NUMERO : 175

DATA : 03/11/2011

# A P P U N T I

STUDENTE : Stoppa

MATERIA : Elettrotecnica
Prof. Gilli

# Elettrotecnica

prof. Gilli AA 2009/10

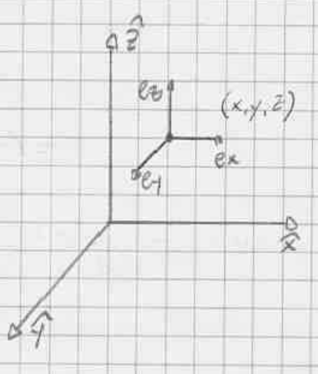
**Indice:**

L'elettromagnetismo classico fa riferimento a due grandezze fondamentali:

- Campo elettrico $\vec{e}(x,y,z,t)$ 4 componenti di cui una è il tempo

$$\vec{e} = e_x\hat{x} + e_y\hat{y} + e_z\hat{z}$$

$\mathbb{R}^4 \longrightarrow \mathbb{R}^3$




l'elettrostatica non dipende dal tempo

- Campo magnetico $\vec{h}(x,y,z,t) = h_x\hat{x} + h_y\hat{y} + h_z\hat{z}$

una funzione che va da $\mathbb{R}^4 \to \mathbb{R}^3$

Maxwell andò a capire che dove c'era un campo elettrico c'era anche un campo magnetico e viceversa
Una grandezza matematica: rotore

Eq. di MAXWELL

1) $\nabla \times \vec{e} = -\dfrac{\partial \vec{b}}{\partial t}$

$\vec{b}$ è l'induzione magnetica (Tesla)
se il sistema è lineare, omogeneo, ...

$\vec{b} = \mu \vec{h}$
$\mu$ permeabilità magnetica

se il campo è statico ovvero non dipende dal tempo il rotore è uguale a $\emptyset$

2) $\nabla \times \vec{h} = \vec{J}_e + \dfrac{\partial \vec{d}}{\partial t}$

$\vec{J}_e$ densità superficiale di corrente elettrica
$\vec{d}$ induzione elettrica $(d = \varepsilon \vec{e})$
quando il sistema è lineare

Legge di Ampere: lega la corrente elettrica al campo magnetico.

In alcuni casi si possono trascurare le dipendenze spaziali lasciando solo il tempo
Immaginiamo di avere un campo elettrico che dipende solo da $t$ e $x$
$\vec{e}(x,t)$

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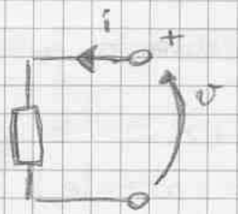
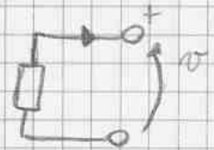
MANCA UN PEZZO 30/09

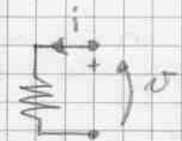



convenzione utilizzatore

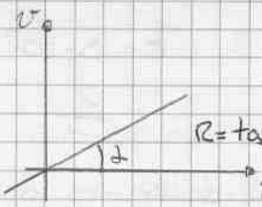



convenzione generatore

## RESISTORE IDEALE




$\boxed{v(t) = R\,i(t)}$ R ⟶ resistenza $[\Omega]$



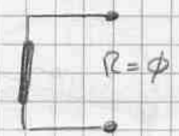

$R = \tan\alpha$

• Se $R \neq \phi$ $\exists\ \frac{1}{R} = G$ ⟶ conduttanza $[\Omega^{-1} = \text{Siemens}]$

• Se $R \neq \phi$ $v(t) = \phi$ $\forall\, i(t)$

⇓

corto circuito ideale




$\boxed{i(t) = G\,v(t)}$

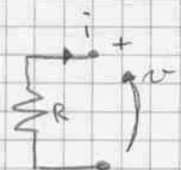
• Se $G = \phi$ $i(t) = \phi$ $\forall\, v(t)$

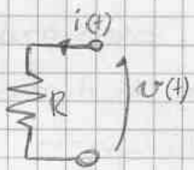
⇓

circuito aperto


Se, invece, uso la convenzione di generatore

$v(t) = -R\,i(t)$

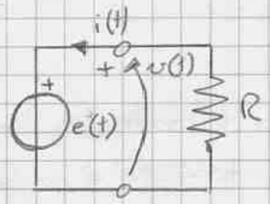



$p(t) = v(t)\,i(t)$ $p(t) = R\,i^2(t)$ — se $R > \phi \Rightarrow p(t) \geq \phi$ $\forall\, i(t)$

La potenza positiva è la potenza entrante

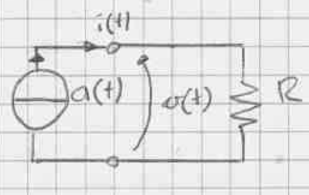
2



$v(t) = -R\,i(t)$

$v(t) = e(t)$ $\Rightarrow$ $\boxed{i(t) = -\frac{e(t)}{R}}$

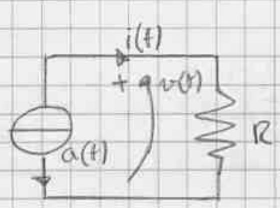
3



$v(t) = R\,i(t)$

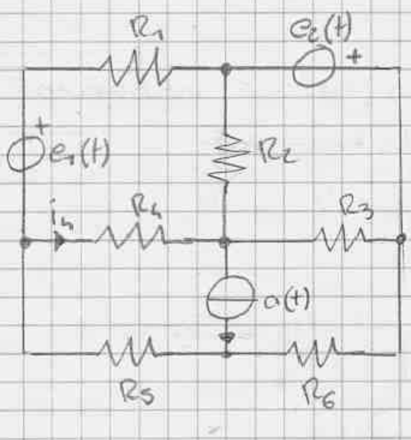
$i(t) = a(t)$ $\Rightarrow$ $\boxed{v(t) = R\,a(t)}$

4



$v(t) = R\,i(t)$

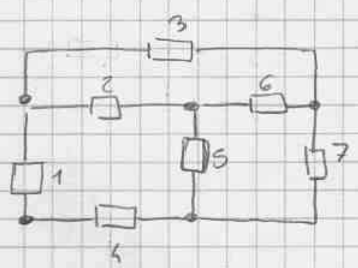
$i(t) = -a(t)$ $\Rightarrow$ $\boxed{V(t) = -R\,a(t)}$



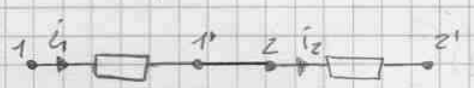
CONNESSIONE SERIE di BIPOLI

1- hanno un terminale in comune

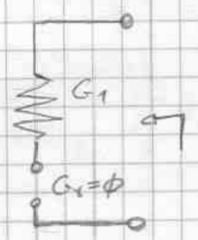
2- In corrispondenza del terminale comune non è connesso nessun altro terminale della rete (eccetto al più C.A.)



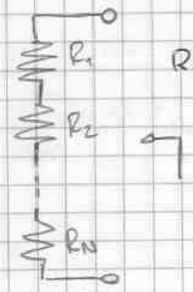
1 e 4 in serie



$i_1 = i_2$ conseguenza

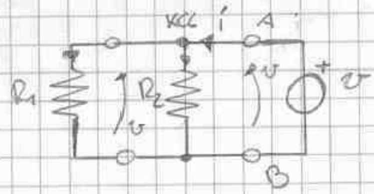



$$G_{eq} = \frac{G_1 G_2}{G_1+G_2} = \phi \Rightarrow$$




$$R_{eq} = R_1 + R_2 + \ldots + R_N$$

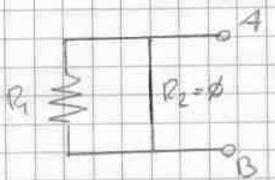
CONNESSIONE in PARALLELO di RESISTORI




$$i = i_1 + i_2 \qquad v = R_1 i_1$$
$$v = R_2 i_2$$

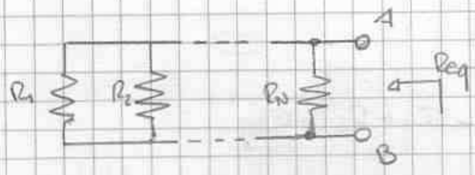
$$i = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \Rightarrow R_{eq} = \frac{v}{i} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}} =$$
$$= \frac{R_1 R_2}{R_1 + R_2}$$

$$G_{eq} = \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = G_1 + G_2$$




$$R_{eq} = \frac{R_1 R_2}{R_1+R_2} = \frac{R_1 \phi}{R_1+\phi} = \phi \Rightarrow$$


$$G_{eq} = G_1 + \phi = G_1$$



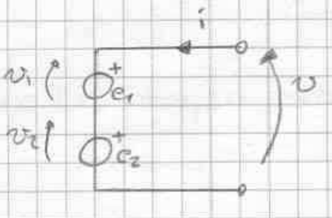

$$G_{eq} = G_1 + G_2 + \ldots + G_N$$

$$R_{eq} = \frac{1}{G_{eq}} = \frac{1}{G_1+G_2+\ldots+G_N} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\ldots+\frac{1}{R_N}}$$

sdo se le resistenze sono 2

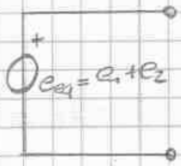
$$\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}} = \frac{R_1 R_2}{R_1+R_2}$$ vale qst seconda formula

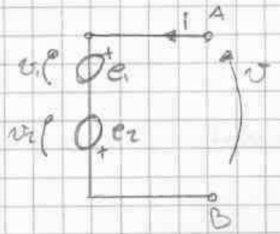
## CONNESSIONE SERIE di GENERATORI IDEALI di TENSIONE



