359
1.28	.3708	.4923	.7532	.8679	.9451	.4253	5.627
1.29	.3658	.4876	.7503	.8662	.9415	.4262	5.898
1.30	.3609	.4829	.7474	.8645	.9378	.4270	6.170
1.31	.3560	.4782	.7445	.8628	.9341	.4277	6.445
1.32	.3512	.4736	.7416	.8611	.9302	.4283	6.721
1.33	.3464	.4690	.7387	.8595	.9263	.4289	7.000
1.34	.3417	.4644	.7358	.8578	.9223	.4294	7.279

TABLE III (*Continued*)

Flow Parameters versus M for Supersonic Flow

M	$\frac{p}{p_0}$	$\frac{\rho}{\rho_0}$	$\frac{T}{T_0}$	$\frac{a}{a_0}$	$\frac{A^*}{A}$	$\frac{\frac{\rho}{2}V^2}{p_0}$	θ
1.35	.3370	.4598	.7329	.8561	.9182	.4299	7.561
1.36	.3323	.4553	.7300	.8544	.9141	.4303	7.844
1.37	.3277	.4508	.7271	.8527	.9099	.4306	8.128
1.38	.3232	.4463	.7242	.8510	.9056	.4308	8.413
1.39	.3187	.4418	.7213	.8493	.9013	.4310	8.699
1.40	.3142	.4374	.7184	.8476	.8969	.4311	8.987
1.41	.3098	.4330	.7155	.8459	.8925	.4312	9.276
1.42	.3055	.4287	.7126	.8442	.8880	.4312	9.565
1.43	.3012	.4244	.7097	.8425	.8834	.4311	9.855
1.44	.2969	.4201	.7069	.8407	.8788	.4310	10.15
1.45	.2927	.4158	.7040	.8390	.8742	.4308	10.44
1.46	.2886	.4116	.7011	.8373	.8695	.4306	10.73
1.47	.2845	.4074	.6982	.8356	.8647	.4303	11.02
1.48	.2804	.4032	.6954	.8339	.8599	.4299	11.32
1.49	.2764	.3991	.6925	.8322	.8551	.4295	11.61
1.50	.2724	.3950	.6897	.8305	.8502	.4290	11.91
1.51	.2685	.3909	.6868	.8287	.8453	.4285	12.20
1.52	.2646	.3869	.6840	.8270	.8404	.4279	12.49
1.53	.2608	.3829	.6811	.8253	.8354	.4273	12.79
1.54	.2570	.3789	.6783	.8236	.8304	.4266	13.09
1.55	.2533	.3750	.6754	.8219	.8254	.4259	13.38
1.56	.2496	.3710	.6726	.8201	.8203	.4252	13.68
1.57	.2459	.3672	.6698	.8184	.8152	.4243	13.97
1.58	.2423	.3633	.6670	.8167	.8101	.4235	14.27
1.59	.2388	.3595	.6642	.8150	.8050	.4226	14.56
1.60	.2353	.3557	.6614	.8133	.7998	.4216	14.86
1.61	.2318	.3520	.6586	.8115	.7947	.4206	15.16
1.62	.2284	.3483	.6558	.8098	.7895	.4196	15.45
1.63	.2250	.3446	.6530	.8081	.7843	.4185	15.75
1.64	.2217	.3409	.6502	.8064	.7791	.4174	16.04
1.65	.2184	.3373	.6475	.8046	.7739	.4162	16.34
1.66	.2151	.3337	.6447	.8029	.7686	.4150	16.63
1.67	.2119	.3302	.6419	.8012	.7634	.4138	16.93
1.68	.2088	.3266	.6392	.7995	.7581	.4125	17.22
1.69	.2057	.3232	.6364	.7978	.7529	.4112	17.52

TABLE III (*Continued*)

Flow Parameters versus M for Supersonic Flow

M	$\frac{p}{p_0}$	$\frac{\rho}{\rho_0}$	$\frac{T}{T_0}$	$\frac{a}{a_0}$	$\frac{A^*}{A}$	$\frac{\frac{\rho}{2}V^2}{p_0}$	θ
1.70	.2026	.3197	.6337	.7961	.7476	.4098	17.81
1.71	.1996	.3163	.6310	.7943	.7423	.4086	18.10
1.72	.1966	.3129	.6283	.7926	.7371	.4071	18.40
1.73	.1936	.3095	.6256	.7909	.7318	.4056	18.69
1.74	.1907	.3062	.6229	.7892	.7265	.4041	18.98
1.75	.1878	.3029	.6202	.7875	.7212	.4026	19.27
1.76	.1850	.2996	.6175	.7858	.7160	.4011	19.56
1.77	.1822	.2964	.6148	.7841	.7107	.3996	19.86
1.78	.1794	.2932	.6121	.7824	.7054	.3980	20.15
1.79	.1767	.2900	.6095	.7807	.7002	.3964	20.44
1.80	.1740	.2868	.6068	.7790	.6949	.3947	20.73
1.81	.1714	.2837	.6041	.7773	.6897	.3931	21.01
1.82	.1688	.2806	.6015	.7756	.6845	.3914	21.30
1.83	.1662	.2776	.5989	.7739	.6792	.3897	21.59
1.84	.1637	.2745	.5963	.7722	.6740	.3879	21.88
1.85	.1612	.2715	.5936	.7705	.6688	.3862	22.16
1.86	.1587	.2686	.5910	.7688	.6636	.3844	22.45
1.87	.1563	.2656	.5884	.7671	.6584	.3826	22.73
1.88	.1539	.2627	.5859	.7654	.6533	.3808	23.02
1.89	.1516	.2598	.5833	.7637	.6481	.3790	23.30
1.90	.1492	.2570	.5807	.7620	.6430	.3771	23.59
1.91	.1470	.2542	.5782	.7604	.6379	.3753	23.87
1.92	.1447	.2514	.5756	.7587	.6328	.3734	24.15
1.93	.1425	.2486	.5731	.7570	.6277	.3715	24.43
1.94	.1403	.2459	.5705	.7553	.6226	.3696	24.71
1.95	.1381	.2432	.5680	.7537	.6175	.3677	24.99
1.96	.1360	.2405	.5655	.7520	.6125	.3657	25.27
1.97	.1339	.2378	.5630	.7503	.6075	.3638	25.55
1.98	.1318	.2352	.5605	.7487	.6025	.3618	25.83
1.99	.1298	.2326	.5580	.7470	.5975	.3598	26.10
2.00	.1278	.2300	.5556	.7454	.5926	.3579	26.38
2.01	.1258	.2275	.5531	.7437	.5877	.3559	26.66
2.02	.1239	.2250	.5506	.7420	.5828	.3539	26.93
2.03	.1220	.2225	.5482	.7404	.5779	.3518	27.20
2.04	.1201	.2200	.5458	.7388	.5730	.3498	27.48

