NUMERO : 411

DATA : 02/11/2012

A P P U N T I

STUDENTE :

MATERIA : Fisica II appunti
Prof. Barbero

- FORZE (CAMPI) CONSERVATIVI
- COORDINATE POLARI SFERICHE
- TEOREMA DELLA DIVERGENZA
- TEOREMA DI STOKES

$\boxed{\text{MATEMATICA}}$

SE $W_{A \to B}$ È INDIPENDENTE DA γ

$$(*) \quad W^{\gamma_1}_{A \to B} = W^{\gamma_2}_{A \to B} = f(A,B)$$

IL CAMPO DI FORZE SI DICE CONSERVATIVO.

NB

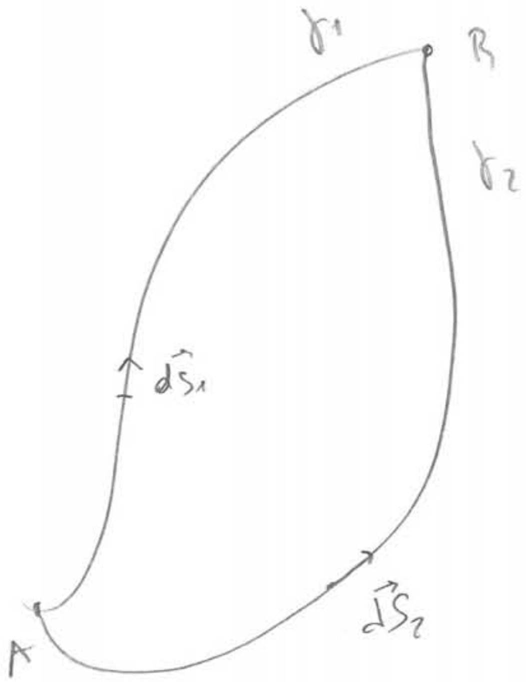
$$W^{\gamma_1}_{A \to B} = \int_{A,\gamma_1}^{B} \vec{F} \cdot d\vec{S}_1 \qquad d\vec{S}_1 = d\vec{S} \text{ su } \gamma_1$$

$$W^{\gamma_2}_{A \to B} = \int_{A,\gamma_2}^{B} \vec{F} \cdot d\vec{S}_2 \qquad d\vec{S}_2 = d\vec{S} \text{ su } \gamma_2$$

SE VALE LA (*)

$$\int_{A,\gamma_1}^{B} \vec{F} \cdot d\vec{S}_1 = \int_{A,\gamma_2}^{B} \vec{F} \cdot d\vec{S}_2$$

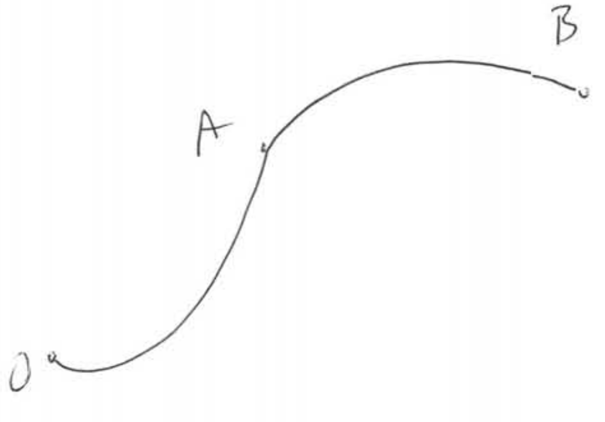
$\Downarrow$



1) $$\oint_{\gamma_1+\gamma_2} \vec{F}\cdot d\vec{s} = \oint_A^A \vec{F}\cdot d\vec{s} = f(A,A) = 0$$

NOTA : SE $\vec{F}$ È CONSERVATIVA

$$\int_A^B \vec{F}\cdot d\vec{s} = f(A,B)$$



(**) $$\int_O^B \vec{F}\, d\vec{s} = \int_O^A + \int_A^B \vec{F}\cdot d\vec{s}$$

DOVE $$\int_O^B \vec{F}\cdot d\vec{s} = f(O,B)\,, \quad \int_O^A \vec{F}\cdot d\vec{s} = f(O,A)$$

CONDIZIONI DI CONSERVATIVITÀ

SE UNA FORZA E' CONSERVATIVA

$$\int_{A\,\gamma}^{B} \vec{F} \cdot d\vec{s} = U(A) - U(B) \qquad \forall \gamma$$

SI HA

$$U(A) - U(B) = -\int_{A}^{B} dU$$

QUINDI

$$\int_{A\,\gamma}^{B} \left(\vec{F} \cdot d\vec{s} + dU \right) = 0 \qquad \forall \gamma$$

$$\vec{F} \cdot d\vec{s} + dU = 0 \qquad \forall \gamma$$

$$(\alpha) \qquad \boxed{\vec{F} \cdot d\vec{s} = -dU} \qquad \forall \gamma$$

IN COORDINATE CARTESIANE

$$\vec{F} = F_x \vec{u}_x + F_y \vec{u}_y + F_z \vec{u}_z \, , \quad d\vec{s} = dx\, \vec{u}_x + dy\, \vec{u}_y + dz\, \vec{u}_z$$

$$\vec{F} = F_x \vec{u}_x + F_y \vec{u}_y + F_z \vec{u}_z \Rightarrow$$

$$\vec{F} = -\vec{u}_x \frac{\partial u}{\partial x} - \vec{u}_y \frac{\partial u}{\partial y} - \vec{u}_z \frac{\partial u}{\partial z}$$

CHE VIENE SCRITTA, IN MODO SIMBOLICO, COME

$$\vec{F} = -\vec{\nabla} u$$

$\vec{\nabla}$ = OPERATORE GRADIENTE

IN COORDINATE CARTESIANE

$$\vec{\nabla} = \vec{u}_x \frac{\partial}{\partial x} + \vec{u}_y \frac{\partial}{\partial y} + \vec{u}_z \frac{\partial}{\partial z}$$

QUANDO SI USANO ALTRI SISTEMI DI COORDINATE L'ESPRESSIONE DI $\vec{\nabla}$ CAMBIA.

TEOREMA SULL'INVERSIONE DELL'ORDINE DELLE DERIVATE

PER UNA FUNZIONE $F = F(x, y, z)$ SI HA

$$\frac{\partial^2 F}{\partial x \partial y} = \frac{\partial^2 F}{\partial y \partial x} \qquad \frac{\partial^2 F}{\partial x \partial z} = \frac{\partial^2 F}{\partial z \partial x} \qquad \frac{\partial^2 F}{\partial y \partial z} = \frac{\partial^2 F}{\partial z \partial y}$$

(TH. INVERSIONE O DI SCHWARTZ)

NOTA SUL PRODOTTO ESTERNO

$$\vec{u}_x \times \vec{u}_y = \vec{u}_z \qquad \vec{u}_z \times \vec{u}_x = \vec{u}_y \qquad \vec{u}_y \times \vec{u}_z = \vec{u}_x$$

$$\vec{A} = A_x \vec{u}_x + A_y \vec{u}_y + A_z \vec{u}_z$$

$$\vec{B} = B_x \vec{u}_x + B_y \vec{u}_y + B_z \vec{u}_z$$

$$\vec{C} = \vec{A} \times \vec{B} = \left(A_x \vec{u}_x + A_y \vec{u}_y + A_z \vec{u}_z\right) \times \left(B_x \vec{u}_x + B_y \vec{u}_y + B_z \vec{u}_z\right)$$

$$= A_x B_y \left(\vec{u}_x \times \vec{u}_y\right) + A_x B_z \left(\vec{u}_x \times \vec{u}_z\right) +$$

$$+ A_y B_x \left(\vec{u}_y \times \vec{u}_x\right) + A_y B_z \left(\vec{u}_y \times \vec{u}_z\right) +$$

$$+ A_z B_x \left(\vec{u}_z \times \vec{u}_x\right) + A_z B_y \left(\vec{u}_z \times \vec{u}_y\right) =$$

$$= A_x B_y \vec{u}_z - A_x B_z \vec{u}_y - A_y B_x \vec{u}_z + A_y B_z \vec{u}_x$$

$$+ A_z B_x \vec{u}_y - A_z B_y \vec{u}_x =$$

$$= \left(A_y B_z - A_z B_y\right) \vec{u}_x + \left(A_z B_x - A_x B_z\right) \vec{u}_y +$$

$$+ \left(A_x B_y - A_y B_x\right) \vec{u}_z$$

SE LA FORZA È CONSERVATIVA

$$\vec{R} \equiv 0$$

$\vec{R}$ SI CHIAMA ROTORE DI $\vec{F}$.

<u>NB</u> SE $\vec{\nabla} \times \vec{F} = 0$ LA FORZA È CONSERVATIVA

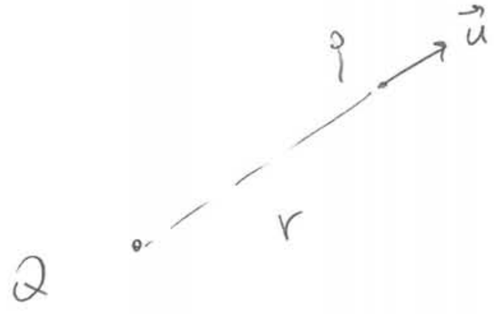
SE LA FORZA È CONSERVATIVA $\vec{\nabla} \times \vec{F} = 0$?

<u>PROBLEMA</u>

LA FORZA $\vec{F} = h \dfrac{qQ}{r^2} \vec{u}$ È CONSERVATIVA?

SCRIVIAMO

$$\vec{F} = h\, qQ \frac{\vec{r}}{r^3}$$



PONIAMO Q IN O

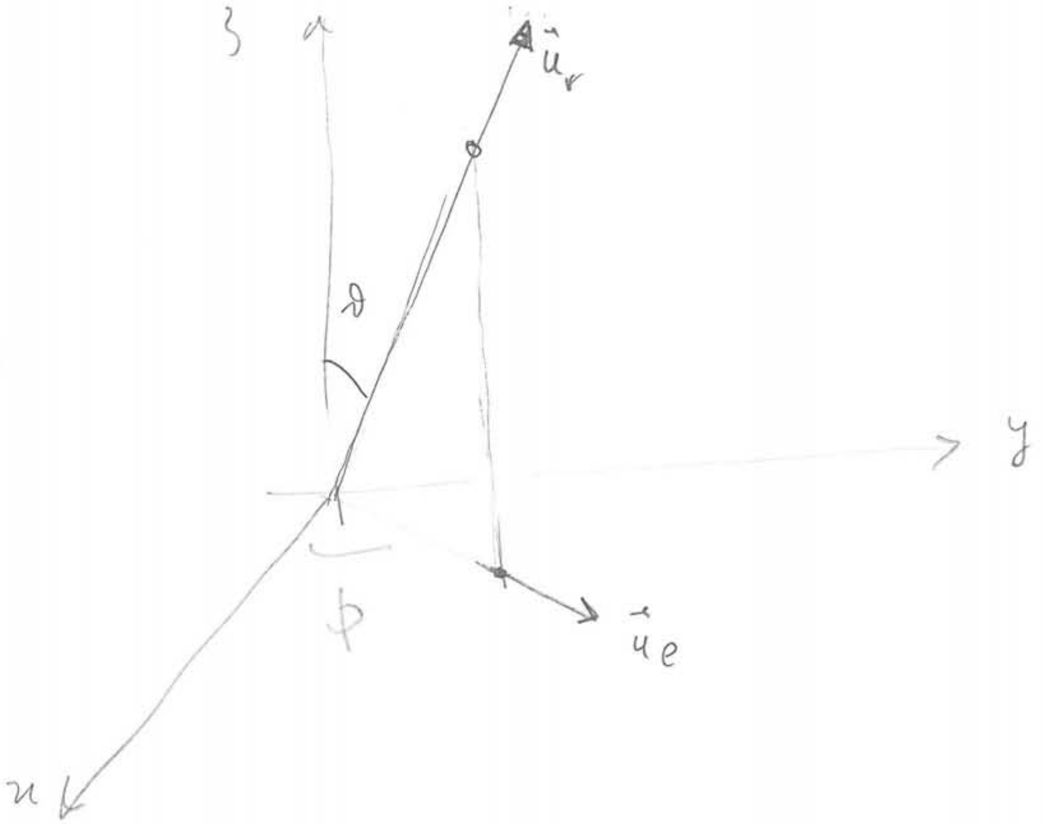
E q in $P \equiv (x, y, z)$.