$v_1 = e_1 \quad \forall i$

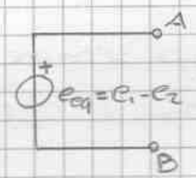
$v_2 = e_2 \quad \forall i \qquad v = v_1 + v_2 = e_1 + e_2 \quad \forall i$

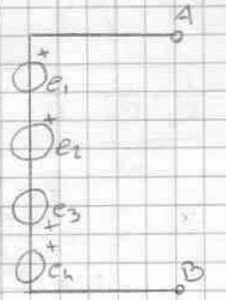


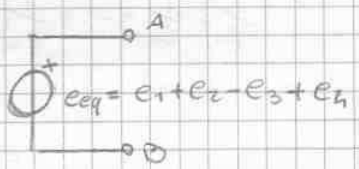



$v_1 = e_1 \quad \forall i$

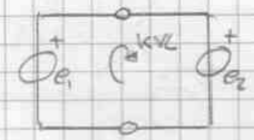
$v_2 = -e_2 \qquad v = v_1 + v_2 = e_1 - e_2 \quad \forall i$






La connessione in parallelo di generatori di tensione non ha senso



$e_1 - e_2 = 0$

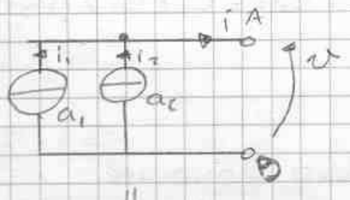
$e_1 = e_2$

La connessione in serie di generatori ideali di corrente non ha senso

$a_1 - a_2 = 0$

$a_1 = a_2$

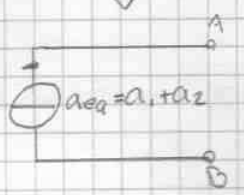
## COLLEGAMENTO PARALLELO di GENERATORI di CORRENTE

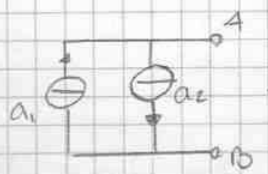


$i_1 = a_1 \quad i_2 = a_2 \quad i = i_1 + i_2 = a_1 + a_2$

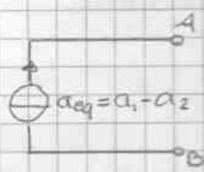
$\forall v \quad \forall v$

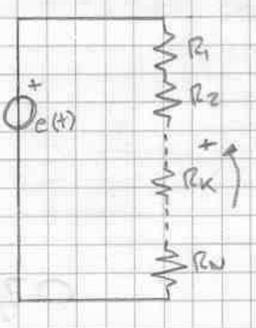
$\Downarrow$



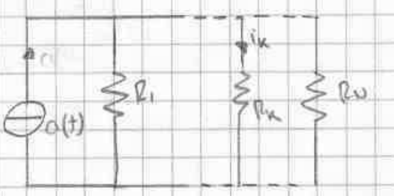



$\Rightarrow$



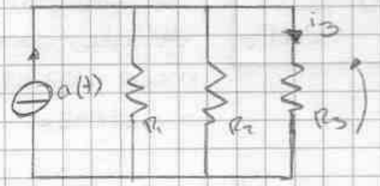



$$V_K = e(t)\,\frac{R_K}{R_1+R_2+\dots+R_K+\dots+R_N}$$




$$i_K = a(t)\,\frac{G_K}{G_1+G_2+\dots+G_N} =$$

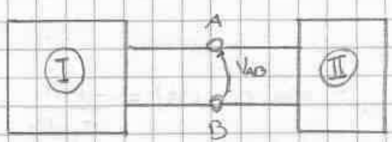
$$= a(t)\,\frac{R_1 \| R_2 \dots \| R_{K-1} \| R_{K+1} \| \dots \| R_N}{R_K + R_1 \| R_2 \dots \| R_{K-1} \| R_{K+1} \| \dots \| R_N}$$



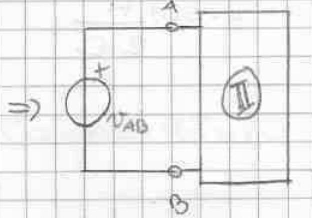

$$i_3 = a(t)\,\frac{R_1 \| R_2}{R_1 \| R_2 + R_3}$$

## TEOREMA di SOSTITUZIONE

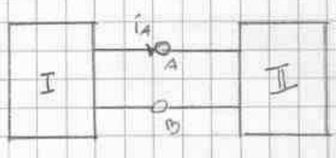
I



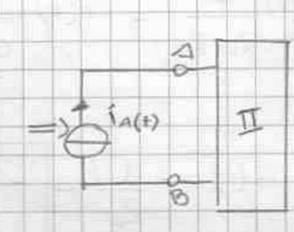

Dati 2 bipoli connessi in parallelo tra 2 terminali A-B e nota la tensione $V_{AB}$, ciascuno dei 2 bipoli si può studiare sostituendo l'altro bipolo con un generatore ideale di tensione pari a $V_{AB}$




II




Dati 2 bipoli connessi in ∥ tra 2 terminali A-B e nota la corrente nel terminale A ($i_A$), ciascuno dei 2 bipoli si può studiare sostituendo l'altro bipolo con un generatore ideale di corrente $i_A(t)$

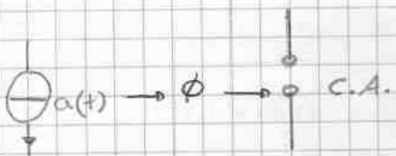


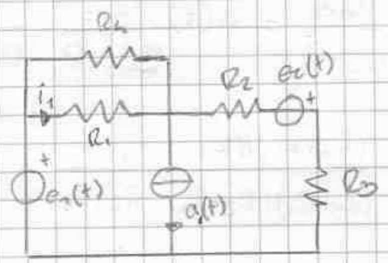
dove i coefficienti $\alpha_i$ $(1 \leq i \leq N)$ $\beta_j$ $(1 \leq j \leq M)$

sono costanti reali, che dipendono soltanto dai resistori presenti nel circuito e si calcolano come segue

$$\alpha_i = \frac{y(t)}{e_i(t)} \Bigg|_{\substack{e_k(t) = \phi \quad \forall k \neq i \\ a_e(t) = \phi \quad \forall e \neq j}} \qquad \beta_j = \frac{y(t)}{a_j(t)} \Bigg|_{\substack{e_k(t) = \phi \quad \forall k \\ a_e(t) \neq \phi \quad \forall e \neq j}}$$

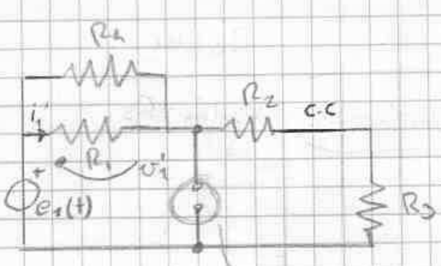
ANNULLAMENTO di un GENERATORE


$$i_1(t) = \overbrace{\alpha_1 e_1(t)}^{i^{I}} + \overbrace{\alpha_2 e_2(t)}^{i^{II}} + \overbrace{\beta_1 a_1(t)}^{i^{III}}$$

$$\alpha_1 = \frac{i_1(t)}{e_1(t)} \Bigg|_{\substack{e_2(t) = \phi \\ a_1(t) = \phi}} = \frac{i_1'}{e_1}$$



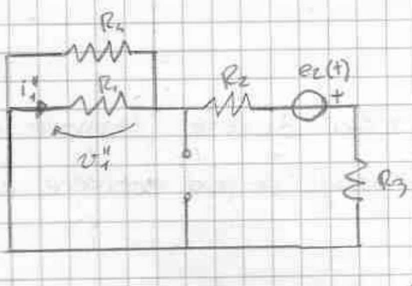

posso eliminarlo perché in || con una resistenza

$$i_1' = i_1 \Big|_{\substack{e_2 = \phi \\ a_1 = \phi}}$$

$$i_1' = \frac{v_1'}{R_1} = \underbrace{e_1 \frac{R_1 \| R_4}{R_1 \| R_4 + R_2 + R_3}}_{} \cdot \frac{1}{R_1}$$

$$\alpha_1 = \frac{i_1'}{e_1} = \frac{R_1 \| R_4}{R_1 \| R_4 + R_2 + R_3} \cdot \frac{1}{R_1}$$

$$\alpha_2 = \frac{i_1}{e_2} \Bigg|_{\substack{e_1 = \phi \\ a_1 = \phi}} = \frac{i_1''}{e_2}$$

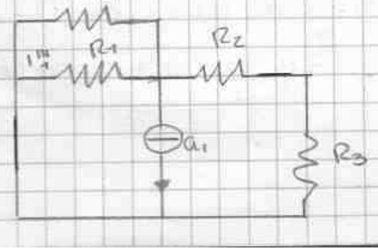



$$i_1'' = i_1 \Big|_{\substack{e_1 = \phi \\ a_1 = \phi}}$$

$$i_1'' = \frac{v_1''}{R_1} = \frac{1}{R_1} e_2(t) \frac{R_1 \| R_4}{R_1 \| R_4 + R_2 + R_3}$$

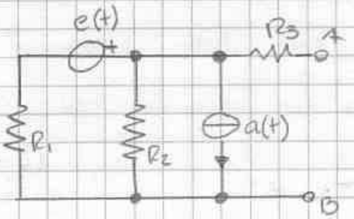
$$\alpha_2 = \frac{i_1''}{e_2} = \frac{R_1 \| R_4}{R_1 \| R_4 + R_2 + R_3} \cdot \frac{1}{R_1}$$