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TABLE III (*Continued*)

FLOW PARAMETERS VERSUS M FOR SUPERSONIC FLOW

M	$\frac{p}{p_0}$	$\frac{\rho}{\rho_0}$	$\frac{T}{T_0}$	$\frac{a}{a_0}$	$\frac{A^*}{A}$	$\frac{\frac{\rho}{2}V^2}{p_0}$	θ
2.05	.1182	.2176	.5433	.7371	.5682	.3478	27.75
2.06	.1164	.2152	.5409	.7355	.5634	.3458	28.02
2.07	.1146	.2128	.5385	.7338	.5586	.3437	28.29
2.08	.1128	.2104	.5361	.7322	.5538	.3417	28.56
2.09	.1111	.2081	.5337	.7306	.5491	.3396	28.83
2.10	.1094	.2058	.5313	.7289	.5444	.3376	29.10
2.11	.1077	.2035	.5290	.7273	.5397	.3355	29.36
2.12	.1060	.2013	.5266	.7257	.5350	.3334	29.63
2.13	.1043	.1990	.5243	.7241	.5304	.3314	29.90
2.14	.1027	.1968	.5219	.7225	.5258	.3293	30.16
2.15	.1011	.1946	.5196	.7208	.5212	.3272	30.43
2.16	.09956	.1925	.5173	.7192	.5167	.3252	30.69
2.17	.09802	.1903	.5150	.7176	.5122	.3231	30.95
2.18	.09650	.1882	.5127	.7160	.5077	.3210	31.21
2.19	.09500	.1861	.5104	.7144	.5032	.3189	31.47
2.20	.09352	.1841	.5081	.7128	.4988	.3169	31.73
2.21	.09207	.1820	.5059	.7112	.4944	.3148	31.99
2.22	.09064	.1800	.5036	.7097	.4900	.3127	32.25
2.23	.08923	.1780	.5014	.7081	.4856	.3106	32.51
2.24	.08785	.1760	.4991	.7065	.4813	.3085	32.76
2.25	.08648	.1740	.4969	.7049	.4770	.3065	33.02
2.26	.08514	.1721	.4947	.7033	.4727	.3044	33.27
2.27	.08382	.1702	.4925	.7018	.4685	.3023	33.53
2.28	.08252	.1683	.4903	.7002	.4643	.3003	33.78
2.29	.08123	.1664	.4881	.6986	.4601	.2982	34.03
2.30	.07997	.1646	.4859	.6971	.4560	.2961	34.28
2.31	.07873	.1628	.4837	.6955	.4519	.2941	34.53
2.32	.07751	.1609	.4816	.6940	.4478	.2920	34.78
2.33	.07631	.1592	.4794	.6924	.4437	.2900	35.03
2.34	.07512	.1574	.4773	.6909	.4397	.2879	35.28
2.35	.07396	.1556	.4752	.6893	.4357	.2859	35.53
2.36	.07281	.1539	.4731	.6878	.4317	.2839	35.77
2.37	.07168	.1522	.4709	.6863	.4278	.2818	36.02
2.38	.07057	.1505	.4688	.6847	.4239	.2798	36.26
2.39	.06948	.1488	.4668	.6832	.4200	.2778	36.50

TABLE III (*Continued*)

FLOW PARAMETERS VERSUS M FOR SUPERSONIC FLOW

M	$\frac{p}{p_0}$	$\frac{\rho}{\rho_0}$	$\frac{T}{T_0}$	$\frac{a}{a_0}$	$\frac{A^*}{A}$	$\frac{\frac{\rho}{2}V^2}{p_0}$	θ
2.40	.06840	.1472	.4647	.6817	.4161	.2758	36.75
2.41	.06734	.1456	.4626	.6802	.4123	.2738	36.99
2.42	.06630	.1439	.4606	.6786	.4085	.2718	37.23
2.43	.06527	.1424	.4585	.6771	.4048	.2698	37.47
2.44	.06426	.1408	.4565	.6756	.4010	.2678	37.71
2.45	.06327	.1392	.4544	.6741	.3973	.2658	37.95
2.46	.06229	.1377	.4524	.6726	.3937	.2639	38.18
2.47	.06133	.1362	.4504	.6711	.3900	.2619	38.42
2.48	.06038	.1347	.4484	.6696	.3864	.2599	38.66
2.49	.05945	.1332	.4464	.6681	.3828	.2580	38.89
2.50	.05853	.1317	.4444	.6667	.3793	.2561	39.12
2.51	.05762	.1302	.4425	.6652	.3757	.2541	39.36
2.52	.05674	.1288	.4405	.6637	.3722	.2522	39.59
2.53	.05586	.1274	.4386	.6622	.3688	.2503	39.82
2.54	.05500	.1260	.4366	.6608	.3653	.2484	40.05
2.55	.05415	.1246	.4347	.6593	.3619	.2465	40.28
2.56	.05332	.1232	.4328	.6579	.3585	.2446	40.51
2.57	.05250	.1218	.4309	.6564	.3552	.2427	40.75
2.58	.05169	.1205	.4289	.6549	.3519	.2409	40.96
2.59	.05090	.1192	.4271	.6535	.3486	.2390	41.19
2.60	.05012	.1179	.4252	.6521	.3453	.2371	41.41
2.61	.04935	.1166	.4233	.6506	.3421	.2353	41.64
2.62	.04859	.1153	.4214	.6492	.3389	.2335	41.86
2.63	.04784	.1140	.4196	.6477	.3357	.2317	42.09
2.64	.04711	.1128	.4177	.6463	.3325	.2298	42.31
2.65	.04639	.1115	.4159	.6449	.3294	.2280	42.53
2.66	.04568	.1103	.4141	.6435	.3263	.2262	42.75
2.67	.04498	.1091	.4122	.6421	.3232	.2245	42.97
2.68	.04429	.1079	.4104	.6406	.3202	.2227	43.19
2.69	.04362	.1067	.4086	.6392	.3172	.2209	43.40
2.70	.04295	.1056	.4068	.6378	.3142	.2192	43.62
2.71	.04229	.1044	.4051	.6364	.3112	.2174	43.84
2.72	.04165	.1033	.4033	.6350	.3083	.2157	44.05
2.73	.04102	.1022	.4015	.6337	.3054	.2140	44.27
2.74	.04039	.1010	.3998	.6323	.3025	.2123	44.48