$$F_x = h\, qQ \frac{x}{r^3} \qquad F_y = h\, qQ \frac{y}{r^3} \qquad F_z = h\, qQ \frac{z}{r^3}$$

CON $r = \sqrt{x^2 + y^2 + z^2}$

$$\vec{u}_r = \sin\vartheta\left(\cos\phi\,\vec{u}_x + \sin\vartheta\,\vec{u}_y\right) + \cos\vartheta\,\vec{u}_z$$

NB1 $\vec{u}_\rho = \cos\phi\,\vec{u}_x + \sin\vartheta\,\vec{u}_y$

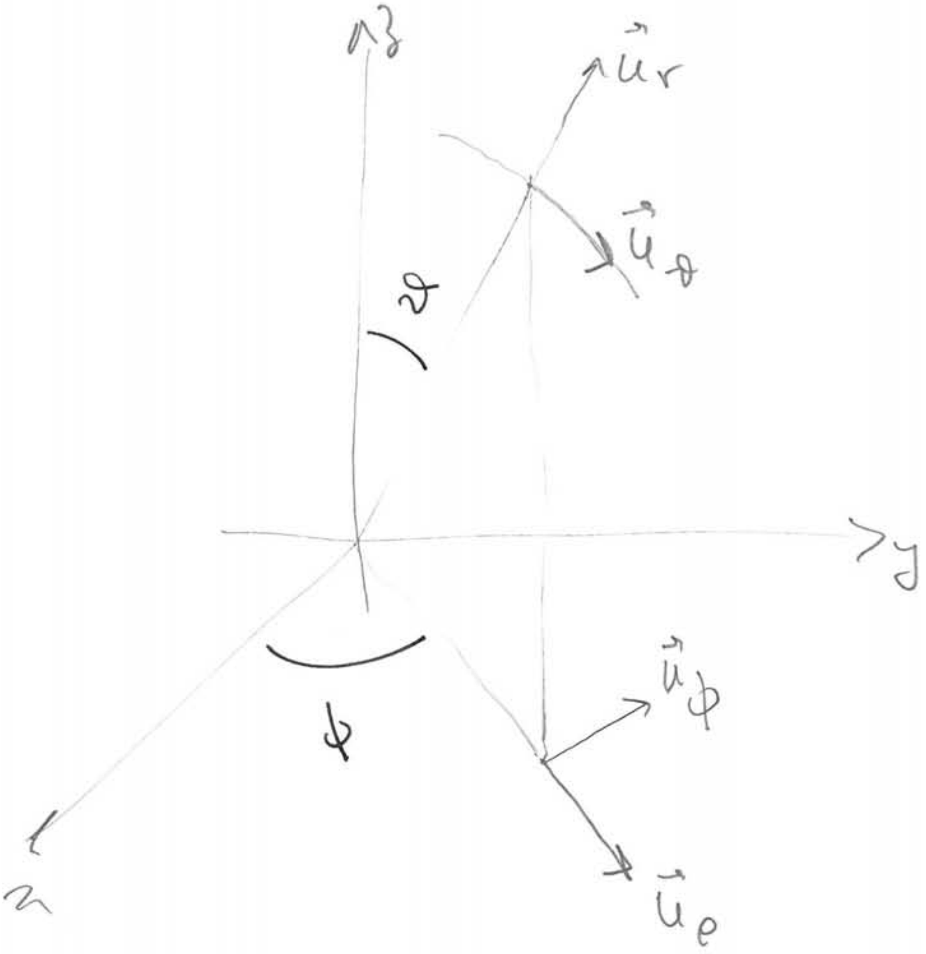


$$\vec{u}_\rho \cdot \vec{u}_\rho = \cos^2\phi + \sin^2\phi = 1$$

$$\vec{u}_r = \sin\vartheta\,\vec{u}_\rho + \cos\vartheta\,\vec{u}_z$$

$$\vec{u}_r \cdot \vec{u}_r = \sin^2\vartheta + \cos^2\vartheta = 1$$

NB2 $\vec{u}_\rho$ ed $\vec{u}_r$ individuano un piano



NB 3 $\quad \vec{u}_\vartheta \cdot \vec{u}_\vartheta = 1$

$\vec{u}_\phi \cdot \vec{u}_\phi = 1$

$\vec{u}_r \cdot \vec{u}_\vartheta = 0 \qquad \vec{u}_r \cdot \vec{u}_\phi = 0 \qquad \vec{u}_\phi \, \vec{u}_\vartheta = 0$

L'ELEMENTO DI LINEA È QUINDI

$$\boxed{\vec{ds} = dr\,\vec{u}_r + r\,d\vartheta\,\vec{u}_\vartheta + r\sin\vartheta\,d\phi\,\vec{u}_\phi}$$

$$W^{(r)}_{A\to B} = \int_A^B \vec{F}\cdot d\vec{s} = \int_A^B f(r)\,dr = \int_{r_A}^{r_B} f(r)\,dr$$

NON DIPENDE DA γ: è conservativa.

ALTRO MODO

SE $\vec{F} = f(r)\,\vec{u}_r$, TENENDO CONTO CHE

$\vec{u}_r = \frac{\vec{r}}{r}$ SI PUÒ ANCHE SCRIVERE

$$\vec{F} = g(r)\,\vec{r}$$

CON $g(r) = f(r)/r$. PER IL LAVORO AVREMO, USANDO COORDINATE CARTESIANE

$$(*)\quad W^{(r)}_{A\to B} = \int_A^B \vec{F}\cdot d\vec{s} = \int_A^B \left(F_x\,dx + F_y\,dy + F_z\,dz\right) =$$

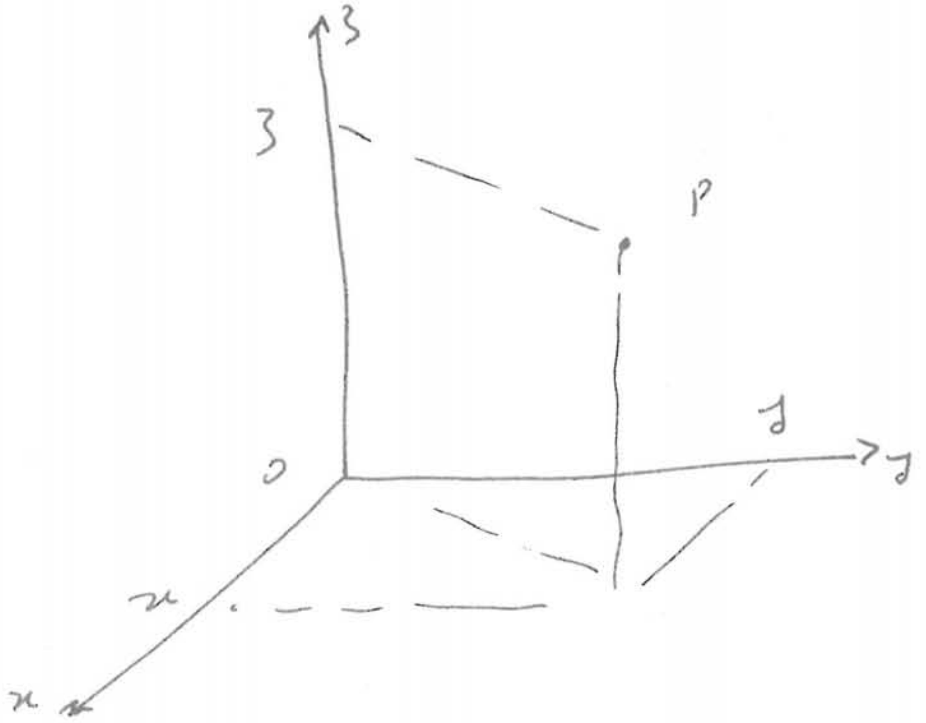
$$= \int_A^B g(r)\left(x\,dx + y\,dy + z\,dz\right) =$$

TEOREMA DI GAUSS

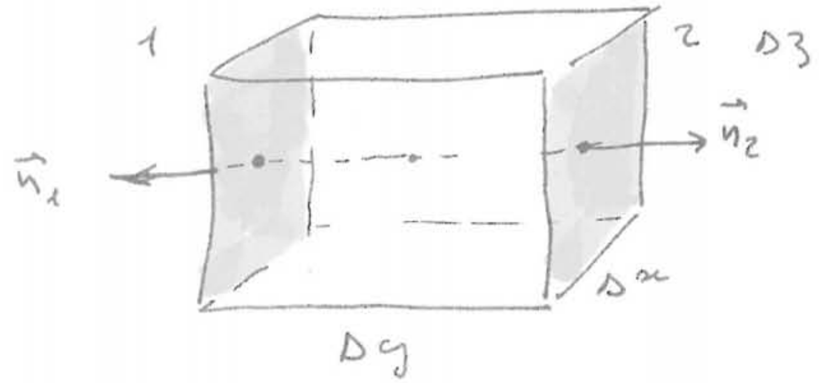
(O DELLA DIVERGENZA)

VETTORE FUNZIONE DEL PUNTO

$$\vec{F} = \vec{F}(x, y, z)$$



COSTRUIAMO UN PARALLELEPIPEDO RETTANGOLO ATTORNO A $P \equiv P(x, y, z)$ DI LATI Δx, Δy, Δz



ANALOGAMENTE

$$\Delta\phi_{(z)} = \frac{\partial F_z}{\partial z}\Delta\tau$$

$$\Delta\phi_{(x)} = \frac{\partial F_x}{\partial x}\Delta\tau$$

IL FLUSSO TOTALE USCENTE DAL VOLUME ELEMENTARE È QUINDI

$$\Delta\phi = \Delta\phi_{(x)} + \Delta\phi_{(y)} + \Delta\phi_{(z)}$$

CHE PER LE PRECEDENTI SI SCRIVE

$$\Delta\phi = \left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}\right)\Delta\tau$$

SI PONE

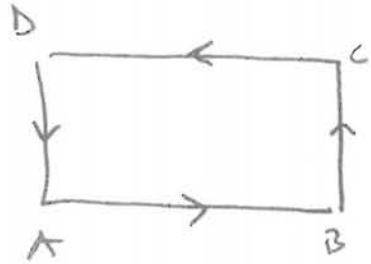
$$\vec{\nabla}\cdot\vec{F} = \operatorname{div}\vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