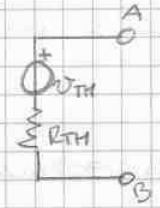
$$\beta_1 = \frac{i_1(t)}{a_1(t)} \Bigg|_{\substack{e_1 = \phi \\ e_2 = \phi}} = \frac{i_1'''}{a_1}$$




$$i_1''' = i_1 \Big|_{\substack{e_1 = 0 \\ e_2 = \phi}} = a_1 \frac{(R_2 + R_3) \| R_4}{(R_2 + R_3) \| R_4 + R_1}$$

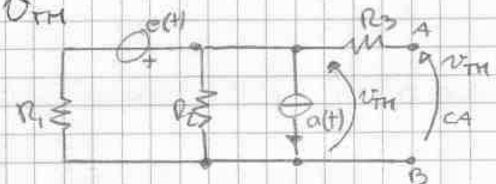
# Teorema di Thevenin





$R_{TH} = R_1 \parallel R_2 + R_3$

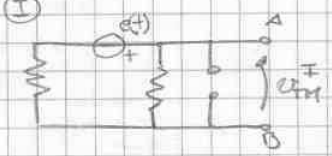
$v_{TH}$



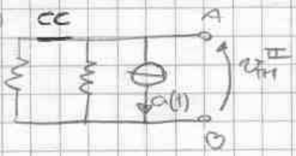
il circuito è aperto e posso eliminare $R_3$

$$v_{TH}^{I} = e(t) \underbrace{\frac{R_2}{R_1+R_2}}_{\text{partitore di tensione}}$$

I

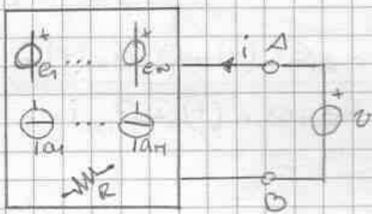


II



$$v_{TH}^{II} = -a(t)\, R_1 \parallel R_2$$

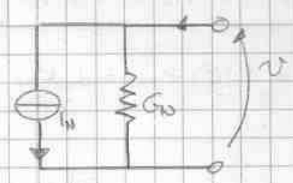
$$v_{TH} = e(t) \frac{R_2}{R_1+R_2} - a(t)\, R_1 \parallel R_2$$



$$i = \overbrace{\alpha_1' e_1 + \ldots + \alpha_N' e_N + \beta_1' a_1 + \ldots + \beta_M' a_M}^{i_N} + G_N v$$

$N \triangleq$ Norton

$$i_N = \alpha_1' e_1 + \ldots + \alpha_N' e_N + \beta_1' a_1 + \ldots + \beta_M' a_M$$

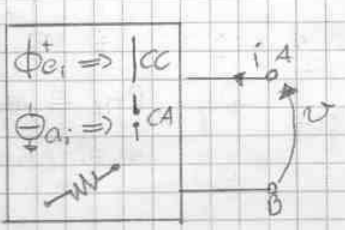


$$\boxed{i = i_N + G_N v}$$

Rappresentazione (bipolo, circuito) equivalente di NORTON

Rappresentazione (bipolo, circuito) equivalente parallelo

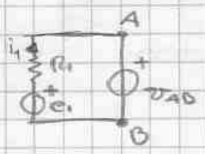
$$G_N = \frac{i}{v} \bigg|_{\substack{e_1 = \ldots = e_N = 0 \\ a_1 = \ldots = a_M = 0}}$$



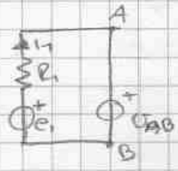
$$\frac{v}{i} = R_{TH} \qquad \frac{i}{v} = G_N \triangleq \frac{1}{R_{TH}}$$

la conduttanza equivalente di Norton

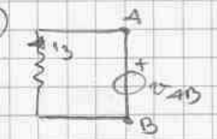
Si assume che $V_{AB}$ sia nota

I



$i_1 = \frac{e_1 - V_{AB}}{R_1}$

II



$i_1 = \frac{e_1 - V_{AB}}{R_1}$

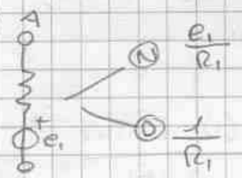
III


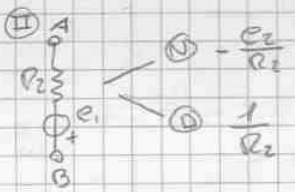

$i_3 = -\frac{V_{AB}}{R_3}$

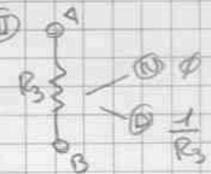
IV $i_4 = a_1$

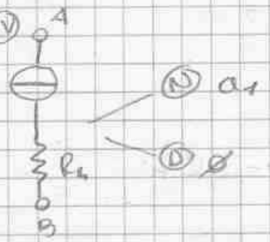
V $i_5 = -a_2$

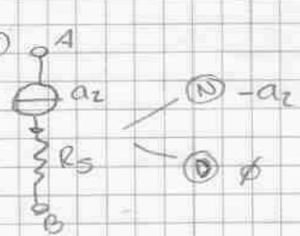
$$\frac{e_1 - V_{AB}}{R_1} + \frac{-e_2 - V_{AB}}{R_2} - \frac{V_{AB}}{R_3} + a_1 - a_2 \Rightarrow V_{AB} = \frac{\frac{e_1}{R_1} - \frac{e_2}{R_2} + a_1 - a_2}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

I



II



III



IV



V



Calcolo riferito al 1° disegno

$$V_{AB} = \frac{-\frac{a_2}{R_1} + a_1 + a_2 - \frac{e_2}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \qquad i_3 = \frac{-e_2 - V_{AB}}{R_3}$$

ALTRI TIPI di BIPOLI: GENERATORI PILOTATI (controllati, indipendenti)

- Generatore di tensione pilotato in tensione

$\hat{e}(t) = \alpha v_p(t)$ $\alpha \in \mathbb{R}$ la tensione dipende da un coeff. $\alpha$ e dalla tensione entrante da un'altra parte

$V_p(t) \triangleq$ tensione pilota

$\hat{e}(t) = r_m i_p$ è noto

(4) teorema di sostituzione (reale) perché $\hat{e}(t)$ è noto e calcolo $i_3$

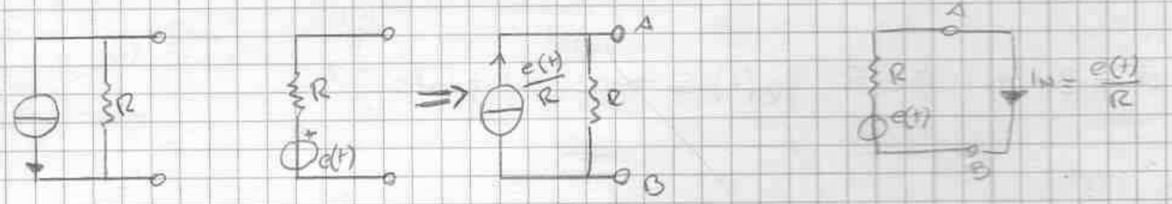
$$i_3 = \frac{e(t)}{R_1 + R_2 \| R_3} \frac{R_2}{R_2 + R_3} - a(t) \frac{R_1 \| R_2}{R_1 \| R_2 + R_3} + \frac{r_m i_p}{R_1 \| R_2 + R_3}$$

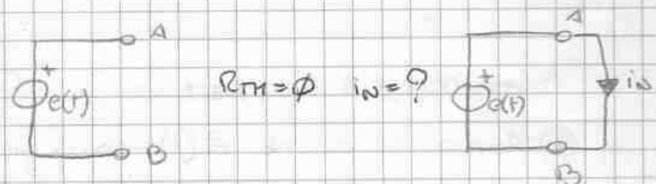
## METODO dei NODI

28-10

ES

Rete elettrica composta di lati che ammettono una rappresentazione di tipo Norton e non contenente generatori pilotati.



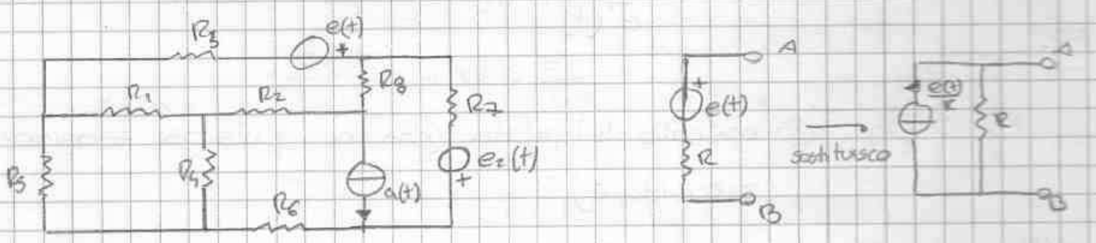


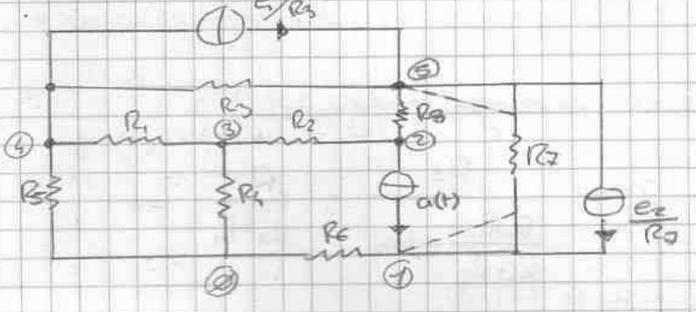
KVL $e(t) = \emptyset$ qst è una contraddizione

⇓

Non esiste una rappresentazione di tipo Norton

In conclusione non ci devono essere lati con soli gen. di tensione





Nodo: punto di afferenza di 2 o più terminali

Nodi effettivi: punto di afferenza di almeno 3 terminali, con la convenzione che 2 nodi connessi da c.c. rappresentano un solo nodo.