TABLE III (*Continued*)

FLOW PARAMETERS VERSUS M FOR SUPERSONIC FLOW

M	$\frac{p}{p_0}$	$\frac{\rho}{\rho_0}$	$\frac{T}{T_0}$	$\frac{a}{a_0}$	$\frac{A^*}{A}$	$\frac{\frac{\rho}{2}V^2}{p_0}$	θ
2.75	.03978	.09994	.3980	.6309	.2996	.2106	44.69
2.76	.03917	.09885	.3963	.6295	.2968	.2089	44.91
2.77	.03858	.09778	.3945	.6281	.2940	.2072	45.12
2.78	.03799	.09671	.3928	.6268	.2912	.2055	45.33
2.79	.03742	.09566	.3911	.6254	.2884	.2039	45.54
2.80	.03685	.09463	.3894	.6240	.2857	.2022	45.75
2.81	.03629	.09360	.3877	.6227	.2830	.2006	45.95
2.82	.03574	.09259	.3860	.6213	.2803	.1990	46.16
2.83	.03520	.09158	.3844	.6200	.2777	.1973	46.37
2.84	.03467	.09059	.3827	.6186	.2750	.1957	46.57
2.85	.03415	.08962	.3810	.6173	.2724	.1941	46.78
2.86	.03363	.08865	.3794	.6159	.2698	.1926	46.98
2.87	.03312	.08769	.3777	.6146	.2673	.1910	47.19
2.88	.03263	.08675	.3761	.6133	.2648	.1894	47.39
2.89	.03213	.08581	.3745	.6119	.2622	.1879	47.59
2.90	.03165	.08489	.3729	.6106	.2598	.1863	47.79
2.91	.03118	.08398	.3712	.6093	.2573	.1848	47.99
2.92	.03071	.08307	.3696	.6080	.2549	.1833	48.19
2.93	.03025	.08218	.3681	.6067	.2524	.1818	48.39
2.94	.02980	.08130	.3665	.6054	.2500	.1803	48.59
2.95	.02935	.08043	.3649	.6041	.2477	.1788	48.78
2.96	.02891	.07957	.3633	.6028	.2453	.1773	48.98
2.97	.02848	.07872	.3618	.6015	.2430	.1758	49.18
2.98	.02805	.07788	.3602	.6002	.2407	.1744	49.37
2.99	.02764	.97705	.3587	.5989	.2384	.1729	49.56
3.00	.02722	.07623	.3571	.5976	.2362	.1715	49.76
3.01	.02682	.07541	.3556	.5963	.2339	.1701	49.95
3.02	.02642	.07461	.3541	.5951	.2317	.1687	50.14
3.03	.02603	.07382	.3526	.5938	.2295	.1673	50.33
3.04	.02564	.07303	.3511	.5925	.2273	.1659	50.52
3.05	.02526	.07226	.3496	.5913	.2252	.1645	50.71
3.06	.02489	.07149	.3481	.5900	.2230	.1631	50.90
3.07	.02452	.07074	.3466	.5887	.2209	.1618	51.09
3.08	.02416	.06999	.3452	.5875	.2188	.1604	51.28
3.09	.02380	.06925	.3437	.5862	.2168	.1591	51.46

TABLE III (*Continued*)

FLOW PARAMETERS VERSUS M FOR SUPERSONIC FLOW

M	$\frac{p}{p_0}$	$\frac{\rho}{\rho_0}$	$\frac{T}{T_0}$	$\frac{a}{a_0}$	$\frac{A^*}{A}$	$\frac{\frac{\rho}{2}V^2}{p_0}$	θ
3.10	.02345	.06852	.3422	.5850	.2147	.1577	51.65
3.11	.02310	.06779	.3408	.5838	.2127	.1564	51.84
3.12	.02276	.06708	.3393	.5825	.2107	.1551	52.02
3.13	.02243	.06637	.3379	.5813	.2087	.1538	52.20
3.14	.02210	.06568	.3365	.5801	.2067	.1525	52.39
3.15	.02177	.06499	.3351	.5788	.2048	.1512	52.57
3.16	.02146	.06430	.3337	.5776	.2028	.1500	52.75
3.17	.02114	.06363	.3323	.5764	.2009	.1487	52.93
3.18	.02083	.06296	.3309	.5752	.1990	.1475	53.11
3.19	.02053	.06231	.3295	.5740	.1971	.1462	53.29
3.20	.02023	.06165	.3281	.5728	.1953	.1450	53.47
3.21	.01993	.06101	.3267	.5716	.1934	.1438	53.65
3.22	.01964	.06037	.3253	.5704	.1916	.1426	53.83
3.23	.01936	.05975	.3240	.5692	.1898	.1414	54.00
3.24	.01908	.05912	.3226	.5680	.1880	.1402	54.18
3.25	.01880	.05851	.3213	.5668	.1863	.1390	54.35
3.26	.01853	.05790	.3199	.5656	.1845	.1378	54.53
3.27	.01826	.05730	.3186	.5645	.1828	.1367	54.71
3.28	.01799	.05671	.3173	.5633	.1810	.1355	54.88
3.29	.01773	.05612	.3160	.5621	.1793	.1344	55.05
3.30	.01748	.05554	.3147	.5609	.1777	.1332	55.22
3.31	.01722	.05497	.3134	.5598	.1760	.1321	55.39
3.32	.01698	.05440	.3121	.5586	.1743	.1310	55.56
3.33	.01673	.05384	.3108	.5575	.1727	.1299	55.73
3.34	.01649	.05329	.3095	.5563	.1711	.1288	55.90
3.35	.01625	.05274	.3082	.5552	.1695	.1277	56.07
3.36	.01602	.05220	.3069	.5540	.1679	.1266	56.24
3.37	.01579	.05166	.3057	.5529	.1663	.1255	56.41
3.38	.01557	.05113	.3044	.5517	.1648	.1245	56.58
3.39	.01534	.05061	.3032	.5506	.1632	.1234	56.75
3.40	.01513	.05009	.3019	.5495	.1617	.1224	56.91
3.41	.01491	.04958	.3007	.5484	.1602	.1214	57.07
3.42	.01470	.04908	.2995	.5472	.1587	.1203	57.24
3.43	.01449	.04858	.2982	.5461	.1572	.1193	57.40
3.44	.01428	.04808	.2970	.5450	.1558	.1183	57.56