E QUINDI

$$\Delta\phi = \vec{\nabla}\cdot\vec{F}\cdot\Delta\tau$$

STOKES

S-1



$A \equiv (x, y) \qquad B \equiv (x+\Delta x,\ y)$

$C \equiv (x+\Delta x,\ y+\Delta y) \qquad D \equiv (x,\ y+\Delta y)$

$\vec{F} = F_x \vec{u}_x + F_y \vec{u}_y$

$F_x = F_x(x,y) \qquad F_y = F_y(x,y)$

(1) $$\oint \vec{F}\cdot d\vec{r} = \int_A^B F_x\,dx + \int_B^C F_y\,dy + \int_D^C -F_x\,dx + \int_A^D -F_y\,dy =$$

$$= \langle F_x\rangle_{AB}\,\Delta x + \langle F_y\rangle_{BC}\,\Delta y - \langle F_x\rangle_{DC}\,\Delta x - \langle F_y\rangle_{AD}\,\Delta y =$$

$$= \left(\langle F_x\rangle_{AB} - \langle F_x\rangle_{DC}\right)\Delta x + \left(\langle F_y\rangle_{BC} - \langle F_y\rangle_{AD}\right)\Delta y$$

(2) $$\langle F_x\rangle_{AB} = \frac{1}{2}\left(F_x(A) + F_x(B)\right) = \frac{1}{2}\left(F_x(x,y) + F_x(x+\Delta x, y)\right)$$

$$= \frac{1}{2}\left(F_x(x,y) + F_x(x,y) + \frac{\partial F_x}{\partial x}\Delta x\right) = \underline{F_x(x,y) + \frac{1}{2}\frac{\partial F_x}{\partial x}\Delta x}$$

(3) $$\langle F_x\rangle_{DC} = \frac{1}{2}\left(F_x(D) + F_x(C)\right) = \frac{1}{2}\left(F_x(x, y+\Delta y) + F_x(x+\Delta x, y+\Delta y)\right)$$

$$= \frac{1}{2}\left(F_x(x,y) + \frac{\partial F_x}{\partial y}\Delta y + F_x(x,y) + \frac{\partial F_x}{\partial x}\Delta x + \frac{\partial F_x}{\partial y}\Delta y\right)$$

$$= \underline{F_x(x,y) + \frac{\partial F_x}{\partial y}\Delta y + \frac{1}{2}\frac{\partial F_x}{\partial x}\Delta x}$$

Quindi

Tenendo conto di (4) e (7) la (1) diventa:

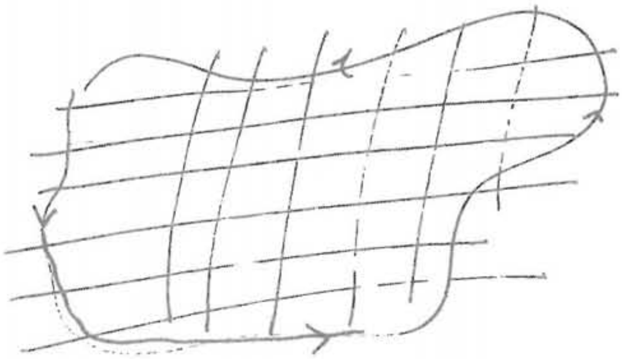
$$\oint \vec{F}\cdot d\vec{r} = \left(-\frac{\partial F_x}{\partial y} + \frac{\partial F_y}{\partial x}\right)\Delta x \Delta y$$

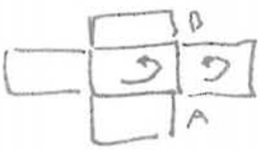
$$\Delta S = \Delta x \, \Delta y$$

$$\oint \vec{F}\cdot d\vec{r} = \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\Delta S$$

$\forall f \in x, y$

$$\boxed{\oint \vec{F}\cdot d\vec{r} = \iint \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right) dS} \quad (8)$$





$\uparrow\downarrow \Rightarrow$ contributo nullo

S - S

(12) $$\boxed{\langle F_x \rangle_{AB} - \langle F_x \rangle_{AC} = -\frac{1}{2} \frac{\partial F_x}{\partial y} \Delta y}$$

Analogamente

(13) $$\langle F_y \rangle_{BC} = \frac{1}{2} \left(F_y(B) + F_y(C) \right) =$$

$$= \frac{1}{2} \left(F_y(x+\Delta x, y) + F_y(x+\Delta x, y+\Delta y) \right) =$$

$$= \frac{1}{2} \left(F_y(x,y) + \frac{\partial F_y}{\partial x} \Delta x + F_y(x,y) + \frac{\partial F_y}{\partial x} \Delta x + \frac{\partial F_y}{\partial y} \Delta y \right)$$

$$= \underline{F_y(x,y) + \frac{\partial F_y}{\partial x} \Delta x + \frac{1}{2} \frac{\partial F_y}{\partial y} \Delta y}$$

(14) $$\langle F_y \rangle_{AC} = \frac{1}{2} \left(F_y(A) + F_y(C) \right) =$$

$$= \frac{1}{2} \left(F_y(x,y) + F_y(x+\Delta x, y+\Delta y) \right) =$$

$$= \frac{1}{2} \left(F_y(x,y) + F_y(x,y) + \frac{\partial F_y}{\partial x} \Delta x + \frac{\partial F_y}{\partial y} \Delta y \right)$$

$$= \underline{F_y(x,y) + \frac{1}{2} \frac{\partial F_y}{\partial x} \Delta x + \frac{1}{2} \frac{\partial F_y}{\partial y} \Delta y}$$

e quindi

(15) $$\boxed{\langle F_y \rangle_{BC} - \langle F_y \rangle_{AC} = \frac{1}{2} \frac{\partial F_y}{\partial x} \Delta x}$$

$$\oint_{ABC} \vec{F}\cdot d\vec{r} = \oint_{ABO} + \oint_{BCO} + \oint_{CAO}$$

$$\oint_{ABO} \vec{F}\cdot d\vec{r} = \iint \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right) dS_z$$

$$\oint_{BCO} \vec{F}\cdot d\vec{r} = \iint \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) dS_x$$

$$\oint_{CAO} \vec{F}\cdot d\vec{r} = \iint \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right) dS_y$$

$$\vec{\nabla}\times\vec{F} = \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\vec{u}_z + \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\vec{u}_y + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\vec{u}_z$$

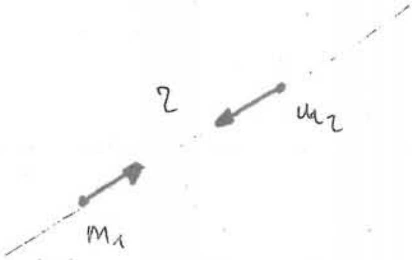
$$\boxed{\oint \vec{F}\cdot d\vec{r} = \iint (\vec{\nabla}\times\vec{F})\cdot d\vec{s}}$$ Th. di Stokes

$$\vec{ds} = dS_x\,\vec{u}_x + dS_y\,\vec{u}_y + dS_z\,\vec{u}_z$$

NB

$$\begin{vmatrix} \vec{u}_x & \vec{u}_y & \vec{u}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} = \vec{u}_x\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) + \vec{u}_y\left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right) + \vec{u}_z\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)$$

LEGGE DI NEWTON



$$F_g = \gamma \frac{m_1 m_2}{r^2}$$

$$\gamma = 6.67 \times 10^{-11} \frac{N m^2}{Kg^2}$$

FORZA ELETTRICA

CARICA ELETTRICA

MATERIALI $\begin{cases} \text{ISOLANTI} \\ \text{CONDUTTORI} \end{cases}$

VETRO +
BACHELITE −

ELETTROSCOPIO

STRUTTURA DELLA MATERIA

PRINCIPIO DI CONSERVAZIONE DELLA CARICA

INDUZIONE ELETTROSTATICA (UNA CARICA VICINA AD UN METALLO)

PROCESSO DI CARICA PER INDUZIONE

SE $q = 10^{-7}$ C

$$n = \frac{q}{e} = \frac{10^{-7}}{1.6\times10^{-19}} = \frac{10^{12}}{1.6} \sim 6.2\times10^{11} \text{ elettroni}$$

Se $r = 1$ cm $= 10^{-2}$ m, $q_1 = q_2 = 10^{-7}$ C

$$F = 9\times10^{9} \times \frac{10^{-14}}{10^{-4}} = 0.9 \text{ N}$$

ANCHE SE $q \sim 10^{-7}$ C, $n \sim 10^{12}$ elettroni: LA QUANTIZZAZIONE NON INFLUENZA L'INTERPRETAZIONE (SI PUÒ IMMAGINARE CHE LA CARICA SIA UNA GRANDEZZA CONTINUA)

FORZA GRAVITAZIONALE ED ELETTRICA TRA UN PROTONE ED UN ELETTRONE

$q_p = 1.6\times10^{-19}$ C

$q_e = -1.6\times10^{-19}$ C $\qquad r = 0.5\times10^{-10}$ m

$m_p = 1.6\times10^{-27}$ kg

$m_e = 9.1\times10^{-31}$ kg

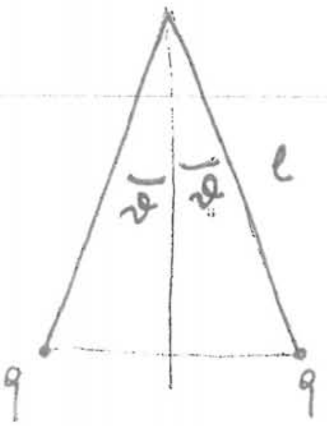
$$F_g = 6.6\times10^{-11} \times \frac{1.6\times10^{-27}\times 9.1\times10^{-31}}{(0.5\times10^{-10})^2} \sim 3.6\times10^{-47} \text{ N}$$

$$F_e = 9\times10^{9} \times \frac{(1.6\times10^{-19})^2}{(0.5\times10^{-10})^2} \sim 8.2\times10^{-8} \text{ N}$$

$$\frac{F_e}{F_g} \sim 2.3\times10^{39}$$

ESERCIZIO

DUE SFERETTE UGUALI CON CARICA q SU DUE FILI IDENTICI
QUANTO VALE ϑ ALL'EQUILIBRIO?



$$r = 2l \sin\vartheta$$

$$F_e = k\,\frac{q^2}{r^2} = k\,\frac{q^2}{(2l\sin\vartheta)^2}$$

$$\operatorname{tg}\vartheta = k\,\frac{q^2}{mg\,(2l\sin\vartheta)^2}$$

$$\operatorname{tg}\vartheta\,\sin^2\vartheta = k\,\frac{q^2}{4l^2mg}$$

se $\vartheta \to 0$ $\quad \operatorname{tg}\vartheta \sim \vartheta$, $\sin\vartheta \sim \vartheta$ $\Rightarrow$

$$\vartheta^3 = k\,\frac{q^2}{4l^2mg} \qquad \vartheta = \sqrt[3]{k\,\frac{q^2}{4l^2mg}}$$