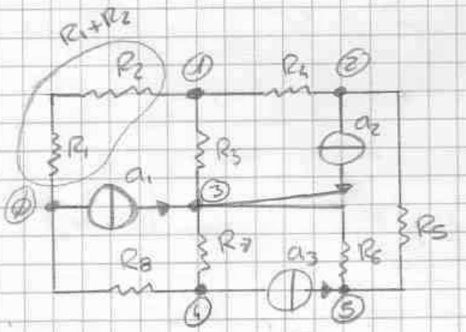
conduttanze posta tra il nodo K e il nodo $\ell$

$\underline{\underline{G_n}}(K,K)$ = somma di tutte le conduttanze che afferiscono al nodo K

$\underline{A}(K)$ = somma di tutti i generatori di corrente che afferiscono al nodo K, assumendo con segno + quelli entranti, con segno - quelli uscenti.

La matrice è invertibile purché ci sia almeno una delle disuguaglianze valida con lo strettamente maggiore

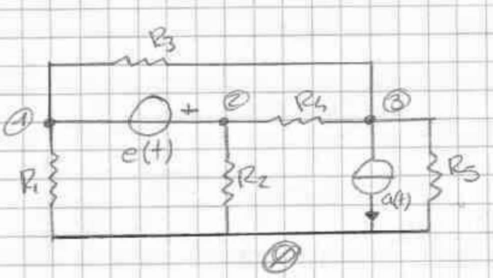
29-10




$$\begin{pmatrix} G_1+G_3+G_4 & -G_4 & -G_3 & \varnothing & \varnothing \\ -G_4 & G_4+G_5 & \varnothing & \varnothing & -G_5 \\ -G_3 & \varnothing & G_3+G_6+G_7 & -G_7 & -G_6 \\ \varnothing & \varnothing & -G_7 & G_7+G_8 & \varnothing \\ \varnothing & -G_5 & -G_6 & \varnothing & G_5+G_6 \end{pmatrix}$$

$$G_{12} = \frac{1}{R_1+R_2}$$

$$\begin{pmatrix} \hat{V}_1 \\ \hat{V}_2 \\ \hat{V}_3 \\ \hat{V}_4 \\ \hat{V}_5 \end{pmatrix} = \begin{pmatrix} \varnothing \\ -a_2 \\ a_1+a_2 \\ -a_3 \\ a_3 \end{pmatrix}$$

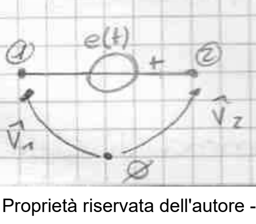



mediante il principio di sostituzione

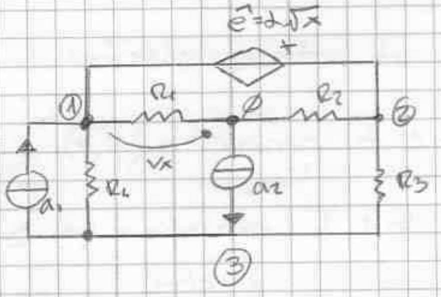


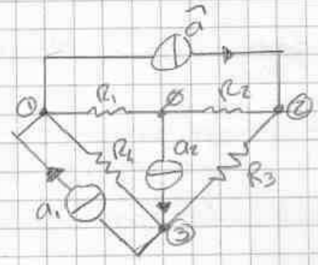
$$\begin{pmatrix} G_1+G_3 & 0 & -G_3 \\ 0 & G_2+G_4 & -G_4 \\ 0 & 0 & G_3+G_4+G_5 \end{pmatrix} \begin{pmatrix} \hat{V}_1 \\ \hat{V}_2 \\ \hat{V}_3 \end{pmatrix} = \begin{pmatrix} -\hat{a} \\ \hat{a} \\ -a(t) \end{pmatrix}$$

in qst caso ho 4 incognite



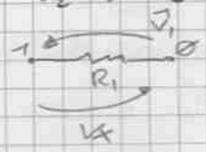

$$e(t) = \hat{V}_2 - \hat{V}_1$$




$$\begin{pmatrix} G_1+G_4 & \emptyset & -G_4 \\ \emptyset & G_2+G_3 & -G_3 \\ -G_4 & -G_3 & G_3+G_4 \end{pmatrix} \begin{pmatrix} \hat{V}_1 \\ \hat{V}_2 \\ \hat{V}_3 \end{pmatrix} = \begin{pmatrix} a_1-\hat{a} \\ \hat{a} \\ a_2-a_1 \end{pmatrix}$$

$\hat{V}_2 - \hat{V}_1 = \vec{e} = \alpha V_x$



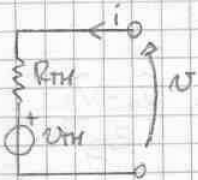

$V_x = -\hat{V}_1$

$\hat{V}_2 - \hat{V}_1 = -\alpha \hat{V}_1$

$\hat{V}_2 + (\alpha - 1)\hat{V}_1 = \emptyset$

04-11

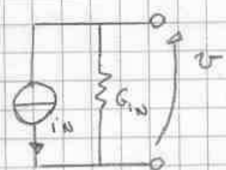

$$\boxed{M v(t) + N i(t) + s(t) = \emptyset}$$




$v = v_{TH} + R_{TH}\, i$

$v - R_{TH}\, i(t) - v_{TH}(t) = \emptyset \qquad M = 1 \quad N = -R_{TH}$

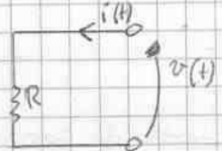
$s(t) = -v_{TH}$




$i(t) = G_N v(t) + i_N(t)$

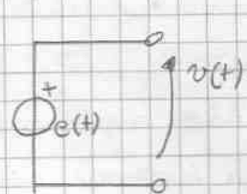
$G_N v(t) - i(t) + i_N(t) = \emptyset \qquad M = G_N \quad s(t) = i_N(t)$

$N = -1$




$v(t) = R\, i(t)$

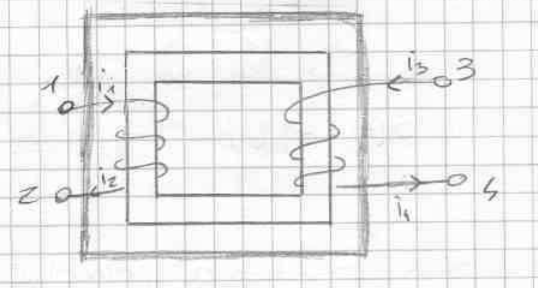
$v(t) - R\, i(t) = \emptyset \quad M = 1 \quad N = -R \quad s(t) = \emptyset$




$v(t) = e(t) \qquad \forall i(t)$

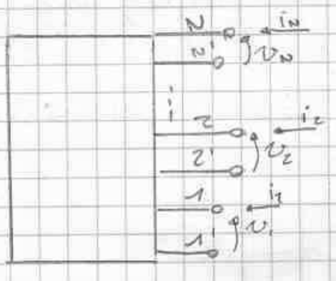
$v(t) - e(t) = \emptyset \quad M = 1 \quad N = \emptyset \quad s(t) = -e(t)$

# TRASFORMATORE




Un multipolo in cui i terminale può essere considerato a coppie è detto MULTIPORTA (per ragioni costruttive). Nelle coppie ~~di~~ le correnti entranti in un terminale è uguale a gli uscenti, e ciascuna coppia di terminali si chiama PORTA

Multipolo con 2N terminali



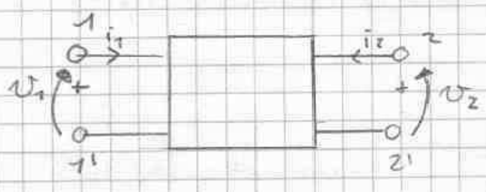

le correnti indipendenti in un multiporta son N. Condizione di utilizzatore

$$\underline{\underline{M}}\,\overline{v} + \underline{\underline{N}}\,\overline{i} + \overline{s}(t) = \emptyset$$

$$\overline{v}, \overline{i} \in \mathbb{R}^N \qquad \overline{s}(t) : \mathbb{R} \to \mathbb{R}^N$$

$$\underline{\underline{M}}, \underline{\underline{N}} \in \mathbb{R}^{N,N}$$

## Doppio BIPOLO (4 terminali, 2 porte)




$$\underline{\underline{M}}\,\underline{v} + \underline{\underline{N}}\,\underline{i} + \underline{s}(t) = \emptyset$$

$$\overline{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \qquad \overline{i} = \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} \qquad \underline{\underline{M}}, \underline{\underline{N}} \in \mathbb{R}^{N,N}$$

$$s(t) \quad \mathbb{R} \to \mathbb{R}^2$$

Se $\overline{s}(t) = \emptyset$ il doppio bipolo è lineare (inerte

4 grandezze elettriche $v_1, v_2, i_1, i_2$

Le possibilità di raggruppare le grandezze

$$\binom{4}{2} = 6$$

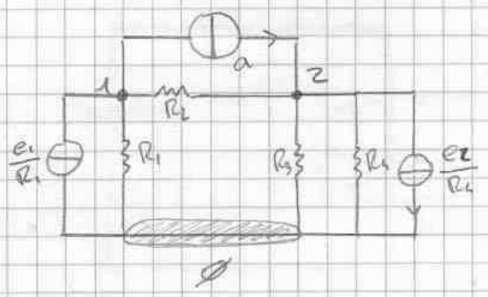
I) Rappresentazione attraverso matrice delle resistenze $\underline{\underline{R}}$

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}$$

$$\overline{v} = \underline{\underline{R}} \quad \overline{i}$$

II) Rappresentazione attraverso la matrice delle conduttanze

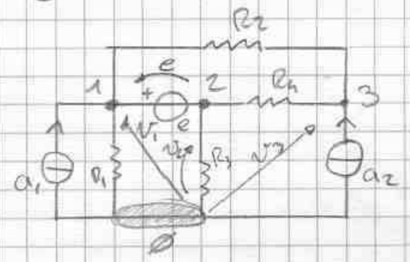
$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \qquad \overline{i} = \underline{\underline{G}}\,\overline{v}$$