TABLE III (*Continued*)

FLOW PARAMETERS VERSUS M FOR SUPERSONIC FLOW

M	$\frac{p}{p_0}$	$\frac{\rho}{\rho_0}$	$\frac{T}{T_0}$	$\frac{a}{a_0}$	$\frac{A^*}{A}$	$\frac{\frac{\rho}{2}V^2}{p_0}$	θ
3.45	.01408	.04759	.2958	.5439	.1543	.1173	57.73
3.46	.01388	.04711	.2946	.5428	.1529	.1163	57.89
3.47	.01368	.04663	.2934	.5417	.1515	.1153	58.05
3.48	.01349	.04616	.2922	.5406	.1501	.1144	58.21
3.49	.01330	.04569	.2910	.5395	.1487	.1134	58.37
3.50	.01311	.04523	.2899	.5384	.1473	.1124	58.53
3.60	.01138	.04089	.2784	.5276	.1342	.1033	60.09
3.70	9.903×10^{-3}	.03702	.2675	.5172	.1224	.09490	61.60
3.80	8.629×10^{-3}	.03355	.2572	.5072	.1117	.08722	63.04
3.90	7.532×10^{-3}	.03044	.2474	.4974	.1021	.08019	64.44
4.00	6.586×10^{-3}	.02766	.2381	.4880	.09329	.07376	65.78
4.10	5.769×10^{-3}	.02516	.2293	.4788	.08536	.06788	67.08
4.20	5.062×10^{-3}	.02292	.2208	.4699	.07818	.06251	68.33
4.30	4.449×10^{-3}	.02090	.2129	.4614	.07166	.05759	69.54
4.40	3.918×10^{-3}	.01909	.2053	.4531	.06575	.05309	70.71
4.50	3.455×10^{-3}	.01745	.1980	.4450	.06038	.04898	71.83
4.60	3.053×10^{-3}	.01597	.1911	.4372	.05550	.04521	72.92
4.70	2.701×10^{-3}	.01464	.1846	.4296	.05107	.04177	73.97
4.80	2.394×10^{-3}	.01343	.1783	.4223	.04703	.03861	74.99
4.90	2.126×10^{-3}	.01233	.1724	.4152	.04335	.03572	75.97
5.00	1.890×10^{-3}	.01134	.1667	.4082	.04000	.03308	76.92
6.00	6.334×10^{-4}	5.194×10^{-3}	.1220	.3492	.01880	.01596	84.96
7.00	2.416×10^{-4}	2.609×10^{-3}	.09259	.3043	9.602×10^{-3}	8.285×10^{-3}	90.97

TABLE III (*Continued*)

FLOW PARAMETERS VERSUS M FOR SUPERSONIC FLOW

M	$\frac{p}{p_0}$	$\frac{\rho}{\rho_0}$	$\frac{T}{T_0}$	$\frac{a}{a_0}$	$\frac{A^*}{A}$	$\frac{\frac{\rho}{2}V^2}{p_0}$	θ
8.00	1.024×10^{-4}	1.414×10^{-3}	.07246	.2692	5.260×10^{-3}	4.589×10^{-3}	95.62
9.00	4.739×10^{-5}	8.150×10^{-4}	.05814	.2411	3.056×10^{-3}	2.687×10^{-3}	99.32
10.00	2.356×10^{-5}	4.948×10^{-4}	.04762	.2182	1.866×10^{-3}	1.649×10^{-3}	102.3
100.00	2.790×10^{-12}	5.583×10^{-9}	4.998×10^{-4}	.02236	2.157×10^{-8}	1.953×10^{-8}	127.6
∞	0	0	0	0	0	0	130.5

Numerical values taken from NACA TN 1428, courtesy of the National Advisory Committee for Aeronautics. Setup from A. M. Kuethe and J. D. Schetzer, *Foundations of Aerodynamics*, John Wiley & Sons, New York, 1950.

TABLE IV

Parameters for Shock Flow

M_{1n}	p_2/p_1	ρ_2/ρ_1	T_2/T_1	a_2/a_1	p_2^0/p_1^0	M_2 for Normal Shocks Only
1.00	1.000	1.000	1.000	1.000	1.0000	1.0000
1.01	1.023	1.017	1.007	1.003	1.0000	.9901
1.02	1.047	1.033	1.013	1.007	1.0000	.9805
1.03	1.071	1.050	1.020	1.010	1.0000	.9712
1.04	1.095	1.067	1.026	1.013	.9999	.9620
1.05	1.120	1.084	1.033	1.016	.9999	.9531
1.06	1.144	1.101	1.039	1.019	.9998	.9444
1.07	1.169	1.118	1.046	1.023	.9996	.9360
1.08	1.194	1.135	1.052	1.026	.9994	.9277
1.09	1.219	1.152	1.059	1.029	.9992	.9196
1.10	1.245	1.169	1.065	1.032	.9989	.9118
1.11	1.271	1.186	1.071	1.035	.9986	.9041
1.12	1.297	1.203	1.078	1.038	.9982	.8966
1.13	1.323	1.221	1.084	1.041	.9978	.8892
1.14	1.350	1.238	1.090	1.044	.9973	.8820
1.15	1.376	1.255	1.097	1.047	.9967	.8750
1.16	1.403	1.272	1.103	1.050	.9961	.8682
1.17	1.430	1.290	1.109	1.053	.9953	.8615
1.18	1.458	1.307	1.115	1.056	.9946	.8549
1.19	1.485	1.324	1.122	1.059	.9937	.8485
1.20	1.513	1.342	1.128	1.062	.9928	.8422
1.21	1.541	1.359	1.134	1.065	.9918	.8360
1.22	1.570	1.376	1.141	1.068	.9907	.8300
1.23	1.598	1.394	1.147	1.071	.9896	.8241
1.24	1.627	1.411	1.153	1.074	.9884	.8183
1.25	1.656	1.429	1.159	1.077	.9871	.8126
1.26	1.686	1.446	1.166	1.080	.9857	.8071
1.27	1.715	1.463	1.172	1.083	.9842	.8016
1.28	1.745	1.481	1.178	1.085	.9827	.7963
1.29	1.775	1.498	1.185	1.088	.9811	.7911
1.30	1.805	1.516	1.191	1.091	.9794	.7860
1.31	1.835	1.533	1.197	1.094	.9776	.7809
1.32	1.866	1.551	1.204	1.097	.9758	.7760
1.33	1.897	1.568	1.210	1.100	.9738	.7712
1.34	1.928	1.585	1.216	1.103	.9718	.7664