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CAMPO CREATO DA UNA CARICA PUNTIFORME

$$\vec{F} = k \frac{\rho \rho_0}{r^2} \vec{u}$$

$$\vec{E} = \frac{\vec{F}}{\rho_0} = k \frac{\rho}{r^2} \vec{u}$$

$$\vec{E} = k \rho \frac{\vec{r}}{r^3}$$

IN GENERALE:

$$\vec{F}(x, y, z) = \rho_0 \vec{E}(x, y, z)$$

$\vec{r} = \vec{R} - \vec{R}'$ ← punto sorgente

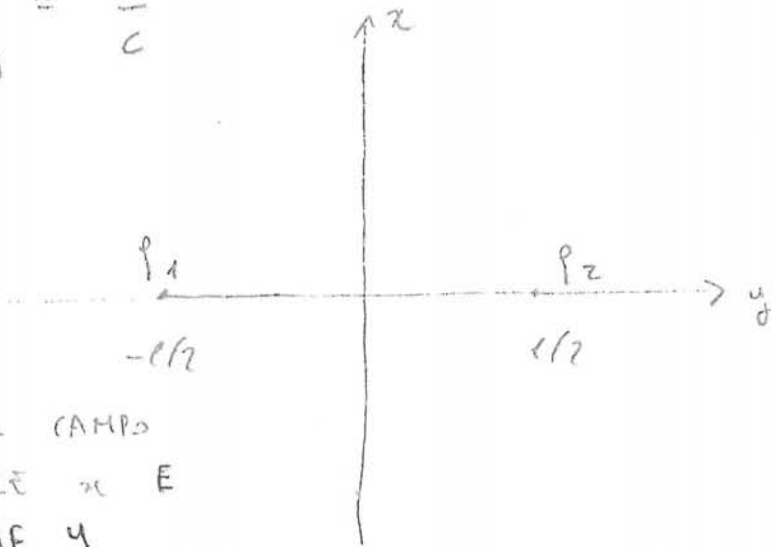
↑ punto campo

UNITA' DI MISURA

$$[E] = \frac{[F]}{[\rho]} = \frac{N}{C}$$

ESERCIZIO

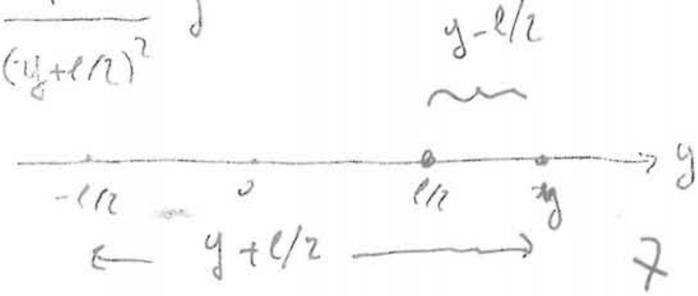
$\rho_1 = \rho_2 = \rho$



CALCOLARE IL CAMPO SULL'ASSE DELLE x E SULL'ASSE DELLE y

$y > l/2$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \underbrace{k \frac{\rho}{(y-l/2)^2} \vec{j}}_{E_2} + \overbrace{k \frac{\rho}{(y+l/2)^2} \vec{j}}^{E_1}$$



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$$\vec{E} = \vec{E}_1 + \vec{E}_2 = k \frac{q}{(l/2)^2 + y^2} \, 2 \cos\vartheta \, \vec{i}$$

CALCOLO DI $\cos\vartheta$

$$\overline{AB} \cdot \cos\vartheta = \overline{OA}$$

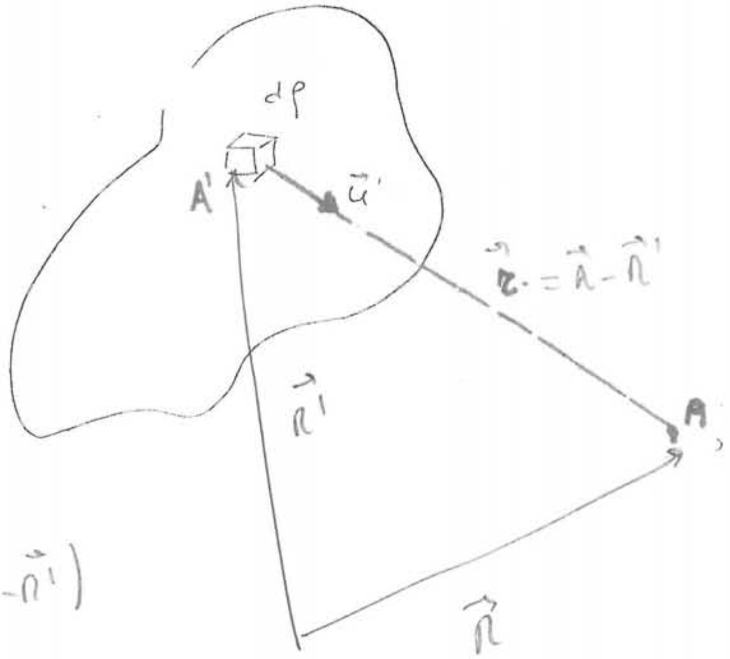
$$\cos\vartheta = \frac{\overline{OA}}{\overline{AB}} = \frac{x}{\sqrt{(l/2)^2 + x^2}}$$

$$\vec{E} = 2 k q \frac{x}{\left[(l/2)^2 + x^2\right]^{3/2}} \vec{i}$$

CAMPO ELETTROSTATICO CREATO DA UNA DISTRIBUZIONE CONTINUA DI CARICHE

$$\vec{E} = \sum_{i=1}^{N} k \frac{q_i}{r_i^2} \vec{u}_i$$

$$\vec{E} = \int k \frac{dq}{r'^2} \vec{u}'$$



$$\vec{E} = \int k \frac{dq(\vec{R}')}{|\vec{R} - \vec{R}'|^3} (\vec{R} - \vec{R}')$$

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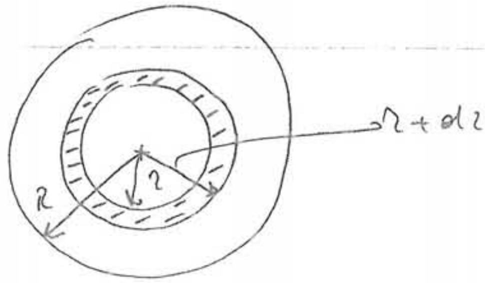
DISCO: CAMPO SULL'ASSE

Q DISTRIBUITA UNIFORMEMENTE SU UN DISCO SOTTILE

DISCO = SUCCESSIONE DI ANELLI

$$\sigma = \frac{Q}{\pi R^2}$$



$$d\Sigma = 2\pi r\, dr$$

r → b

R → B

$$dQ = \sigma\, d\Sigma = 2\pi r\, dr\, \sigma$$

$$d\vec{E}(x) = k\, dQ \frac{x}{(r^2+x^2)^{3/2}} \vec{i} =$$

$$= k\sigma\, 2\pi r\, dr \frac{x}{(r^2+x^2)^{3/2}} \vec{i}$$

$$\vec{E}(x) = k\sigma\, 2\pi x\, \vec{i} \int_0^R \frac{r\, dr}{(r^2+x^2)^{3/2}} \Rightarrow$$

$$\vec{E}(x) = \frac{\sigma}{2\varepsilon_0} x\, \vec{i} \int_0^R \frac{r\, dr}{(r^2+x^2)^{3/2}}$$

si ha

$$I = \int_0^R \frac{r\, dr}{(r^2+x^2)^{3/2}} = \frac{1}{2}\int_0^R \frac{d\, r^2}{(r^2+x^2)^{3/2}} = \frac{1}{2}\int_0^R \frac{d\,(r^2+x^2)}{(r^2+x^2)^{3/2}}$$

$$\boxed{\vec{E}(x) = -\frac{\sigma}{2\varepsilon_0}\vec{i}\left\{1 - \frac{|x|}{\sqrt{x^2+R^2}}\right\}} \qquad x<0$$

Quindi

$$\boxed{\vec{E}(x) = \pm\frac{\sigma}{2\varepsilon_0}\vec{i}\left\{1 - \frac{|x|}{\sqrt{x^2+R^2}}\right\}} \qquad \begin{matrix} + & x>0 \\ - & x<0 \end{matrix}$$

$$\vec{E}_+ = \vec{E}(0^+) = \frac{\sigma}{2\varepsilon_0}\vec{i}$$

$$\vec{E}_- = \vec{E}(0^-) = -\frac{\sigma}{2\varepsilon_0}\vec{i}$$

$$\vec{E}_+ - \vec{E}_- = \frac{\sigma}{\varepsilon_0}\vec{i}$$

$\boxed{\text{SE } |x| >> R}$ TENENDO CONTO CHE

$$\frac{|x|}{\sqrt{x^2+R^2}} = \frac{|x|}{|x|\sqrt{1+\left(\frac{R}{x}\right)^2}} = \frac{1}{\sqrt{1+\left(\frac{R}{x}\right)^2}} = \frac{1}{1+\frac{1}{2}\frac{R^2}{x^2}} =$$

$$= 1 - \frac{1}{2}\frac{R^2}{x^2}$$

AVREMO

$$\vec{E}(x) = \pm\frac{\sigma}{2\varepsilon_0}\vec{i}\left\{1 - 1 + \frac{1}{2}\frac{R^2}{x^2}\right\} = \pm\frac{\sigma}{4\varepsilon_0}\frac{R^2}{x^2}\vec{i} = \pm\frac{\sigma}{4\pi\varepsilon_0}\frac{\pi R^2}{x^2}\vec{i} =$$

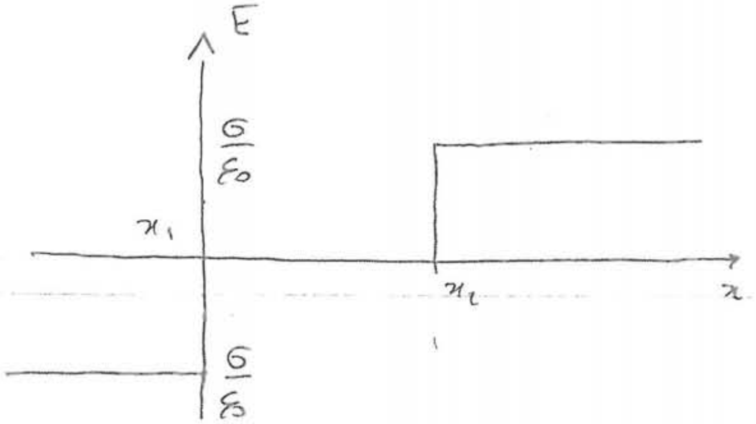
$$= \pm k\frac{Q}{x^2}\vec{i}$$

se $\sigma_1 = \sigma_2 = \sigma$

$x > x_2$ $\quad \vec{E} = \frac{\sigma}{\varepsilon_0} \vec{i}$

$x_1 < x < x_2$ $\quad \vec{E} = 0$

$x < x_2$ $\quad \vec{E} = -\frac{\sigma}{\varepsilon_0} \vec{i}$

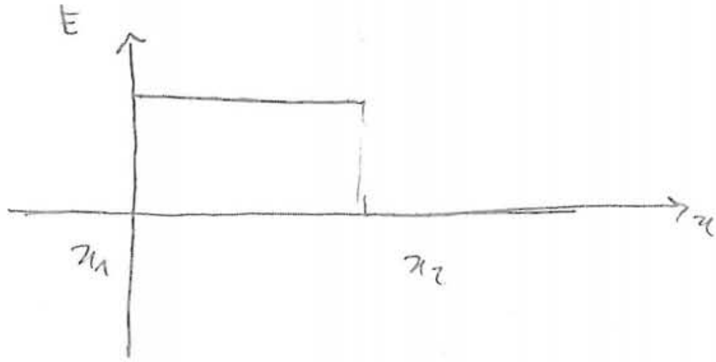


se $\sigma_1 = \sigma$, $\sigma_2 = -\sigma$

$x > x_2$ $\quad \vec{E} = 0$

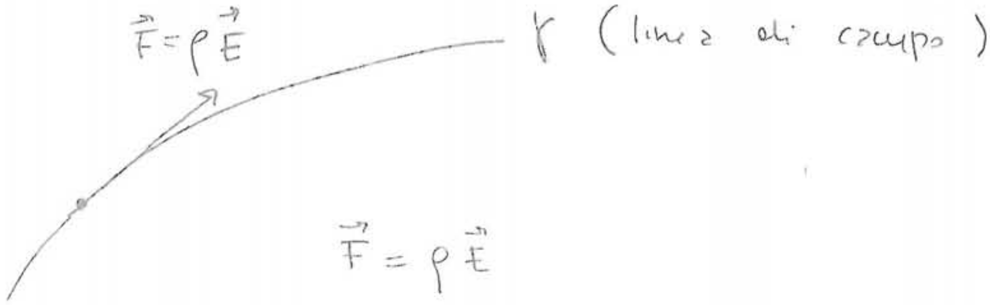
$x_1 < x < x_2$ $\quad \vec{E} = \frac{\sigma}{\varepsilon_0} \vec{i}$