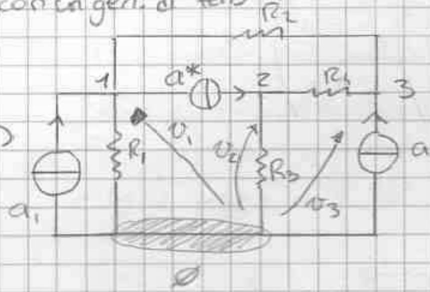

in qst modo nel circuito i bipoli sono di tipo Norton

$$\begin{pmatrix} G_1+G_2 & -G_2 \\ -G_2 & G_2+G_3+G_4 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} \frac{e_1}{R_1} - a \\ a - \frac{e_2}{R_4} \end{pmatrix}$$

② non abbiamo R in serie con un gen. di tensione




$\Rightarrow$




incognite $\begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ a^* \end{pmatrix}$

$v_1 - v_2 = e$

$$\begin{pmatrix} G_1+G_2 & \emptyset & -G_2 \\ \emptyset & G_3+G_4 & -G_4 \\ -G_2 & -G_4 & G_2+G_4 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} a_1 - a^* \\ a^* \\ a_2 \end{pmatrix}$$

3 equazioni nelle incognite $(v_1, v_2, v_3, a^*)^T$

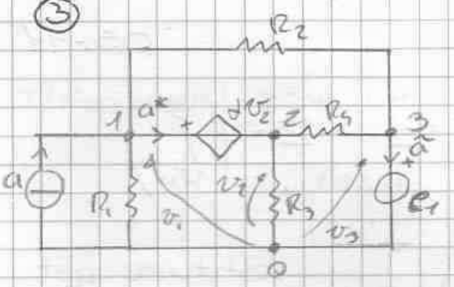
coeff. di $a^*$

$$v_1 - v_2 = e \rightarrow \left(\begin{array}{ccc|c} G_1+G_2 & 0 & -G_2 & +1 \\ 0 & G_3+G_4 & -G_4 & -1 \\ G_2 & -G_4 & G_2+G_4 & 0 \\ \hline 1 & -1 & 0 & 0 \end{array}\right) \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ \hline a^* \end{pmatrix} = \begin{pmatrix} a_1 \\ 0 \\ a_2 \\ \hline e \end{pmatrix}$$

$(G_1+G_2)v_1 + 0v_2 + G_2 v_3 = a_1 - a^*$

$(G_1+G_2)v_1 + 0v_2 - G_2 v_3 + a^* = a_1$

③



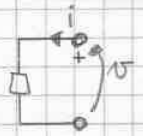

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \rightarrow \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ \hline a^* \\ \tilde{a} \end{pmatrix}$$

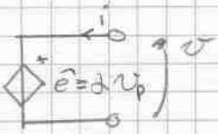
$$\begin{pmatrix} G_1+G_2 & 0 & -G_2 \\ 0 & G_3+G_4 & -G_4 \\ -G_2 & -G_4 & G_4+G_2 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} a - a^* \\ a^* \\ \tilde{a} \end{pmatrix}$$

3 equazioni nelle incognite $(v_1, v_2, v_3, a^*, \tilde{a})^T$

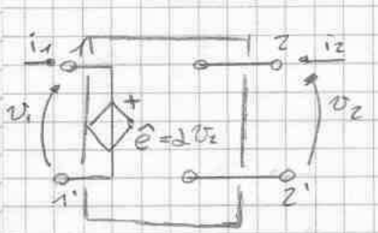
$$\begin{pmatrix} G_1+G_2+G_3 & -G_3 \\ -(G_3+\alpha G_4) & G_3+G_4 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} \frac{e}{R_1} - a \\ a - \alpha G_N e \end{pmatrix}$$

Continuazione dei doppi bipoli 11-11





non è un bipolo perché non ho nessuna informazione riguardo a $v_p$



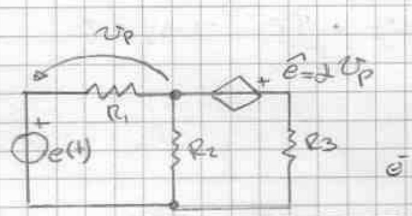
quindi si che è un bipolo doppio

le relazioni costitutive sono $\begin{cases} v_1 = \alpha v_2 \\ i_2 = \emptyset \end{cases}$

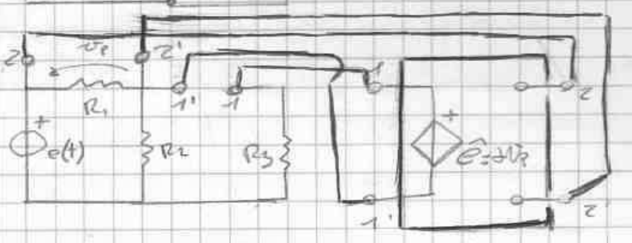
non ammette una rappresentazione in termini di resistenze e conduttanze ma posso usare una matrice ibrida di tensione pilotato in tensione

$$\begin{pmatrix} v_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} 0 & \alpha \\ 0 & 0 \end{pmatrix} \begin{pmatrix} i_1 \\ v_2 \end{pmatrix}$$ ammette anche $\underline{\underline{I}}'$

$\underline{\underline{H}}$

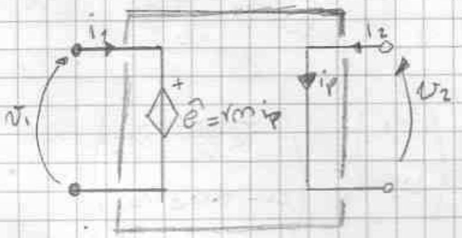


è un doppio bipolo secondo una matrice ibrida



per vedere che il circuito pilotato è un doppio bipolo

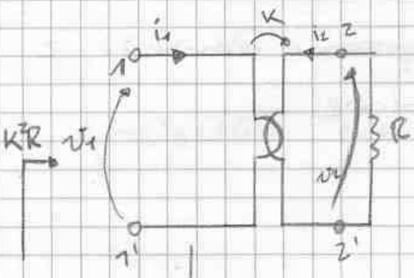
doppio bipolo xk 4 terminali



$$\begin{cases} v_1 = \hat{e} = r_m i_p \\ i_2 = i_p \\ v_2 = \emptyset \end{cases} \Rightarrow \begin{cases} v_1 = r_m i_2 \\ v_2 = 0 \end{cases}$$

generatore di tensione pilotato in corrente

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 & r_m \\ 0 & 0 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}$$

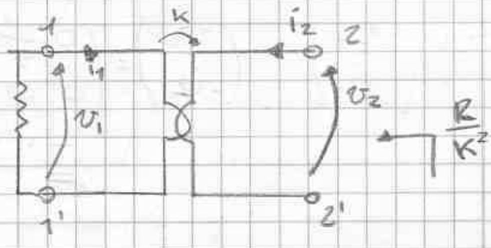



$$\begin{cases} v_1 = K v_2 \\ i_1 = -\frac{1}{K} i_2 \end{cases}$$

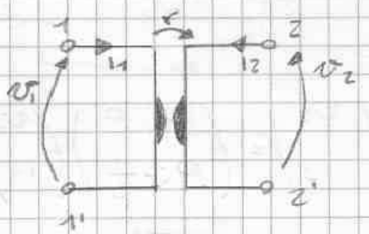
$$v_2 = -R i_2$$

$$v_1 = K v_2 = K(-R i_2) =$$
$$= K(-R)(-K i_1) = K^2 R i_1$$

qui vedo un valore della R maggiore




GIRATORE IDEALE




$$\begin{cases} v_1 = r i_2 \\ v_2 = -r i_1 \end{cases}$$

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 & r \\ -r & 0 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}$$

$\underline{\underline{R}}$

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{r} \\ \frac{1}{r} & 0 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

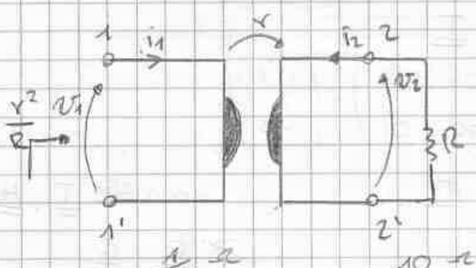
$\underline{\underline{G}}$

$$p(t) = v_1 i_1 + v_2 i_2 =$$
$$= v_1\left(-\frac{v_2}{r}\right) + v_2\left(\frac{v_1}{r}\right) = 0$$

potenza assorbita = ceduta

$$\begin{pmatrix} v_1 \\ i_1 \end{pmatrix} = \begin{pmatrix} 0 & -r \\ -\frac{1}{R} & 0 \end{pmatrix} \begin{pmatrix} v_2 \\ -i_2 \end{pmatrix}$$

$\underline{\underline{T}} \Rightarrow \exists\, \underline{\underline{T'}}$




$$\begin{cases} v_1 = r i_2 \\ v_2 = -r i_1 \end{cases} \qquad v_2 = -R i_2$$

$$-R i_2 = -r i_1 \Rightarrow i_2 = \frac{r}{R} i_1$$

$\frac{1}{10}\ \Omega$ 10 $\Omega$

$\frac{1}{50}\ \Omega$ 50 $\Omega$

ecc. ecc.