TABLE IV (*Continued*)

Parameters for Shock Flow

M_{1n}	p_2/p_1	ρ_2/ρ_1	T_2/T_1	a_2/a_1	p_2^0/p_1^0	M_2 for Normal Shocks Only
1.35	1.960	1.603	1.223	1.106	.9697	.7618
1.36	1.991	1.620	1.229	1.109	.9676	.7572
1.37	2.023	1.638	1.235	1.111	.9653	.7527
1.38	2.055	1.655	1.242	1.114	.9630	.7483
1.39	2.087	1.672	1.248	1.117	.9606	.7440
1.40	2.120	1.690	1.255	1.120	.9582	.7397
1.41	2.153	1.707	1.261	1.123	.9557	.7355
1.42	2.186	1.724	1.268	1.126	.9531	.7314
1.43	2.219	1.742	1.274	1.129	.9504	.7274
1.44	2.253	1.759	1.281	1.132	.9476	.7235
1.45	2.286	1.776	1.287	1.135	.9448	.7196
1.46	2.320	1.793	1.294	1.137	.9420	.7157
1.47	2.354	1.811	1.300	1.140	.9390	.7120
1.48	2.389	1.828	1.307	1.143	.9360	.7083
1.49	2.423	1.845	1.314	1.146	.9329	.7047
1.50	2.458	1.862	1.320	1.149	.9298	.7011
1.51	2.493	1.879	1.327	1.152	.9266	.6976
1.52	2.529	1.896	1.334	1.155	.9233	.6941
1.53	2.564	1.913	1.340	1.158	.9200	.6907
1.54	2.600	1.930	1.347	1.161	.9166	.6874
1.55	2.636	1.947	1.354	1.164	.9132	.6841
1.56	2.673	1.964	1.361	1.166	.9097	.6809
1.57	2.709	1.981	1.367	1.169	.9061	.6777
1.58	2.746	1.998	1.374	1.172	.9026	.6746
1.59	2.783	2.015	1.381	1.175	.8989	.6715
1.60	2.820	2.032	1.388	1.178	.8952	.6684
1.61	2.857	2.049	1.395	1.181	.8914	.6655
1.62	2.895	2.065	1.402	1.184	.8877	.6625
1.63	2.933	2.082	1.409	1.187	.8838	.6596
1.64	2.971	2.099	1.416	1.190	.8799	.6568
1.65	3.010	2.115	1.423	1.193	.8760	.6540
1.66	3.048	2.132	1.430	1.196	.8720	.6512
1.67	3.087	2.148	1.437	1.199	.8680	.6485
1.68	3.126	2.165	1.444	1.202	.8640	.6458
1.69	3.165	2.181	1.451	1.205	.8599	.6431

TABLE IV (*Continued*)

Parameters for Shock Flow

M_{1n}	p_2/p_1	ρ_2/ρ_1	T_2/T_1	a_2/a_1	p_2^0/p_1^0	M_2 for Normal Shocks Only
1.70	3.205	2.198	1.458	1.208	.8557	.6405
1.71	3.245	2.214	1.466	1.211	.8516	.6380
1.72	3.285	2.230	1.473	1.214	.8474	.6355
1.73	3.325	2.247	1.480	1.217	.8431	.6330
1.74	3.366	2.263	1.487	1.220	.8389	.6305
1.75	3.406	2.279	1.495	1.223	.8346	.6281
1.76	3.447	2.295	1.502	1.226	.8302	.6257
1.77	3.488	2.311	1.509	1.229	.8259	.6234
1.78	3.530	2.327	1.517	1.232	.8215	.6210
1.79	3.571	2.343	1.524	1.235	.8171	.6188
1.80	3.613	2.359	1.532	1.238	.8127	.6165
1.81	3.655	2.375	1.539	1.241	.8082	.6143
1.82	3.698	2.391	1.547	1.244	.8038	.6121
1.83	3.740	2.407	1.554	1.247	.7993	.6099
1.84	3.783	2.422	1.562	1.250	.7948	.6078
1.85	3.826	2.438	1.569	1.253	.7902	.6057
1.86	3.870	2.454	1.577	1.256	.7857	.6036
1.87	3.913	2.469	1.585	1.259	.7811	.6016
1.88	3.957	2.485	1.592	1.262	.7765	.5996
1.89	4.001	2.500	1.600	1.265	.7720	.5976
1.90	4.045	2.516	1.608	1.268	.7674	.5956
1.91	4.089	2.531	1.616	1.271	.7628	.5937
1.92	4.134	2.546	1.624	1.274	.7581	.5918
1.93	4.179	2.562	1.631	1.277	.7535	.5899
1.94	4.224	2.577	1.639	1.280	.7488	.5880
1.95	4.270	2.592	1.647	1.283	.7442	.5862
1.96	4.315	2.607	1.655	1.287	.7395	.5844
1.97	4.361	2.622	1.663	1.290	.7349	.5826
1.98	4.407	2.637	1.671	1.293	.7302	.5808
1.99	4.453	2.652	1.679	1.296	.7255	.5791
2.00	4.500	2.667	1.688	1.299	.7209	.5773
2.01	4.547	2.681	1.696	1.302	.7162	.5757
2.02	4.594	2.696	1.704	1.305	.7115	.5740
2.03	4.641	2.711	1.712	1.308	.7069	.5723
2.04	4.689	2.725	1.720	1.312	.7022	.5707

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TABLE IV (*Continued*)