$x < x_1$ $\quad \vec{E} = 0$



LINEE DI FORZA DEL CAMPO $\vec{E}$

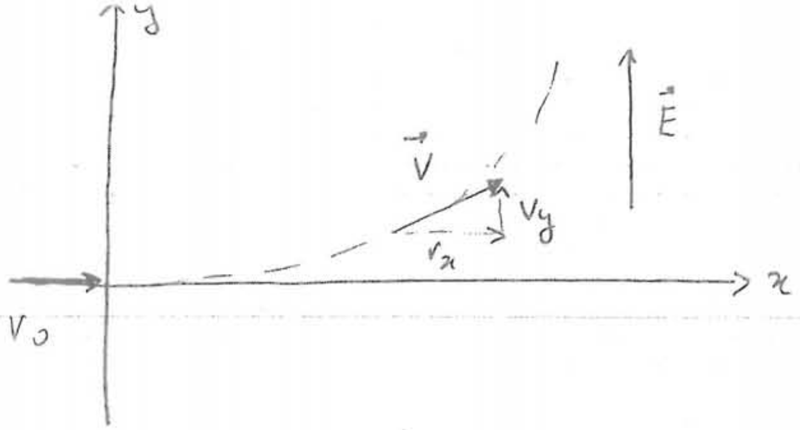
- HANNO PER TANGENTE UN VERSORE $\vec{u}_T \parallel \vec{E}$. LE LINEE DI CAMPO NON SI INCONTRANO MAI (UNA SOLA TANGENTE)
- UNA PARTICELLA CARICA NON SI MUOVE SU UNA LINEA DI FORZA



$$\vec{F} = \rho \vec{E}$$

$$\vec{F} = m\vec{a} = m\left(\frac{dv}{dt}\vec{u}_T + \frac{v^2}{R}\vec{u}_N\right)$$

LA FORZA <u>NON</u> È TANGENTE ALLA TRAIETTORIA DESCRITTA DALLA PARTICELLA



$$\vec{V}(t) = V_x(t)\,\vec{i} + V_y(t)\,\vec{j} = V_0\,\vec{i} + \frac{q}{m} E t\,\vec{j}$$

VARIAZIONE DI ENERGIA CINETICA

$$\Delta E_K = E_K(t) - E_K(0) = \frac{1}{2} m v^2(t) - \frac{1}{2} m v_0^2 =$$

$$= \frac{1}{2} m \left(v^2(t) - v_0^2\right) = \frac{1}{2} m \left\{ v_0^2 + \left(\frac{q}{m} E t\right)^2 - v_0^2 \right\} =$$

$$= \frac{1}{2} m \left(\frac{q}{m} E t\right)^2 = \frac{1}{2} \frac{q}{m} E \cdot t^2 \cdot qE = qEy$$

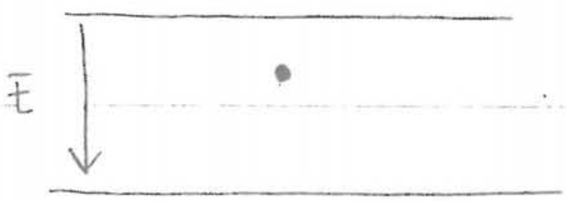
qEy = LAVORO FATTO DALLA FORZA ELETTRICA SULLA CARICA

$$W = \int_A^B \vec{F} \cdot \vec{ds} = \int_A^B q\vec{E} \cdot (dx\,\vec{i} + dy\,\vec{j}) =$$

$$= \int_A^B q E\, dy = qE(y_B - y_A) = qEy$$

ABBIAMO RITROVATO IL TEOREMA DELL'ENERGIA CINETICA

SI CAMPO



$$m a = m g - \frac{4}{3}\pi R^3 \rho_A g + q E - 6\pi\eta R V$$

NUOVA VELOCITÀ LIMITE

$$\frac{4}{3}\pi R^3 (\rho - \rho_A) g + q E - 6\pi\eta R V_1 = 0$$

$$V_1 = \frac{2}{9}\,\frac{\rho - \rho_A}{\eta}\,R^2 g + \frac{q}{6\pi\eta R}\,E$$

$$\boxed{V_1 = V_0 + \frac{q}{6\pi\eta R}\,E}$$

RELAZIONE FONDAMENTALE

$$W^{(C)}_{A\to B} = U(A) - U(B)$$

U = ENERGIA POTENZIALE

CALCOLO DELL'ENERGIA POTENZIALE PER 1 CARICA q

$$\vec{ds} = dr\,\vec{u}_r + r\,d\vartheta\,\vec{u}_\vartheta \qquad \text{COORDINATE POLARI (PIANE)}$$

$$\vec{F}\cdot\vec{ds} = k\frac{q_0 q}{r^2}\vec{u}_r \cdot (dr\,\vec{u}_r + r\,d\vartheta\,\vec{u}_\vartheta)$$

$$= k\frac{q_0 q}{r^2}\,dr$$

$$W^{(C)}_{A\to B} = \int_{A\,C}^{B} \vec{F}\cdot\vec{ds} = \int_A^B k\frac{q_0 q}{r^2}\,dr = -k\,q_0 q\left[\frac{1}{r}\right]_{r_A}^{r_B}$$

$$W^{(C)}_{A\to B} = k\,q_0 q\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

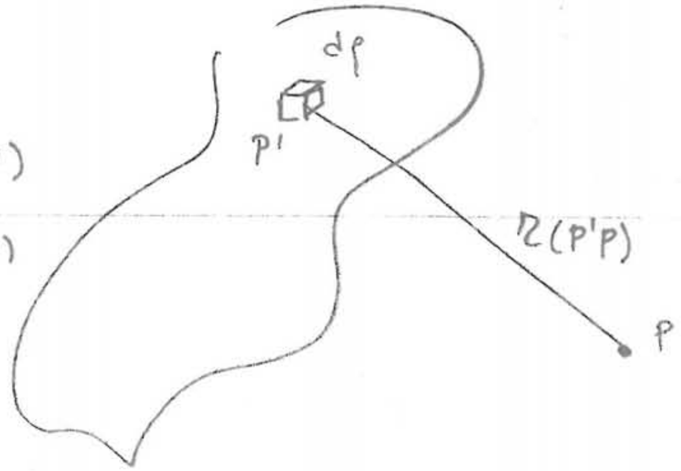
$$U = k\frac{q_0 q}{r} + \text{cost} \qquad = \text{ENERGIA POTENZIALE}$$

$$T^{(C)}_{A\to B} = \int_{A\,C}^{B} \vec{E}\cdot\vec{ds} = \int_{A\,C}^{B} k\frac{q}{r^2}\,dr = k\,q\left(\frac{1}{r_A} - \frac{1}{r_B}\right) = \frac{U(A)}{q_0} - \frac{U(B)}{q_0}$$

$$V = \frac{U}{q} = \text{POTENZIALE ELETTRICO}$$

CASO CONTINUO

$$u = \rho_0 \int_{\text{SUL CORPO}} k \frac{d\rho(P')}{r(P'P)}$$



$$V = \frac{u}{\rho_0} = \int_{\text{SUL CORPO}} k \frac{d\rho(P')}{r(P'P)}$$

UNITA' DI MISURA

$[u] = \text{joule}$

$$[V] = \frac{[u]}{[\rho]} = \frac{\text{joule}}{C} = \text{Volt}$$

$$[E] = \frac{N}{C} = \frac{N\,m}{C\,m} = \frac{\text{Volt}}{m} = \frac{V}{m}$$

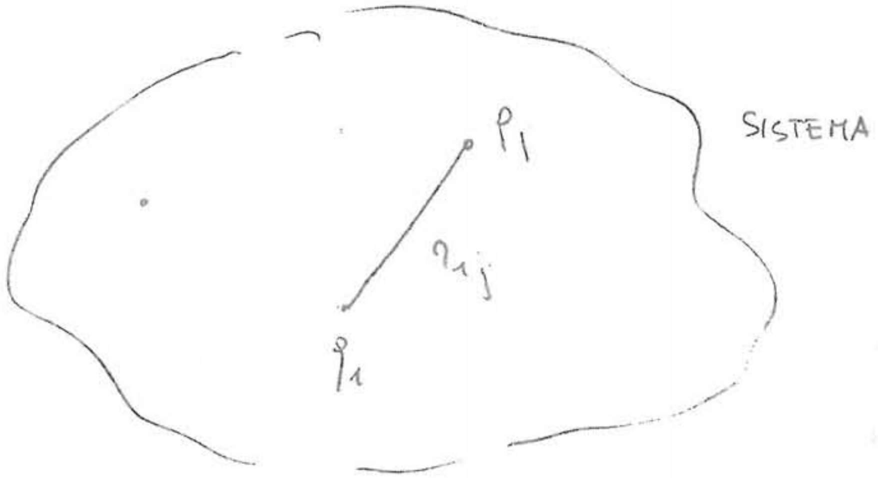
NB $\oint \vec{F} \cdot d\vec{s} = 0$

$$\rho_0 \oint \vec{E} \cdot d\vec{s} = 0 \quad \Rightarrow \quad \oint \vec{E} \cdot d\vec{s} = 0$$

$$U(P_1,P_2,P_3) = U(P_1) + U(P_2) + U(P_3) = 0 + k\frac{q_1 q_2}{r_{12}} + k\frac{q_1 q_3}{r_{13}} + k\frac{q_2 q_3}{r_{23}}$$

IN GENERALE

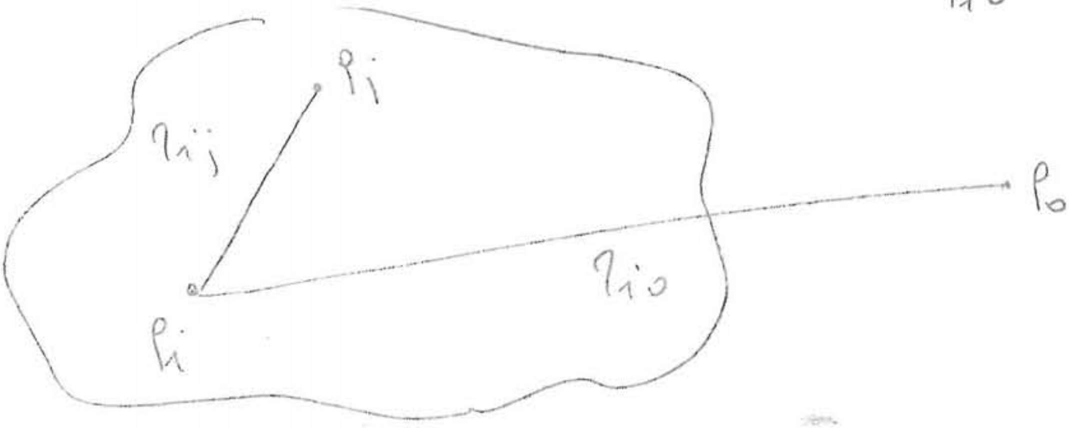
$$U = \frac{1}{2}\sum_{i,j} k\frac{q_i q_j}{r_{ij}}$$ = LAVORO PER COSTRUIRE IL SISTEMA CHE DEVE ESSERE FATTO DALL'ESTERNO



SE AGGIUNGIAMO UNA CARICA q_0 (DIVERSA DALLE PRECEDENTI)
IL LAVORO TOTALE CHE SI E' FATTO E'