$$v_1 = r\,\frac{r i_1}{R} = \frac{r^2}{R} i_1 \qquad \frac{v_1}{i_1} = \frac{r^2}{R} = r^2 G$$

$$\text{se } r = 1\,\Omega^2 \rightarrow \frac{v_1}{i_1} = 1 \cdot G$$

"gira" le resistenze con le conduttanze

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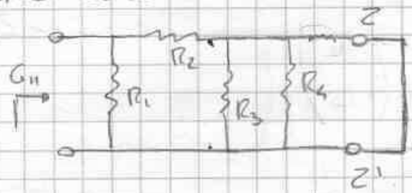
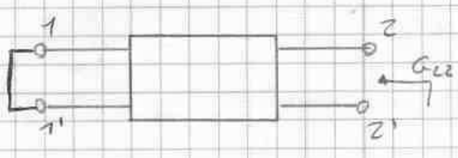
## MATRICE delle ~~RESISTENZE~~ CONDUTTANZE




$$G_{11} = \frac{1}{R_1 \| R_2} = G_1 + G_2$$

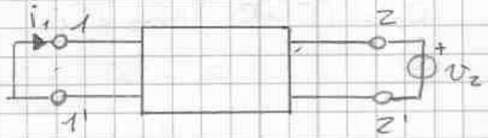
$$G_{22} = \left.\frac{i_2}{v_2}\right|_{v_1 = \emptyset}$$




esempio contrario a sopra

$$G_{22} = \frac{1}{R_2 \| R_3 \| R_4} = G_2 + G_3 + G_4$$

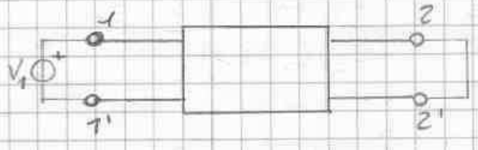
$$G_{12} = \left.\frac{i_1}{v_2}\right|_{v_1 = \emptyset}$$




$$i_1 = \frac{-V_2}{R_2}$$

$$G_{12} = -\frac{1}{R_2} = -G_2$$

$$G_{21} = \left.\frac{i_2}{v_1}\right|_{v_2 = \emptyset}$$




$$i_2 = -\frac{V_1}{R_2} \qquad G_{21} = -\frac{1}{R_2} = -G_2$$

## MATRICE IBRIDA

$$\begin{cases} v_1 = R_{11} i_1 + R_{12} i_2 \\ v_2 = R_{21} i_1 + R_{22} i_2 \end{cases} \qquad \begin{pmatrix} v_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \begin{pmatrix} i_1 \\ v_2 \end{pmatrix}$$

$$i_2 = \frac{v_2 - R_{21} i_1}{R_{22}}$$

$$v_1 = R_{11} i_1 + R_{12} \left( \frac{v_2 - R_{21} i_1}{R_{22}} \right)$$

$$v_1 = \underbrace{\left( R_{11} - \frac{R_{12} R_{21}}{R_{22}} \right)}_{H_{11}} i_1 + \underbrace{\frac{R_{12}}{R_{22}}}_{H_{12}} v_2$$

$$i_2 = \underbrace{-\frac{R_{21}}{R_{22}}}_{H_{21}} i_1 + \underbrace{\frac{1}{R_{22}}}_{H_{22}} v_2$$

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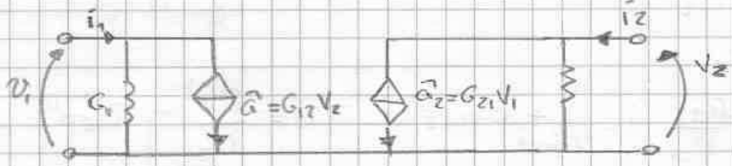
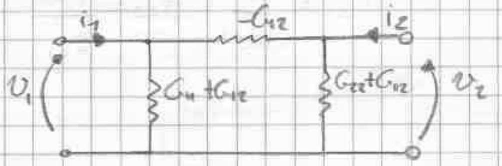
$$\begin{cases} i_1 = G_{11} v_1 + G_{12} v_2 \\ i_2 = G_{21} v_1 + G_{22} v_2 \end{cases}$$

$G_{12} \neq G_{21}$




gen di corrente pilotato in tensione

Se $G_{12} = G_{21}$




Commento sull'esercizio

18-11


$\underline{\underline{G}} = ?$

$\underline{i} = \underline{\underline{G}}\, \underline{v}$ oppure $\underline{v} = \underline{\underline{R}}\, \underline{i}$ $\quad \underline{\underline{G}} = \underline{\underline{R}}^{-1}$

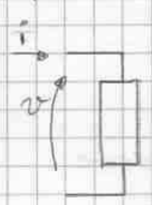
$i = i_1 \quad i^* = i_2 + \beta i_1 \quad \hat{i} = i + i^* - \beta i = i_1 + i_2$

$$v_1 = R_1 i + R_2 \hat{i} = \overbrace{\left[ R_1 + R_2 \right]}^{R_{11}} i_1 + \underbrace{R_2}_{R_{12}} i_2$$

$$v_2 = R_3 i^* + R_2 \hat{i} = \underbrace{\left[ R_3 \beta + R_2 \right]}_{R_{21}} i_1 + \underbrace{\left[ R_3 + R_2 \right]}_{R_{22}} i_2$$

$$\underline{\underline{G}} = \underline{\underline{R}}^{-1} \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} = \frac{1}{\det \underline{\underline{R}}} \begin{pmatrix} R_{22} & -R_{12} \\ -R_{21} & R_{11} \end{pmatrix}$$

## BIPOLI DINAMICI




$\frac{d}{dt} \underline{x} = F(\underline{x}, v, i, t)$ eq. di stato, differenziale

$0 = G(\underline{x}, v, i, t) \qquad \underline{x} \in \mathbb{R}^n$

$\downarrow$

come $v$ e $i$ sono legati allo stato

$v(t_0) \rightarrow v(t_1)$ il lavoro è uguale ad $A$

$v(t_1) \rightarrow v(t_0)$ " " $-A$ → LAVORO CEDUTO

Il condensatore non dissipa lavoro ; il lavoro assorbito viene ceduto e quindi il condensatore è un BIPOLO CONSERVATIVO

$$W_e(t) = C \int v \, dv = \frac{1}{2} C v^2 + \cancel{cost}^{\,0}$$ ENERGIA DIELETTRICA del condensatore

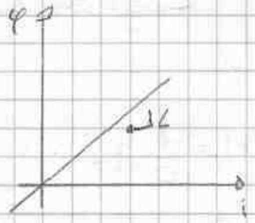
INDUTTORE

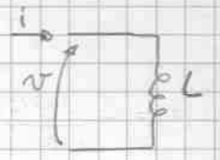
$$f(\varphi, i, t) = 0$$
$$f(\varphi, i) = 0$$

- induttore var nel tempo non lineare
- " " " lineare
- " invar " non lineare
- " " " lineare → INDUTTORE IDEALE

• INDUTTORE IDEALE

$\varphi(t) = L\, i(t)$ $\quad L = [H]$ induttanza





$$i(t) = \frac{1}{L} \varphi(t)$$

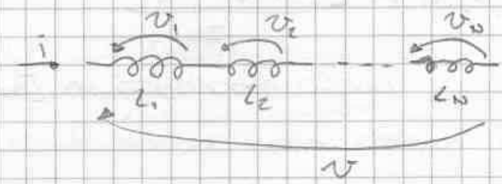
$$\varphi(t) = L\, i(t) \rightarrow \frac{d\varphi}{dt} = L \frac{di(t)}{dt} \qquad \boxed{v(t) = L \frac{di(t)}{dt}}$$

$$v(t) = \frac{1}{C} q(t) \rightarrow \boxed{v(t) = \frac{1}{C} \int_{-\infty}^{t} i(\tau)\, d\tau}$$

$$i(t) = \frac{1}{L} \varphi(t) \Rightarrow \boxed{i(t) = \frac{1}{L} \int_{-\infty}^{t} v(\tau)\, d\tau}$$

$$i(t) = \frac{1}{L} \int_{t_0}^{t} v(\tau)\, d\tau + \frac{\Phi_0}{L}$$ → $i(t_0)$ COND. INIZIALE

INDUTTORE IN SERIE



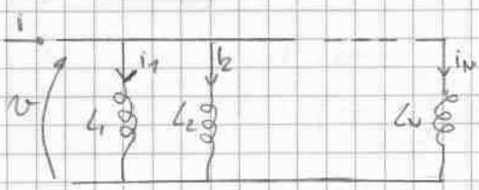

$$v_1 = L_1 \frac{di}{dt} \qquad v_2 = L_2 \frac{di}{dt} \quad , \dots ,$$

$$v_N = L_N \frac{di}{dt}$$

$$LKT \Rightarrow v = v_1 + v_2 + \dots + v_N$$

$$v = \underbrace{(L_1 + L_2 + \dots + L_N)}_{L_{eq}} \frac{di}{dt}$$

• IN PARALLELO




$$\frac{di_1}{dt} = \frac{1}{L_1} v \; , \; \frac{di_2}{dt} = \frac{v}{L_2} \; , \dots , \; \frac{di_N}{dt} = \frac{v}{L_N}$$

$$LKC \Rightarrow i = i_1 + i_2 + \dots + i_N$$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} + \dots + \frac{di_N}{dt}$$

$$\frac{di}{dt} = \left( \frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_N} \right) v(t)$$

$$v(t) = \frac{1}{\frac{1}{L_1} + \dots + \frac{1}{L_N}} \cdot \frac{di}{dt} \qquad L_{eq} = L_1 \| L_2 \| \dots \| L_N$$

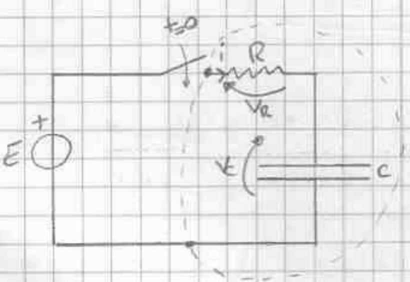
$\to L_{eq}$

RETI DINAMICHE 1° ORDINE 19-11

$$\frac{dx}{dt} = f(\underline{x}, v, i, t) \qquad \underline{x} \in \mathbb{R}^m$$