PARAMETERS FOR SHOCK FLOW

M_{1n}	p_2/p_1	ρ_2/ρ_1	T_2/T_1	a_2/a_1	p_2^0/p_1^0	M_2 for Normal Shocks Only
2.05	4.736	2.740	1.729	1.315	.6975	.5691
2.06	4.784	2.755	1.737	1.318	.6928	.5675
2.07	4.832	2.769	1.745	1.321	.6882	.5659
2.08	4.881	2.783	1.754	1.324	.6835	.5643
2.09	4.929	2.798	1.762	1.327	.6789	.5628
2.10	4.978	2.812	1.770	1.331	.6742	.5613
2.11	5.027	2.826	1.779	1.334	.6696	.5598
2.12	5.077	2.840	1.787	1.337	.6649	.5583
2.13	5.126	2.854	1.796	1.340	.6603	.5568
2.14	5.176	2.868	1.805	1.343	.6557	.5554
2.15	5.226	2.882	1.813	1.347	.6511	.5540
2.16	5.277	2.896	1.822	1.350	.6464	.5525
2.17	5.327	2.910	1.831	1.353	.6419	.5511
2.18	5.378	2.924	1.839	1.356	.6373	.5498
2.19	5.429	2.938	1.848	1.359	.6327	.5484
2.20	5.480	2.951	1.857	1.363	.6281	.5471
2.21	5.531	2.965	1.866	1.366	.6236	.5457
2.22	5.583	2.978	1.875	1.369	.6191	.5444
2.23	5.635	2.992	1.883	1.372	.6145	.5431
2.24	5.687	3.005	1.892	1.376	.6100	.5418
2.25	5.740	3.019	1.901	1.379	.6055	.5406
2.26	5.792	3.032	1.910	1.382	.6011	.5393
2.27	5.845	3.045	1.919	1.385	.5966	.5381
2.28	5.898	3.058	1.929	1.389	.5921	.5368
2.29	5.951	3.071	1.938	1.392	.5877	.5356
2.30	6.005	3.085	1.947	1.395	.5833	.5344
2.31	6.059	3.098	1.956	1.399	.5789	.5332
2.32	6.113	3.110	1.965	1.402	.5745	.5321
2.33	6.167	3.123	1.974	1.405	.5702	.5309
2.34	6.222	3.136	1.984	1.408	.5658	.5297
2.35	6.276	3.149	1.993	1.412	.5615	.5286
2.36	6.331	3.162	2.002	1.415	.5572	.5275
2.37	6.386	3.174	2.012	1.418	.5529	.5264
2.38	6.442	3.187	2.021	1.422	.5486	.5253
2.39	6.497	3.199	2.031	1.425	.5444	.5242

TABLE IV (*Continued*)

PARAMETERS FOR SHOCK FLOW

M_{1n}	p_2/p_1	ρ_2/ρ_1	T_2/T_1	a_2/a_1	p_2^0/p_1^0	M_2 for Normal Shocks Only
2.40	6.553	3.212	2.040	1.428	.5401	.5231
2.41	6.609	3.224	2.050	1.432	.5359	.5221
2.42	6.666	3.237	2.059	1.435	.5317	.5210
2.43	6.722	3.249	2.069	1.438	.5276	.5200
2.44	6.779	3.261	2.079	1.442	.5234	.5189
2.45	6.836	3.273	2.088	1.445	.5193	.5179
2.46	6.894	3.285	2.098	1.449	.5152	.5169
2.47	6.951	3.298	2.108	1.452	.5111	.5159
2.48	7.009	3.310	2.118	1.455	.5071	.5149
2.49	7.067	3.321	2.128	1.459	.5030	.5140
2.50	7.125	3.333	2.138	1.462	.4990	.5130
2.51	7.183	3.345	2.147	1.465	.4950	.5120
2.52	7.242	3.357	2.157	1.469	.4911	.5111
2.53	7.301	3.369	2.167	1.472	.4871	.5102
2.54	7.360	3.380	2.177	1.476	.4832	.5092
2.55	7.420	3.392	2.187	1.479	.4793	.5083
2.56	7.479	3.403	2.198	1.482	.4754	.5074
2.57	7.539	3.415	2.208	1.486	.4715	.5065
2.58	7.599	3.426	2.218	1.489	.4677	.5056
2.59	7.659	3.438	2.228	1.493	.4639	.5047
2.60	7.720	3.449	2.238	1.496	.4601	.5039
2.61	7.781	3.460	2.249	1.500	.4564	.5030
2.62	7.842	3.471	2.259	1.503	.4526	.5022
2.63	7.903	3.483	2.269	1.506	.4489	.5013
2.64	7.965	3.494	2.280	1.510	.4452	.5005
2.65	8.026	3.505	2.290	1.513	.4416	.4996
2.66	8.088	3.516	2.301	1.517	.4379	.4988
2.67	8.150	3.527	2.311	1.520	.4343	.4980
2.68	8.213	3.537	2.322	1.524	.4307	.4972
2.69	8.275	3.548	2.332	1.527	.4271	.4964
2.70	8.338	3.559	2.343	1.531	.4236	.4956
2.71	8.401	3.570	2.354	1.534	.4201	.4949
2.72	8.465	3.580	2.364	1.538	.4166	.4941
2.73	8.528	3.591	2.375	1.541	.4131	.4933
2.74	8.592	3.601	2.386	1.545	.4097	.4926

TABLE IV (*Continued*)

PARAMETERS FOR SHOCK FLOW

M_{1n}	p_2/p_1	ρ_2/ρ_1	T_2/T_1	a_2/a_1	p_2^0/p_1^0	M_2 for Normal Shocks Only
2.75	8.656	3.612	2.397	1.548	.4062	.4918
2.76	8.721	3.622	2.407	1.552	.4028	.4911
2.77	8.785	3.633	2.418	1.555	.3994	.4903
2.78	8.850	3.643	2.429	1.559	.3961	.4896
2.79	8.915	3.653	2.440	1.562	.3928	.4889
2.80	8.980	3.664	2.451	1.566	.3895	.4882
2.81	9.045	3.674	2.462	1.569	.3862	.4875
2.82	9.111	3.684	2.473	1.573	.3829	.4868
2.83	9.177	3.694	2.484	1.576	.3797	.4861
2.84	9.243	3.704	2.496	1.580	.3765	.4854
2.85	9.310	3.714	2.507	1.583	.3733	.4847
2.86	9.376	3.724	2.518	1.587	.3701	.4840
2.87	9.443	3.734	2.529	1.590	.3670	.4833
2.88	9.510	3.743	2.540	1.594	.3639	.4827
2.89	9.577	3.753	2.552	1.597	.3608	.4820
2.90	9.645	3.763	2.563	1.601	.3577	.4814
2.91	9.713	3.773	2.575	1.605	.3547	.4807
2.92	9.781	3.782	2.586	1.608	.3517	.4801
2.93	9.849	3.792	2.598	1.612	.3487	.4795
2.94	9.918	3.801	2.609	1.615	.3457	.4788
2.95	9.986	3.811	2.621	1.619	.3428	.4782
2.96	10.06	3.820	2.632	1.622	.3398	.4776
2.97	10.12	3.829	2.644	1.626	.3369	.4770
2.98	10.19	3.839	2.656	1.630	.3340	.4764
2.99	10.26	3.848	2.667	1.633	.3312	.4758
3.00	10.33	3.857	2.679	1.637	.3283	.4752
3.10	11.05	3.947	2.799	1.673	.3012	.4695
3.20	11.78	4.031	2.922	1.709	.2762	.4643
3.30	12.54	4.112	3.049	1.746	.2533	.4596
3.40	13.32	4.188	3.180	1.783	.2322	.4552
3.50	14.13	4.261	3.315	1.821	.2129	.4512
3.60	14.95	4.330	3.454	1.858	.1953	.4474
3.70	15.80	4.395	3.596	1.896	.1792	.4439
3.80	16.68	4.457	3.743	1.935	.1645	.4407
3.90	17.58	4.516	3.893	1.973	.1510	.4377