W = LAVORO PER COSTRUIRE IL SISTEMA $\left(\frac{1}{2}\sum_{i,j} k\frac{q_i q_j}{r_{ij}}\right)$

+ LAVORO PER PORTARE q_0 $\left(q_0 \sum_i k\frac{q_i}{r_{i0}}\right)$



MOTO DI UNA CARICA IN UN CAMPO ELETTROSTATICO

$$E = E_k + E_p$$

$$E = \frac{1}{2} m v^2 + U \qquad U = \rho_0 V$$

$$E = \frac{1}{2} m v^2 + \rho_0 V = \text{COSTANTE}$$

ESEMPIO 1

MOTO IN CAMPO COSTANTE $\vec{E} = E \vec{k}$

CALCOLO ENERGIA POTENZIALE

$$W_{A \to B} = U(A) - U(B) = \int_A^B \vec{F} \cdot d\vec{s} = \rho_0 \int_A^B \vec{E} \cdot d\vec{s} =$$

$$= \rho_0 \int_A^B E \, dz = \rho_0 E \left[z(B) - z(A) \right]$$

$$U = -\rho_0 E z + \text{cost}$$

$$V = \frac{U}{\rho_0} = -E z + \text{cost}$$

$$E = \frac{1}{2} m v^2 - \rho_0 E z = \text{COSTANTE}$$

CAMPO COME GRADIENTE DEL POTENZIALE

- se $\vec{F}$ E' CONSERVATIVA $\exists\, U$ /

$$\vec{F} = -\vec{\nabla} \cdot U$$

$$\frac{\vec{F}}{q_0} = -\vec{\nabla}\left(\frac{U}{q_0}\right)$$

- $$\vec{E} = \frac{\vec{F}}{q_0}$$

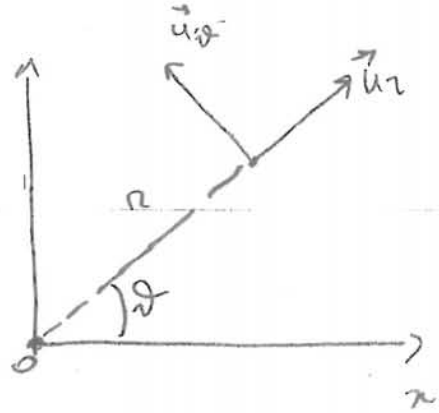
$$V = \frac{U}{q_0}$$

$$\vec{E} = -\vec{\nabla} V$$

- $$\int_A^B \vec{E} \cdot \vec{ds} = V(A) - V(B)$$

$$\vec{E} \cdot \vec{ds} = -dV$$

COORDINATE POLARI (PIANE)



$$\vec{ds} = \vec{u}_r \, dr + \vec{u}_\vartheta \, r \, d\vartheta$$

$$\vec{E} = E_r \vec{u}_r + E_\vartheta \vec{u}_\vartheta$$

$$\vec{E} \cdot \vec{ds} = E_r \, dr + E_\vartheta \, r \, d\vartheta$$

$$dV = \frac{\partial V}{\partial r} dr + \frac{\partial V}{\partial \vartheta} d\vartheta$$

$$\vec{E} \cdot \vec{ds} = -dV$$

$$E_r \, dr + E_\vartheta \, r \, d\vartheta = -\frac{\partial V}{\partial r} dr - \frac{\partial V}{\partial \vartheta} d\vartheta$$

$$E_r = -\frac{\partial V}{\partial r} \qquad E_\vartheta = -\frac{1}{r} \frac{\partial V}{\partial \vartheta}$$

$$\vec{E} = -\frac{\partial V}{\partial r} \vec{u}_r - \frac{1}{r} \frac{\partial V}{\partial \vartheta} \vec{u}_\vartheta$$

$$\vec{\nabla} = \vec{u}_r \frac{\partial}{\partial r} + \vec{u}_\theta \frac{1}{r} \frac{\partial}{\partial \theta}$$

$$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left(k \frac{Q}{\sqrt{x^2+R^2}} \right) =$$

$$= -kQ \frac{\partial}{\partial x}\left[(x^2+R^2)^{-1/2} \right] =$$

$$= +kQ \frac{1}{2} (x^2+R^2)^{-3/2} \cdot 2x$$

$$E_x = kQ \frac{x}{(x^2+R^2)^{3/2}}$$

$$E_y = E_z = 0$$

$$\vec{E} = kQ \frac{x}{(x^2+R^2)^{3/2}} \vec{i}$$

COME GIA' TROVATO

$$\eta = x^2 + r^2$$

$$\eta(0) = x^2 \qquad \eta(R) = x^2 + R^2$$

$$V = \pi h \sigma \, 2 \left[\sqrt{\eta} \right]_{\eta(0)}^{\eta(R)} = 2\pi h \sigma \left(\sqrt{\eta(R)} - \sqrt{\eta(0)} \right) =$$

$$= 2\pi h \sigma \left(\sqrt{x^2 + R^2} - \sqrt{x^2} \right) \Rightarrow$$

$$\boxed{V = 2\pi h \sigma \left(\sqrt{x^2 + R^2} - |x| \right)}$$

SE $|x| >> R$

$$\sqrt{x^2 + R^2} = |x| \sqrt{1 + \left(\frac{R}{x}\right)^2} = |x| \left(1 + \frac{1}{2} \frac{R^2}{x^2} \right)$$

$$\sqrt{x^2 + R^2} - |x| = |x| + \frac{1}{2} \frac{R^2}{|x|} - |x| = \frac{1}{2} \frac{R^2}{|x|}$$

$$V = 2\pi h \sigma \frac{1}{2} \frac{R^2}{|x|} = h \sigma \frac{\pi R^2}{|x|} = h \frac{Q}{|x|}$$

COME SI DEVE

SE $|x| << R$

$$V = 2\pi h \sigma \left(R - |x| \right)$$

$$V = 2\pi h \sigma (R - x) \qquad x > 0$$

$$V = 2\pi h \sigma (R + x) \qquad x < 0$$

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$$\frac{dV}{dx} = -\frac{\sigma}{\varepsilon_0}$$

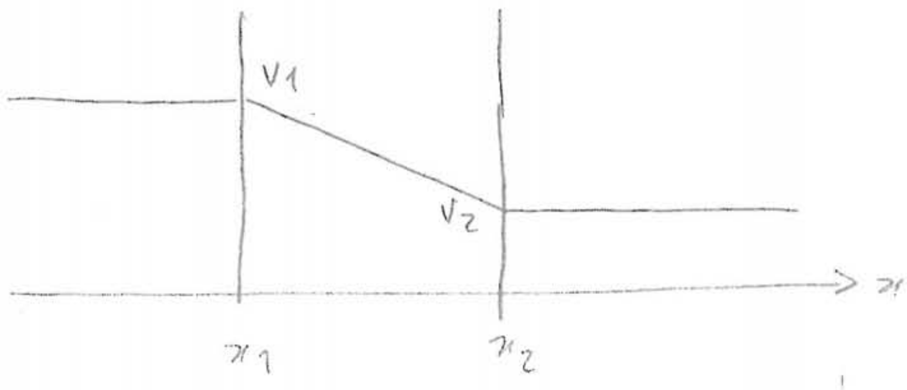
$$V = -\frac{\sigma}{\varepsilon_0} x + \alpha$$

$$\begin{cases} V(x_2) = V'(x_2) \\ V(x_1) = V''(x_1) \end{cases}$$

$$-\frac{\sigma}{\varepsilon_0} x_2 + \alpha = V_2 \quad (*)$$

$$-\frac{\sigma}{\varepsilon_0} x_1 + \alpha = V_1$$

$$-\frac{\sigma}{\varepsilon_0}(x_2 - x_1) = V_2 - V_1$$



PONIAMO $V_2 = 0 \overset{(*)}{\Rightarrow} \alpha = \frac{\sigma}{\varepsilon_0} x_2 \Rightarrow$

$$V = \frac{\sigma}{\varepsilon_0}(x_2 - x)$$

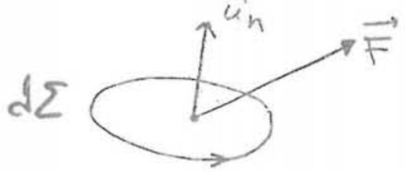
ROTORE DI UN VETTORE $\vec{A}(x,y,z)$

$$\text{rot}\,\vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} =$$

$$= \vec{i}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \vec{j}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \vec{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

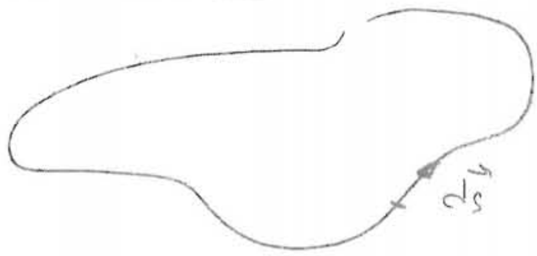
FLUSSO DI UN VETTORE $\vec{F}(x,y,z)$

$$\phi_\Sigma = \iint_\Sigma \vec{F}\cdot\vec{u}_n \, d\Sigma$$



CIRCUITAZIONE DI UN VETTORE $\vec{G}(x,y,z)$

$$C = \oint_\gamma \vec{G}\cdot d\vec{s}$$



TEOREMA DI STOKES

$$\oint_\gamma \vec{A}\cdot d\vec{s} = \iint_\Sigma \text{rot}\,\vec{A}\cdot\vec{u}_n \, d\Sigma$$

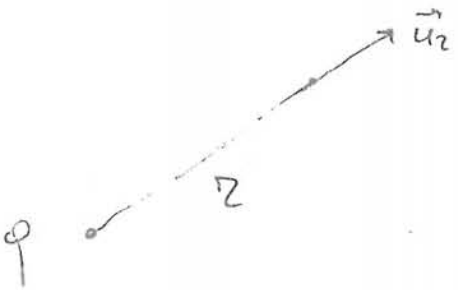
CAMPO ELETTROSTATICO

È CONSERVATIVO $\Rightarrow$ $\vec{\nabla} \times \vec{E} = 0$

PROBLEMA

VERIFICARE CHE IL CAMPO CREATO DA UNA CARICA PUNTIFORME È IRROTAZIONALE

SOLUZIONE



$r = \sqrt{x^2+y^2+z^2}$

$$\vec{E} = k \frac{q}{r^2} \vec{u}_r = k q \frac{\vec{r}}{r^3}$$

$$E_x = k q \frac{x}{r^3} \qquad E_y = k q \frac{y}{r^3} \qquad E_z = k q \frac{z}{r^3}$$

$$\frac{\partial E_x}{\partial y} = \frac{\partial}{\partial y}\left(k q \frac{x}{r^3}\right) = k q x \frac{\partial}{\partial y}\left(r^{-3}\right)$$

$$\begin{cases} \dfrac{\partial E_x}{\partial y} = -3 k q \dfrac{x}{r^4} \dfrac{\partial r}{\partial y} \\[2ex] \dfrac{\partial E_y}{\partial x} = -3 k q \dfrac{y}{r^4} \dfrac{\partial r}{\partial x} \end{cases}$$

$$\frac{\partial r}{\partial y} = \frac{1}{r} y \qquad \frac{\partial r}{\partial x} = \frac{1}{r} x$$