Reti LINEARI, invariabili nel tempo $\boxed{\frac{d\underline{x}}{dt} = \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u}}$ $\underline{x} \in \mathbb{R}^m$, $\underline{u} \in \mathbb{R}^p$

$\underline{u}$ vettore ingressi




$t > 0$

$$\begin{cases} E = v_R + v_C & (LKT) \\ v_R = Ri & (\text{legge di Ohm}) \\ i = C \dfrac{dv_C}{dt} & (\text{eq. costitutiva } C) \end{cases}$$

Determinato $v_c(0^+) = v_c(0^-)$

$$v_c(0^+) = V_{c_0} + E \Rightarrow \boxed{V_{c_0} = v_c(0^+) - E}$$

regime o evoluzione permanente

$$\boxed{v_c(t) = (v_c(0^+) - E)\, e^{-\frac{t}{RC}} + E} \qquad t \geq 0$$

evoluzione transitoria
parte che varia nel tempo

tensione ai capi del condensatore

$$v_c(t) = \underbrace{v_c(0^+)\, e^{-\frac{t}{RC}}}_{} + \underbrace{E\left(1 - e^{-\frac{t}{RC}}\right)}_{}$$

dipende dalla condizione iniziale
Evoluzione libera ($E=0$)

dipende dal generatore
Evoluzione forzata

$$\begin{cases} \frac{dx}{dt} = ax + b \\ x(0) = x_0 \end{cases} \qquad x(t) = x_0 e^{-at} + b \int_0^t e^{-at'} dt'$$

$$i(t) = C \frac{dv_c}{dt} = -\frac{v_c(0^+)}{R} e^{-\frac{t}{RC}} + \frac{E}{R} e^{-\frac{t}{RC}}$$

a regime $\frac{d}{dt} \to 0$ → bipolo a corrente nulla qualunque sia la tensione. Si comporta come un circuito aperto

$$i(t) = C \frac{dv_c}{dt}$$

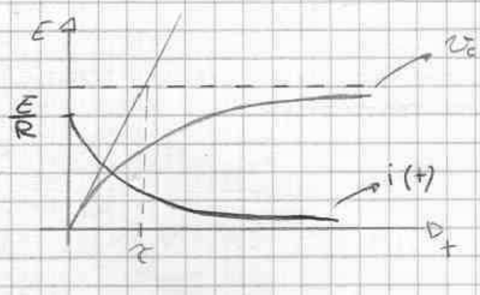
$\Downarrow$

a regime $i(\infty) = 0$ C.A.

- CARICA CONDENSATORE

$v_c(0^-) = 0, \; E \neq 0$

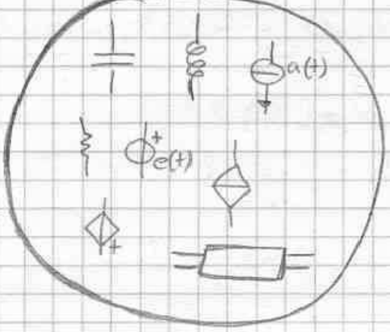
$v_c(t) = (1 - e^{-t/RC})E \qquad i(t) = \frac{E}{R} e^{-t/RC}$

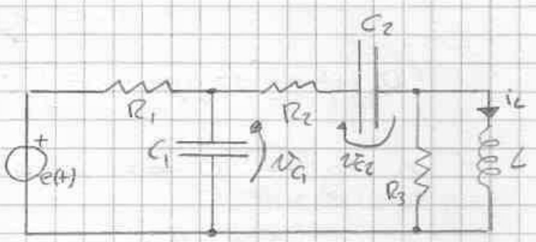



$$\left.\frac{d v_c(t)}{dt}\right|_{t=0} = +\frac{1}{RC} e^{-\frac{t}{RC}} \cdot E \Big|_{t=0} =$$

$$= \frac{E}{RC} \quad \text{pendenza retta}$$

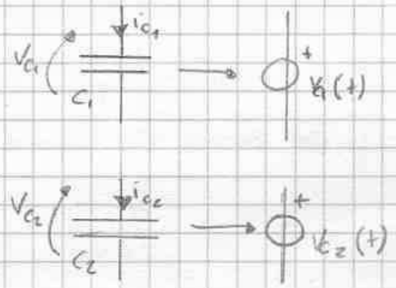
$$\frac{E}{\tau} = \frac{E}{RC} \Rightarrow \boxed{\tau = RC} \quad \text{costante di tempo [s]}$$





esiste una variabile di stato che compare sotto il segno di derivata

$$i_c(t) = C\,\frac{dv_c(t)}{dt} \qquad v_L(t) = L\,\frac{di_L(t)}{dt}$$



pr. sostituzione


$\underline{x} = (v_{C_1}, v_{C_2}, i_L)^T$ variabili di stato

$\hat{\underline{x}} = (i_{C_1}, i_{C_2}, v_L)^T$ variabili di stato coniugate   tensione sugli induttori e corrente sui condensatori

$$i_{C_1}(t) = -v_{C_1}\cdot\frac{1}{R_1 /\!/ (R_2+R_3)} + v_{C_2}\cdot\frac{1}{R_2+R_3} - i_L\,\frac{R_3}{R_3+R_2} + e(t)\,\frac{1}{R_1}$$

$$i_{C_2}(t) = v_{C_1}\cdot\frac{1}{R_2+R_3} - v_{C_2}\,\frac{1}{R_2+R_3} + i_L\,\frac{R_3}{R_3+R_2}$$ ($e(t)$ è nullo xk cc)

$$v_L(t) = v_{C_1}\,\frac{R_3}{R_3+R_2} - \frac{V_{C_2}R_3}{R_2+R_3} - i_L\,R_2 /\!/ R_3$$

---

$$i_{C_1} = C_1\,\frac{dv_{C_1}}{dt} \qquad i_{C_2} = C_2\,\frac{dv_{C_2}}{dt} \qquad v_L = L\,\frac{di_L}{dt}$$

---

$$C_1\,\frac{dv_{C_1}}{dt} = -\frac{V_{C_1}}{R_1 /\!/ (R_2+R_3)} + \frac{V_{C_2}}{R_2+R_3} - i_L\,\frac{R_3}{R_3+R_2} + e(t)\,\frac{1}{R_1}$$

$$C_2\,\frac{dv_{C_2}}{dt} = \frac{v_{C_1}}{R_2+R_3} - \frac{v_{C_2}}{R_2+R_3} + i_L\,\frac{R_3}{R_3+R_2}$$

$$v_L = L\,\frac{di_L}{dt} = v_{C_1}\,\frac{R_3}{R_3+R_2} - \frac{V_{C_2}R_3}{R_2+R_3} - i_L\,R_2 /\!/ R_3$$

$$\underline{x}(t) = e^{\underline{\underline{A}}t}\left[\underline{x}(0) - \underline{x}_r(0)\right] + \underline{x}_r(t)$$

pezzo transitorio — pezzo di regime

il transitorio mi interessa qnd può danneggiare il meccanismo

$$e^{a} = 1 + a + \frac{a^2}{2!} + \frac{a^3}{3!} + \dots + \dots = \sum_{n=0}^{+\infty} \frac{a^n}{n!}$$

$$e^{\underline{\underline{A}}} \triangleq \underline{\underline{I}} + \underline{\underline{A}} + \frac{\underline{\underline{A}}^2}{2!} + \frac{\underline{\underline{A}}^3}{3!} + \dots + \dots = \sum_{n=0}^{+\infty} \underline{\underline{A}}^n \frac{1}{n!}$$

Autovalori matrice $\underline{\underline{A}}$

Se tutti gli autovalori della matrice $\underline{\underline{A}}$ hanno parte reale negativa, allora la rete si dice strettamente passiva e $\lim_{t \to +\infty} \| e^{\underline{\underline{A}}t} \| = \emptyset$

$e^{\underline{\underline{A}}t}(\underline{x}(0) - \underline{x}(0)) \longrightarrow \emptyset$ $\quad x_r(t)$ è il termine di regime

RETI DINAMICHE (lineari, inv. tempo) 26-11

$$\frac{d\vec{x}}{dt} = \underline{\underline{A}}\underline{x} + \underline{\underline{B}}\underline{u}$$

stato $\bar{x} \in \mathbb{R}^n$ $\quad \underline{\underline{A}} \in \mathbb{R}^{n,n}$

ingressi $\bar{u} \in \mathbb{R}^p$ $\quad \underline{\underline{B}} \in \mathbb{R}^{n,p}$

$$\vec{x}(t) = \vec{x}(t) + \vec{x}_r(t)$$

$\longrightarrow e^{\underline{\underline{A}}t}\left[x(0) - x_r(0)\right]$

RETI 2° ORDINE (con ingressi costanti a tratti)

$n = 2$ $\quad \underline{\underline{A}} \in \mathbb{R}^{2,2} \longrightarrow \lambda_1, \lambda_2$

$\underline{x}_r(\infty)$ evoluzione regime

$\underline{x}_{tr}(t) = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$ $\quad A_1, A_2$ costanti che dipendono dalle C.I.

RETI 1° ORDINE

$$\tau \frac{dx}{dt} + x = 0 \longrightarrow \frac{dx}{dt} = -\frac{1}{\tau}x$$

$-\frac{1}{\tau} \longrightarrow \underline{\underline{A}}$

$$\tau\lambda + 1 = 0 \longrightarrow \lambda = -\frac{1}{\tau} \qquad x(0)\,e^{\lambda t}$$

$v_c(t) = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$ $\quad v_c(0^+) \quad \frac{dv}{dt} v(t)\Big|_{t=0^+}$

$\alpha > \omega_0$ (sovrasmorzamento)
$\lambda_{1,2}$ reali dist.