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TABLE IV (*Continued*)

PARAMETERS FOR SHOCK FLOW

M_{1n}	p_2/p_1	ρ_2/ρ_1	T_2/T_1	a_2/a_1	p_2^0/p_1^0	M_2 for Normal Shocks Only
4.00	18.50	4.571	4.047	2.012	.1388	.4350
5.00	29.00	5.000	5.800	2.408	.06172	.4152
6.00	41.83	5.268	7.941	2.818	.02965	.4042
7.00	57.00	5.444	10.47	3.236	.01535	.3974
8.00	74.50	5.565	13.39	3.659	8.488×10^{-3}	.3929
9.00	94.33	5.651	16.69	4.086	4.964×10^{-3}	.3898
10.00	116.5	5.714	20.39	4.515	3.045×10^{-3}	.3876
100.00	11,666.5	5.997	1945.4	44.11	3.593×10^{-8}	.3781
∞	∞	6	∞	∞	0	.3780

Data taken from NACA TN 1428, courtesy of National Advisory Committee of Aeronautics. Setup from A. M. Kuethe and J. D. Schetzer, *Foundations of Aerodynamics*, John Wiley & Sons, New York, 1950.

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TABLE V

MACH NUMBER AND MACH ANGLE VERSUS PRANDTL-MEYER FUNCTION

ν (deg)	M	μ (deg)	ν (deg)	M	μ (deg)
0.0	1.000	90.000	17.5	1.689	36.293
0.5	1.051	72.099	18.0	1.706	35.874
1.0	1.082	67.574	18.5	1.724	35.465
1.5	1.108	64.451	19.0	1.741	35.065
2.0	1.133	61.997	19.5	1.758	34.673
2.5	1.155	59.950	20.0	1.775	34.290
3.0	1.177	58.180	20.5	1.792	33.915
3.5	1.198	56.614	21.0	1.810	33.548
4.0	1.218	55.205	21.5	1.827	33.188
4.5	1.237	53.920	22.0	1.844	32.834
5.0	1.256	52.738	22.5	1.862	32.488
5.5	1.275	51.642	23.0	1.879	32.148
6.0	1.294	50.619	23.5	1.897	31.814
6.5	1.312	49.658	24.0	1.915	31.486
7.0	1.330	48.753	24.5	1.932	31.164
7.5	1.348	47.896	25.0	1.950	30.847
8.0	1.366	47.082	25.5	1.968	30.536
8.5	1.383	46.306	26.0	1.986	30.229
9.0	1.400	45.566	26.5	2.004	29.928
9.5	1.418	44.857	27.0	2.023	29.632
10.0	1.435	44.177	27.5	2.041	29.340
10.5	1.452	43.523	28.0	2.059	29.052
11.0	1.469	42.894	28.5	2.078	28.769
11.5	1.486	42.287	29.0	2.096	28.491
12.0	1.503	41.701	29.5	2.115	28.216
12.5	1.520	41.134	30.0	2.134	27.945
13.0	1.537	40.585	30.5	2.153	27.678
13.5	1.554	40.053	31.0	2.172	27.415
14.0	1.571	39.537	31.5	2.191	27.155
14.5	1.588	39.035	32.0	2.210	26.899
15.0	1.605	38.547	32.5	2.230	26.646
15.5	1.622	38.073	33.0	2.249	26.397
16.0	1.639	37.611	33.5	2.269	26.151
16.5	1.655	37.160	34.0	2.289	25.908
17.0	1.672	36.721	34.5	2.309	25.668

TABLE V (*Continued*)

MACH NUMBER AND MACH ANGLE VERSUS PRANDTL-MEYER FUNCTION

ν (deg)	M	μ (deg)	ν (deg)	M	μ (deg)
35.0	2.329	25.430	52.5	3.146	18.532
35.5	2.349	25.196	53.0	3.174	18.366
36.0	2.369	24.965	53.5	3.202	18.200
36.5	2.390	24.736	54.0	3.230	18.036
37.0	2.410	24.510	54.5	3.258	17.873
37.5	2.431	24.287	55.0	3.287	17.711
38.0	2.452	24.066	55.5	3.316	17.551
38.5	2.473	23.847	56.0	3.346	17.391
39.0	2.495	23.631	56.5	3.375	17.233
39.5	2.516	23.418	57.0	3.406	17.076
40.0	2.538	23.206	57.5	3.436	16.920
40.5	2.560	22.997	58.0	3.467	16.765
41.0	2.582	22.790	58.5	3.498	16.611
41.5	2.604	22.585	59.0	3.530	16.458
42.0	2.626	22.382	59.5	3.562	16.306
42.5	2.649	22.182	60.0	3.594	16.155
43.0	2.671	21.983	60.5	3.627	16.005
43.5	2.694	21.786	61.0	3.660	15.856
44.0	2.718	21.591	61.5	3.694	15.708
44.5	2.741	21.398	62.0	3.728	15.561
45.0	2.764	21.207	62.5	3.762	15.415
45.5	2.788	21.017	63.0	3.797	15.270
46.0	2.812	20.830	63.5	3.832	15.126
46.5	2.836	20.644	64.0	3.868	14.983
47.0	2.861	20.459	64.5	3.904	14.840
47.5	2.886	20.277	65.0	3.941	14.698
48.0	2.910	20.096	65.5	3.979	14.557
48.5	2.936	19.916	66.0	4.016	14.417
49.0	2.961	19.738	66.5	4.055	14.278
49.5	2.987	19.561	67.0	4.094	14.140
50.0	3.013	19.386	67.5	4.133	14.002
50.5	3.039	19.213	68.0	4.173	13.865
51.0	3.065	19.041	68.5	4.214	13.729
51.5	3.092	18.870	69.0	4.255	13.593
52.0	3.119	18.701	69.5	4.297	13.459