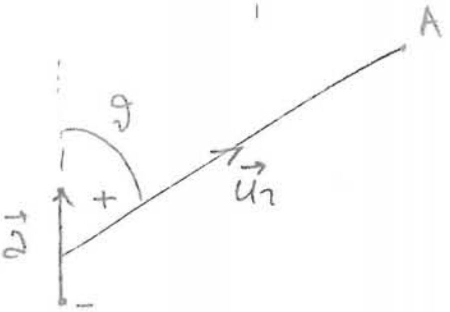
$$\frac{\partial E_x}{\partial y} = -3 k q \frac{xy}{r^5} = \frac{\partial E_y}{\partial x}$$

$$\frac{1}{r_+} - \frac{1}{r_-} = \frac{r_- - r_+}{r_+ r_-}$$

$$r_- - r_+ = a \cos\vartheta$$

$$r_+ r_- = r^2 - \left(\frac{a}{2}\cos\vartheta\right)^2 \sim r^2$$

$$V(A) = k \frac{q a \cos\vartheta}{r^2}$$



$\vec{p} = q\vec{a}$ (MOMENTO DI DIPOLO)

$$V(A) = k \frac{\vec{p} \cdot \vec{u}_r}{r^2}$$

DIPENDE DA r^{-2}

PER UNA CARICA PUNTIFORME

$$V = k \frac{q}{r}$$

DIPENDE DA r^{-1}

$$\frac{\partial V}{\partial x} = h \frac{1}{r^3} \left\{ p_x - 3 \frac{x}{r^2} (\vec{p} \cdot \vec{r}) \right\}$$

ANALOGAMENTE

$$\frac{\partial V}{\partial y} = h \frac{1}{r^3} \left\{ p_y - 3 \frac{y}{r^2} (\vec{p} \cdot \vec{r}) \right\}$$

$$\frac{\partial V}{\partial z} = h \frac{1}{r^3} \left\{ p_z - 3 \frac{z}{r^2} (\vec{p} \cdot \vec{r}) \right\}$$

DI CONSEGUENZA

$$\vec{E} = -\vec{\nabla} V = -\left(\vec{u}_x \frac{\partial V}{\partial x} + \vec{u}_y \frac{\partial V}{\partial y} + \vec{u}_z \frac{\partial V}{\partial z} \right) =$$

$$= -\frac{h}{r^3} \left\{ \vec{u}_x p_x + \vec{u}_y p_y + \vec{u}_z p_z \right.$$

$$\left. -3 \frac{x \vec{u}_x + y \vec{u}_y + z \vec{u}_z}{r^2} (\vec{p} \cdot \vec{r}) \right\} =$$

$$= -\frac{h}{r^3} \left\{ \vec{p} - 3 \frac{\vec{r} (\vec{p} \cdot \vec{r})}{r^2} \right\} \Rightarrow$$

$$\boxed{\vec{E} = h \frac{3(\vec{p} \cdot \vec{u}_r)\vec{u}_r - \vec{p}}{r^3}}$$

$$\boxed{\vec{E} = k \, \frac{3(\vec{p}\cdot\vec{u}_r)\vec{u}_r - \vec{p}}{r^3}}$$

CAMPO SULL'ASSE ($\vartheta = 0$)

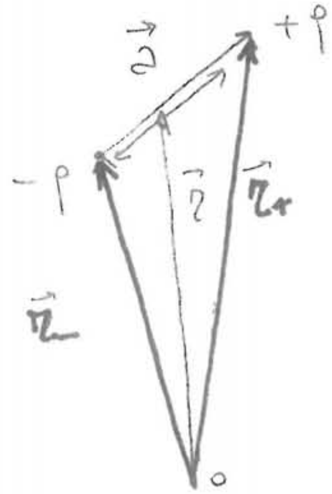
$$\vec{E} = 2 \frac{k}{r^3} p \vec{u}_r = 2 \frac{k}{r^3} \vec{p}$$

CAMPO NEL PIANO MERIDIANO ($\vartheta = \pi/2$)

$$\vec{E} = \frac{k}{r^3} p \vec{u}_\vartheta = - \frac{k}{r^3} \vec{p}$$

DIPOLO IN UN CAMPO ESTERNO

A) CALCOLO DELL'ENERGIA POTENZIALE DEL DIPOLO



$$U = U_- + U_+ = -q V(\vec{r}_-) + q V(\vec{r}_+) =$$

$$= q \left[V(\vec{r}_+) - V(\vec{r}_-) \right]$$

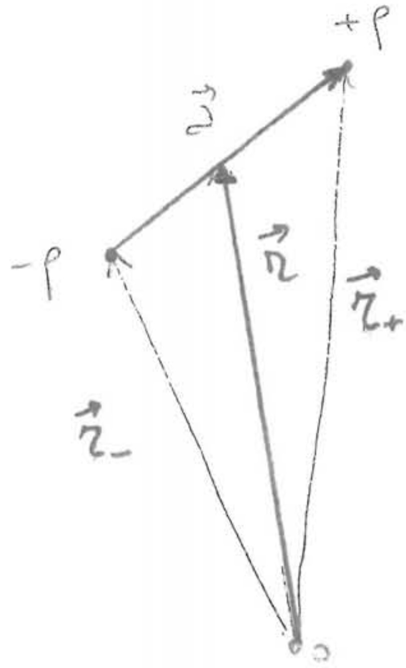
$$\vec{r}_- = \vec{r} - \frac{\vec{a}}{2} \qquad \vec{r}_+ = \vec{r} + \frac{\vec{a}}{2}$$

B) CALCOLO DELLA FORZA

$$\vec{F} = -\vec{\nabla} U = \vec{\nabla}(\vec{p} \cdot \vec{E})$$

NB _ SE $\vec{E}$ = UNIFORME $\quad \vec{F} \equiv 0$

C) CALCOLO DEL MOMENTO SU DI UN DIPOLO IN UN CAMPO ASSEGNATO UNIFORME



$$\vec{M}_O = \vec{r}_- \times \vec{F}_- + \vec{r}_+ \times \vec{F}_+ =$$

$$= \left(\vec{r} - \frac{\vec{a}}{2}\right) \times (-\rho \vec{E}) + \left(\vec{r} + \frac{\vec{a}}{2}\right) \times (\rho \vec{E}) =$$

$$= -\rho\, \vec{r} \times \vec{E} + \frac{1}{2} \rho\, \vec{a} \times \vec{E} + \rho\, \vec{r} \times \vec{E} + \frac{1}{2} \rho\, \vec{a} \times \vec{E}$$

$$= \rho\, \vec{a} \times \vec{E} = \vec{p} \times \vec{E}$$

$\vec{M}_O = \vec{p} \times \vec{E}$ INDIPENDENTE DAL POLO (OVVIO)

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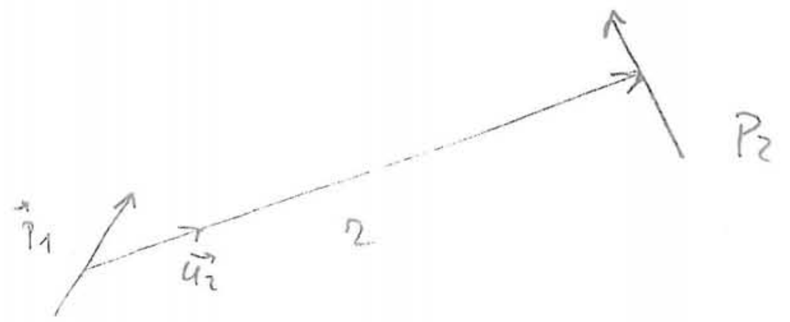
ESERCIZIO

DETERMINARE L'ENERGIA DI INTERAZIONE TRA DUE DIPOLI $\vec{p}_1$ E $\vec{p}_2$.

SOLUZIONE

$$U = -\vec{p}_2 \cdot \vec{E}_1$$

$\vec{E}_1$ = CAMPO ELETTRICO CREATO DAL DIPOLO 1 NEL PUNTO DOVE SI TROVA IL DIPOLO 2



$$\vec{E}_1 = k \, \frac{3(\vec{p}_1 \cdot \vec{u}_r)\vec{u}_r - \vec{p}_1}{r^3}$$

$$U = -k \, \frac{3(\vec{p}_1 \cdot \vec{u}_r)(\vec{p}_2 \cdot \vec{u}_r) - \vec{p}_1 \cdot \vec{p}_2}{r^3}$$

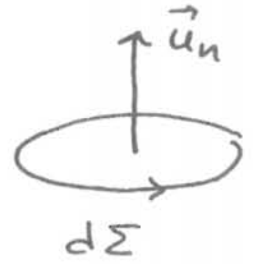
CHE È SIMMETRICA IN $\vec{p}_1$ E $\vec{p}_2$ (COME SI DEVE)

FLUSSO DI UN VETTORE

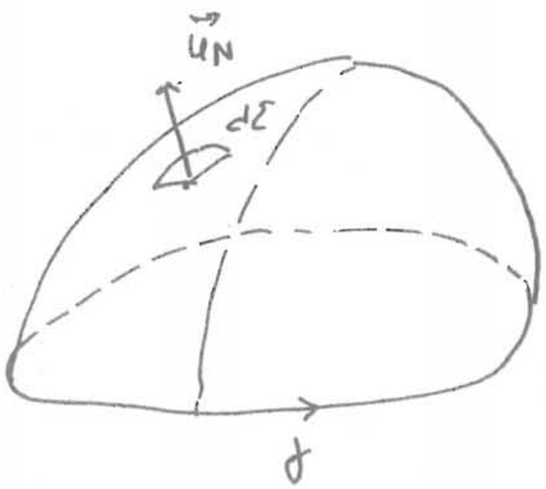
$\vec{F} = \vec{F}(x, y, z)$ = VETTORE FUNZIONE DEL POSTO

$$d\phi = \vec{F} \cdot \vec{u}_n \, d\Sigma$$

FLUSSO ATTRAVERSO $d\Sigma$

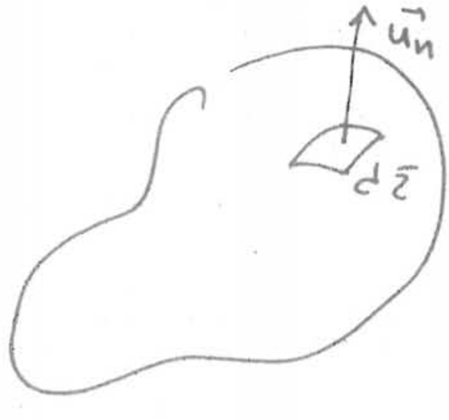


$$\phi_\Sigma = \iint_\Sigma \vec{F} \cdot \vec{u}_n \, d\Sigma$$ = FLUSSO ATTRAVERSO UNA SUPERFICIE FINITA Σ