TABLE V (*Continued*)

MACH NUMBER AND MACH ANGLE VERSUS PRANDTL-MEYER FUNCTION

ν (deg)	M	μ (deg)	ν (deg)	M	μ (deg)
70.0	4.339	13.325	87.5	6.390	9.003
70.5	4.382	13.191	88.0	6.472	8.888
71.0	4.426	13.059	88.5	6.556	8.774
71.5	4.470	12.927	89.0	6.642	8.660
72.0	4.515	12.795	89.5	6.729	8.546
72.5	4.561	12.665	90.0	6.819	8.433
73.0	4.608	12.535	90.5	6.911	8.320
73.5	4.655	12.406	91.0	7.005	8.207
74.0	4.703	12.277	91.5	7.102	8.095
74.5	4.752	12.149	92.0	7.201	7.983
75.0	4.801	12.021	92.5	7.302	7.871
75.5	4.852	11.894	93.0	7.406	7.760
76.0	4.903	11.768	93.5	7.513	7.649
76.5	4.955	11.642	94.0	7.623	7.538
77.0	5.009	11.517	94.5	7.735	7.428
77.5	5.063	11.392	95.0	7.851	7.318
78.0	5.118	11.268	95.5	7.970	7.208
78.5	5.174	11.145	96.0	8.092	7.099
79.0	5.231	11.022	96.5	8.218	6.989
79.5	5.289	10.899	97.0	8.347	6.881
80.0	5.348	10.777	97.5	8.480	6.772
80.5	5.408	10.656	98.0	8.618	6.664
81.0	5.470	10.535	98.5	8.759	6.556
81.5	5.532	10.414	99.0	8.905	6.448
82.0	5.596	10.294	99.5	9.055	6.340
82.5	5.661	10.175	100.0	9.210	6.233
83.0	5.727	10.056	100.5	9.371	6.126
83.5	5.795	9.937	101.0	9.536	6.019
84.0	5.864	9.819	101.5	9.708	5.913
84.5	5.935	9.701	102.0	9.885	5.806
85.0	6.006	9.584			
85.5	6.080	9.467			
86.0	6.155	9.350			
86.5	6.232	9.234			
87.0	6.310	9.119			

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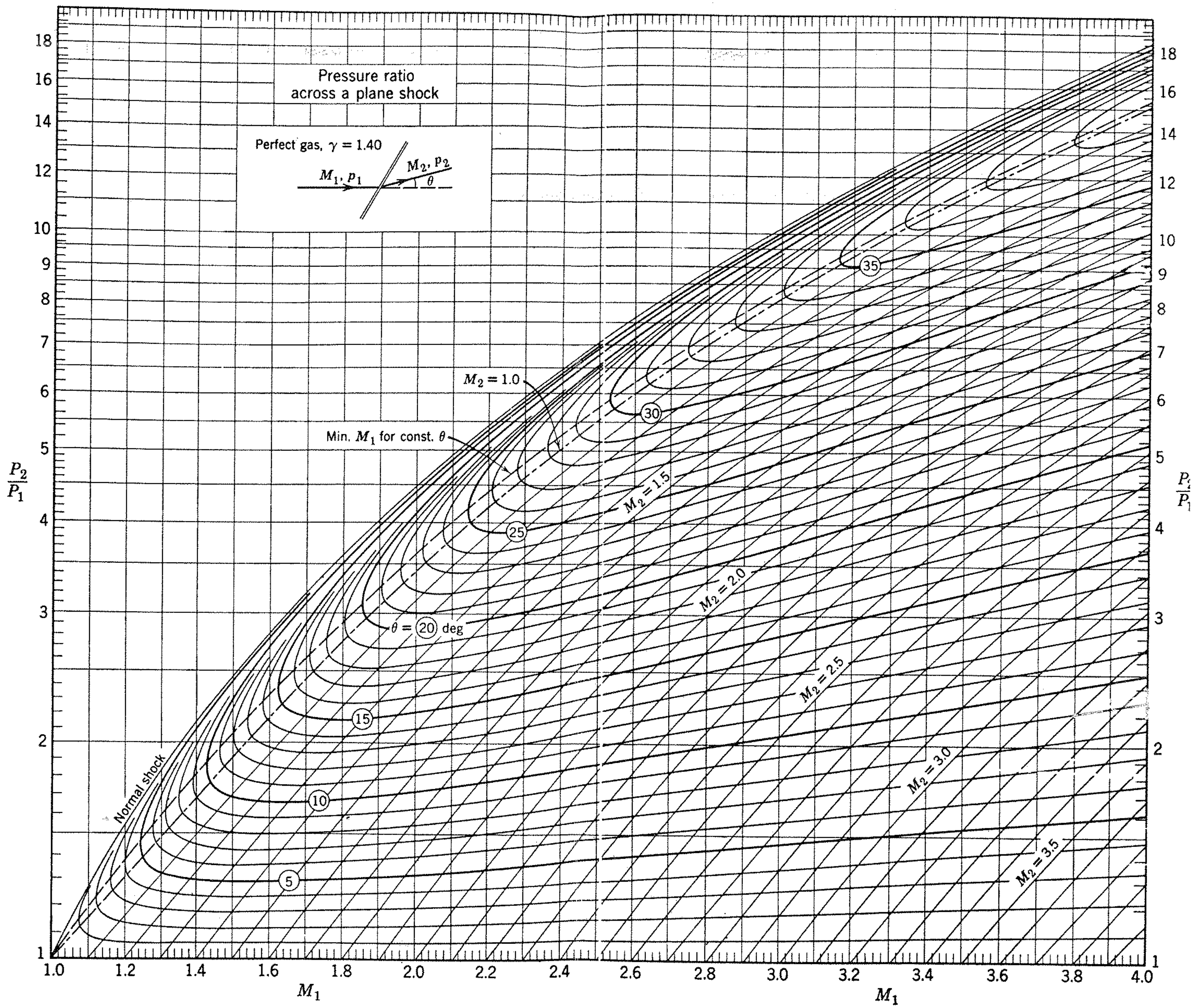

Oblique Shock Chart 2 Variation of pressure ratio and downstream Mach number with flow-deflection angle and upstream Mach number. (Data from C. L. Dailey and F. C. Wood, *Computation Curves for Compressible Flow Problems*, Wiley, 1949.)

Chart 2 (continued)

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