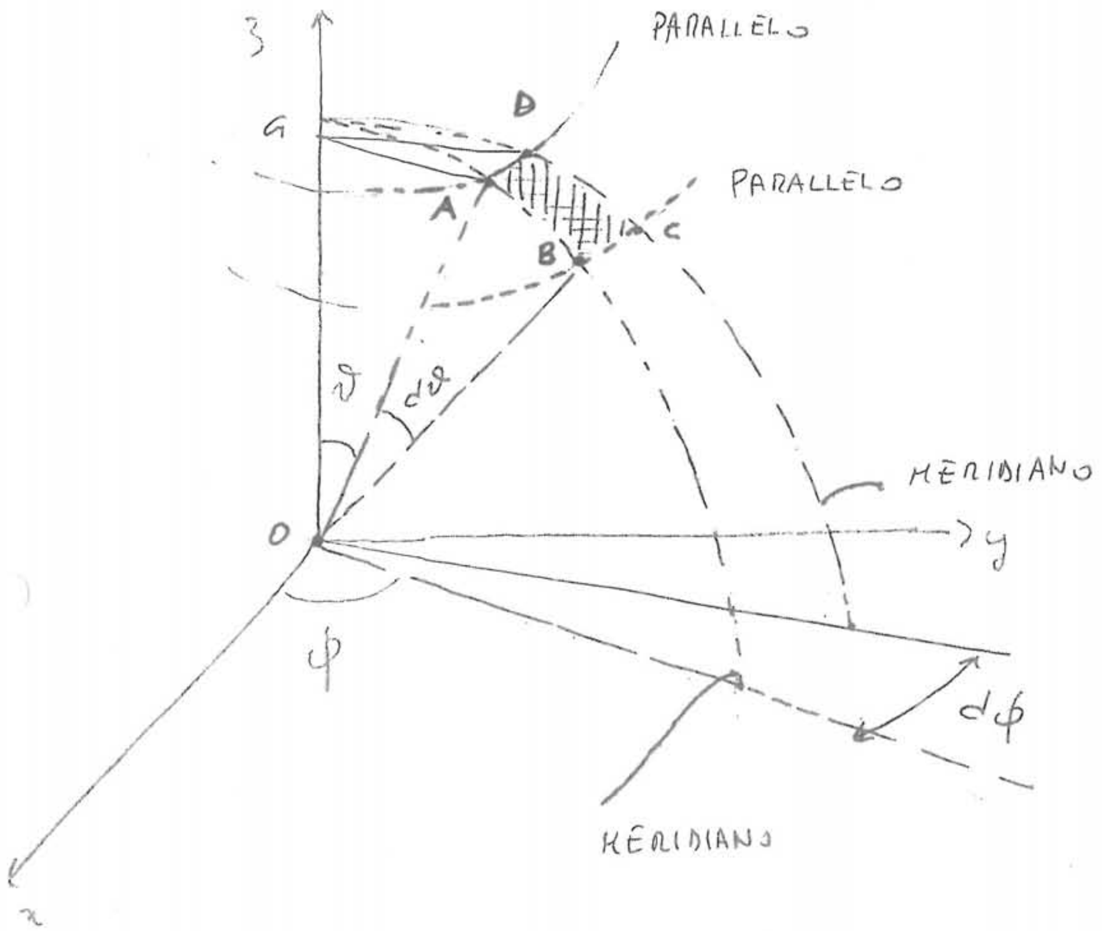


SE LA SUPERFICIE È CHIUSA $\vec{u}_n$ VA VERSO IL FUORI

$$\phi_\Sigma = \oiint_\Sigma \vec{F} \cdot \vec{u}_n \, d\Sigma$$



ANGOLO SOLIDO IN COORDINATE SFERICHE



$$d\Sigma_o = \overline{AB} \times \overline{AD}$$

$$\overline{AB} = r\, d\vartheta$$

$$\overline{AD} = \overline{GA}\, d\phi \qquad \overline{GA} = r \sin\vartheta$$

$$\overline{AD} = r \sin\vartheta\, d\phi$$

$$d\Sigma_o = r^2 \sin\vartheta\, d\vartheta\, d\phi$$

$$d\Omega = \frac{d\Sigma_o}{r^2} = \sin\vartheta\, d\vartheta\, d\phi$$

NON DIPENDE DA r

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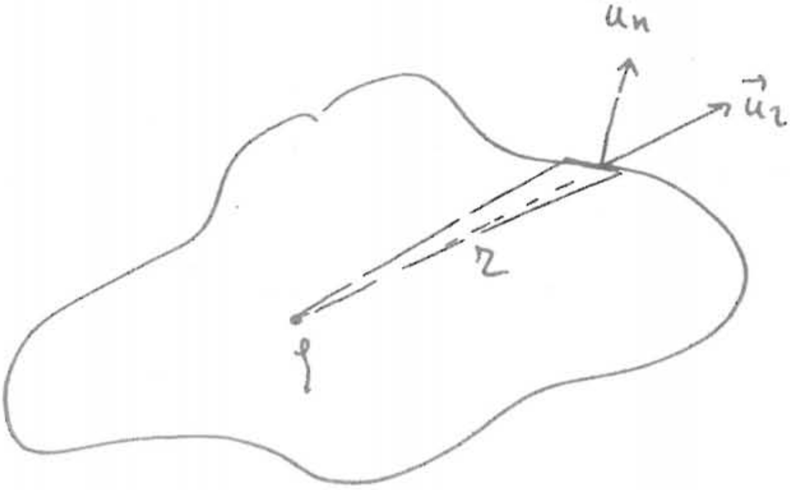
LEGGE DI GAUSS

$$\oiint_{\Sigma} \vec{E} \cdot \vec{u}_n \, d\Sigma = \frac{\left(\sum_i q_i\right)_{int}}{\varepsilon_0}$$

$\left(\sum_i q_i\right)_{int}$ = SOMMA DELLE CARICHE CONTENUTE IN Σ

DIMOSTRAZIONE

q È INTERNA A Σ (q È PUNTIFORME)



$$d\phi = \vec{E} \cdot \vec{u}_n \, d\Sigma$$

$$\vec{E} = k \, \frac{q}{r^2} \, \vec{u}_r$$

$$d\phi = k \, q \, d\Sigma \, \frac{\vec{u}_r \cdot \vec{u}_n}{r^2}$$

$$\frac{\vec{u}_r \cdot \vec{u}_{n1}}{r_1^2} \, d\Sigma_1 = - d\Omega$$

$$\frac{\vec{u}_r \cdot \vec{u}_{n2}}{r_2^2} \, d\Sigma_2 = + d\Omega$$

$$d\phi_1 + d\phi_2 = -kq \, d\Omega + kq \, d\Omega \equiv 0$$

$\phi(\Sigma) = 0$ SE q È ESTERNA A Σ

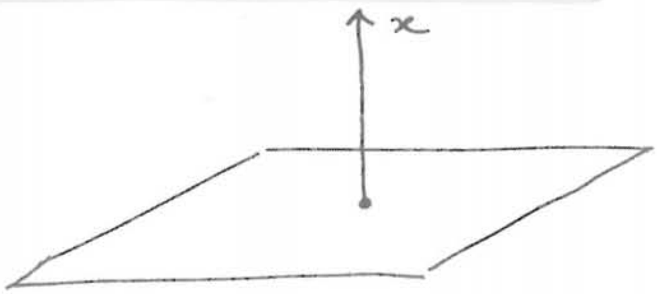
QUINDI PER UNA CARICA PUNTIFORME

$$\phi(\Sigma) = \oint\!\!\oint_{\Sigma} \vec{E} \cdot \vec{u}_n \, d\Sigma = \begin{cases} \dfrac{q}{\varepsilon_0} & \text{INTERNA} \\ 0 & \text{ESTERNA} \end{cases}$$

LEGGE DI GAUSS

ESERCIZIO

CAMPO CREATO DA UN PIANO ILLIMITATO UNIFORME CON DENSITA' σ

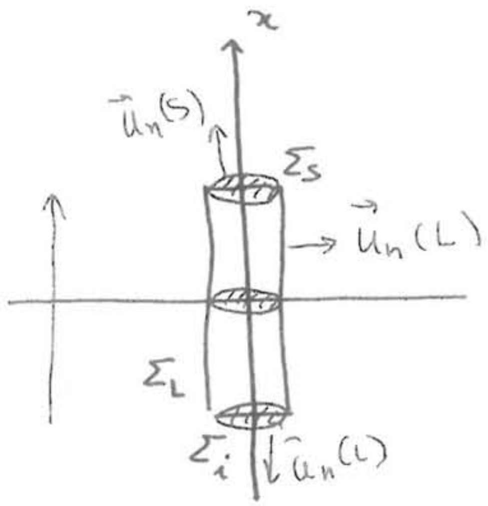


SOLUZIONE

LA SIMMETRIA CI DICE CHE

$$\vec{E} = E(x)\,\vec{i}$$

$$E(x) = -E(-x)$$



$$\phi(\Sigma) = \oint\!\!\!\oint_{\Sigma} \vec{E}\cdot\vec{u}_n\, d\Sigma =$$

$$= \iint_{\Sigma_L} \vec{E}\cdot\vec{u}_n(L)\, d\Sigma + \iint_{\Sigma_S} \vec{E}\cdot\vec{u}_n(S)\, d\Sigma$$

$$+ \iint_{\Sigma_i} \vec{E}\cdot\vec{u}_n(i)\, d\Sigma = \iint_{\Sigma_S} \vec{E}(x)\,\vec{i}\, d\Sigma - \iint_{\Sigma_i} \vec{E}(-x)\,\vec{i}\, d\Sigma =$$

$$= E(x)\,\Sigma_S - E(-x)\,\Sigma_i \qquad \text{MA} \quad \Sigma_i = \Sigma_S = A$$

$$\phi(\Sigma) = [E(x) - E(-x)]\,A = 2E(x)\,A$$

$$\phi(\Sigma) = \frac{\rho_{int}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0} \qquad (\text{GAUSS})$$

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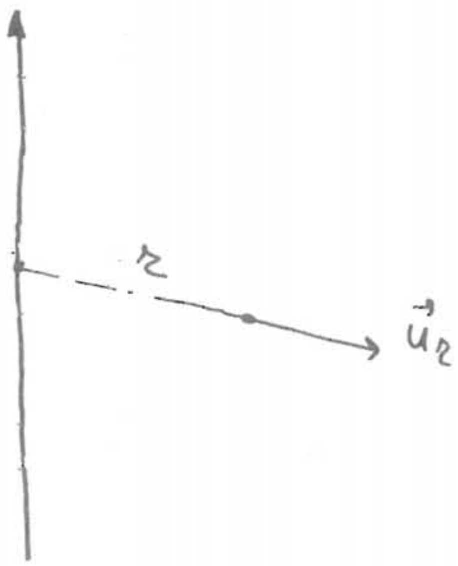
ESERCIZIO

CAMPO CREATO DA UN FILO ILLIMITATO (λ UNIFORME)

SOLUZIONE

LA SIMMETRIA CI DICE CHE

$$\vec{E} = E(r)\,\vec{u}_r$$



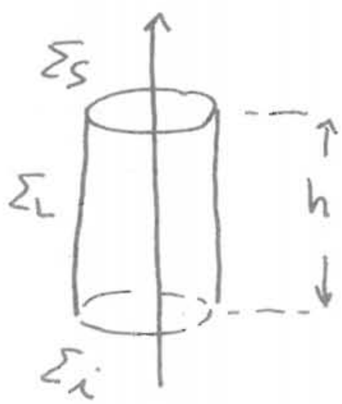
$$\phi(r) = \oiint_{\Sigma} \vec{E}\cdot\vec{u}_n \, d\Sigma =$$

$$= \iint_{\Sigma_i} \vec{E}\cdot\vec{u}_n(i)\, d\Sigma + \iint_{\Sigma_s} \vec{E}\cdot\vec{u}_n(s)\, d\Sigma + \iint_{\Sigma_L} \vec{E}\cdot\vec{u}_n(L)\, d\Sigma$$

$$= \iint_{\Sigma_L} \vec{E}\cdot\vec{u}_n(L)\, d\Sigma = \iint_{\Sigma_L} E(r)\,\vec{u}_r\cdot\vec{u}_r\, d\Sigma =$$

$$= E(r)\iint_{\Sigma_L} d\Sigma = E(r)\,\Sigma_L = 2\pi r h\, E(r)$$

$$\phi(\Sigma) = \frac{Q_{int}}{\varepsilon_0} = \frac{\lambda h}{\varepsilon_0} \qquad \text{GAUSS}$$



$$2\pi r h\, E(r) = \frac{\lambda h}{\varepsilon_0}$$

$$\boxed{E(r) = \frac{\lambda}{2\pi\varepsilon_0 r}}$$