$\alpha = \omega_0$ (smorzamento critico)
$\lambda_1 = \lambda_2 = -\alpha$ reali coincidenti

$\alpha < \omega_0$ (sottosmorzamento)
$\lambda_1, \lambda_2$ radici c, c.

$$\lambda_{1,2} = -\alpha \pm j\sqrt{\omega_0^2 - \alpha^2}$$

$\alpha > \omega_0 \quad (R_1 = 5\,\Omega)$

$\lambda_1 = -1, \; \lambda_2 = -4$

$v_{c_{tr}}(t) = A_1 e^{-t} + A e^{-4t}$

$v_c(0^+) = 4 \rightarrow A_1 + A_2 = 4$

$\frac{d v_c(0^+)}{dt} = 16 \rightarrow -A_1 - 4A_2 = 16$

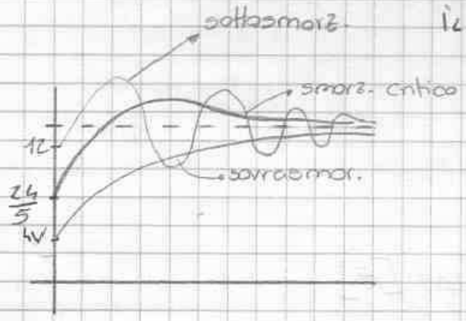
$-3A_2 = 20 \quad A_2 = -\frac{20}{3}$

$A_1 = 4 + \frac{20}{3} = \frac{32}{3}$

$v_{c_{tr}}(t) = \frac{32}{3} e^{-t} - \frac{20}{3} e^{-4t}$

$v_{c_r}(t) = v_c(\infty) = E \qquad v_c(t) = \frac{32}{3} e^{-t} - \frac{20}{3} e^{-4t} + 24 \quad (t \geq 0)$

$i_L(t) = i_c(t) = C \frac{d v_c(t)}{dt} = -\frac{8}{3} e^{-t} + \frac{20}{3} e^{-4t}$ A $(t \geq 0)$




$\alpha = \omega_0 \quad (R_1 = 4\,\Omega)$

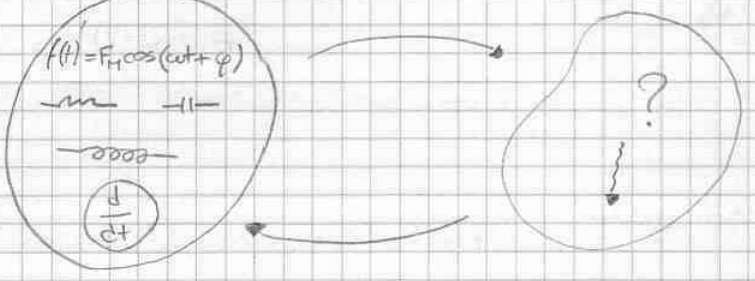
$\lambda_1 = \lambda_2 = -\alpha = -2$

$v_{c_{tr}}(t) = (A_1 + A_2 t) e^{\lambda_1 t} = (A_1 + A_2 t) e^{-2t}$

$v_c(0^+) = \frac{24}{5} \rightarrow A_1 = \frac{24}{5}$

$\frac{dv_c}{dt}(0^+) = \frac{96}{5} \rightarrow -2A_1 + A_2 = \frac{96}{5} \qquad A_2 = \frac{144}{5}$

Dominio Tempo    Dominio Fasori




passo da un dominio all'altro, faccio i calcoli, e ritorno al primo

$$f(t) = F_M \cos(\omega t + \varphi) \xrightarrow[\text{operatore fasore}]{\mathcal{F}} F = F_M e^{j\varphi_F} \qquad j = i = \sqrt{-1}$$

$$e^{j\varphi_F} = \cos(\varphi_F) + j\sin(\varphi_F)$$

$$e^{j\frac{\pi}{2}} = j \qquad j^{(j)} = \left(e^{j\frac{\pi}{2}}\right)^j = e^{-\frac{\pi}{2}}$$

$$e^{2k\pi j} = \cos(2k\pi) + j\sin(2k\pi) = 1 \qquad k \in \mathbb{Z}$$

- $f(t) = 3\cos(4t) \longrightarrow F = 3e^{j\phi} = 3$

  $F_M = 3 \quad \varphi_F = \phi \quad \omega = 4 \text{ rad/sec}$

- $f(t) = \sin(\omega t) = \cos(\omega t - \frac{\pi}{2})$

  $F_M = 1 \quad \varphi_F = -\frac{\pi}{2} \longrightarrow F = e^{-j\frac{\pi}{2}} = -j$

- $f(t) = 2\sin(2t - \frac{\pi}{3}) = 2\cos(2t - \frac{\pi}{3} - \frac{\pi}{2}) = 2\cos(2t - \frac{5}{6}\pi)$

- $F = 2e^{-j\frac{5}{6}\pi} = 2\left[\cos(\frac{5}{6}\pi) - j\sin(\frac{5}{6}\pi)\right] = 2\left[-\frac{\sqrt{3}}{2} - j\frac{1}{2}\right]$

$\mathbb{R} \to \mathbb{R} \longrightarrow \mathbb{C}$

spazio di dimensione 2

Base $\begin{pmatrix}\cos\omega t\\ \sin\omega t\end{pmatrix}$

$$f(t) = F_M \cos(\omega t + \varphi_F) = F_M[\cos(\omega t)\cos\varphi_F - \sin\omega t \sin\varphi_F] =$$

$$= F_M \cos\varphi_F \cos\omega t - F_M \sin\varphi_F \sin\omega t$$

L'operatore Fasore $\mathcal{F}$ è LINEARE (biezione)

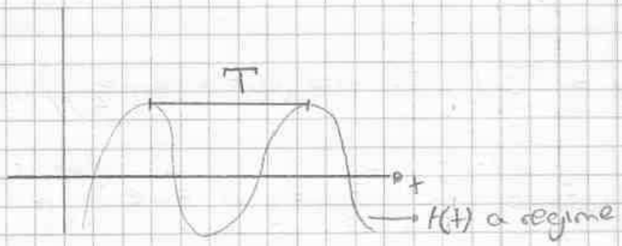
$$\mathcal{F}(c_1 f_1 + c_2 f_2) = c_1 \mathcal{F}(f_1) + c_2 \mathcal{F}(f_2) = c_1 F_1 + c_2 F_2$$

$f_1 = F_{1M}\cos(\omega t + \varphi_1)$

$f_2 = F_{2M}\cos(\omega t + \varphi_2)$ $\Rightarrow \mathcal{F}(c_1 f_1 + c_2 f_2) = c_1 F_{1M} e^{j\varphi_1} + c_2 F_{2M} e^{j\varphi_2}$

# RETI IN REGIME SINUSOIDALE 

I fasori sono uno strumento per rappresentare grandezze sinusoidali




$f(t) = A\cos(\omega t + \varphi) = \mathrm{Re}[F e^{j\omega t}]$

$\updownarrow$

$F \in \mathbb{C} \quad F = Ae^{j\varphi} \quad \omega$ data

$i_1(t) = 4\cos(\omega t + 30°)$ lettere minuscole $\rightarrow$ dominio del tempo

$I_1 = 4e^{j30°} = 4\cos(30) + j4\sin 30 = 2\sqrt{3} + 2 = 2(\sqrt{3} + j)$

$i_2 = 5\sin(\omega t - 20°) =$ $\rightarrow$ bisogna trasformarla al cos $= 5\cos(\omega t - 20° - 90°) =$
$= 5e^{-j110°}$

$\sin(x - 90°) = \sin x \cos 90 - \cos x \sin 90 = -\cos x$
$\sin(x + 90°) = \sin x \cos 90 + \cos x \sin 90 = \cos x$
$\cos(x + 90°) = \cos x \cos 90 - \sin x \sin 90 = -\sin x$
$\cos(x - 90°) = \cos x \cos 90 + \sin x \sin 90 = \sin x$

$V = -3 + j4 \quad (\omega = 2\pi \cdot 50 \quad f = 50\,Hz)$
$= 5e^{j(-\mathrm{atan}(\frac{4}{3}) + \pi)}$

$v(t) = 5\cos(100\pi t - \mathrm{atan}(\frac{4}{3}) + \pi) = -5\cos[100\pi t - \mathrm{atan}(\frac{4}{3})]$

| <u>Dominio tempo</u> | | <u>Dominio Fasori</u> |
|---|---|---|
| $f(t)$ | $\longrightarrow$ | $F \in \mathbb{C}$ |
| | $\longleftarrow$ | |

$\frac{d}{dt} \longrightarrow j\omega$


$v(t) = Ri(t)$
$v(t) = \mathrm{Re}[V e^{j\omega t}]$
$V \in \mathbb{C}$

$V = RI$
$|V| = R|I|$
$\angle V = \angle I$


$i(t) = \mathrm{Re}[I e^{j\omega t}]$
$\mathrm{Re}[V e^{j\omega t}] = \mathrm{Re}[RI e^{j\omega t}]$

L'impedenza di un resistore vale R

$Z = R + jX$ — parte immaginaria (reattanza)

$R$: parte reale (resistenza)

$\mathrm{Re}[Z] = R$ RESISTENZA

$\mathrm{Im}[Z] = X$ REATTANZA

Il resistore ha resistenza nulla $(x=0)$

resist. $R \neq 0$ $\quad x_R = 0$

cond. $R = 0$ $\quad x_C = -\frac{1}{\omega C} < 0$

indutt. $R = 0$ $\quad x_L = \omega L > 0$

$\frac{I}{V} = Y = \frac{1}{Z}$ AMMETTENZA

$Y_R = \frac{1}{R}$

$Y_C = j\omega C$

$Y_L = \frac{1}{j\omega L} = j\frac{1}{\omega L}$

$Y = G + jB$

$G = \mathrm{Re}[Y]$ conduttanza

$B = \mathrm{Im}[Y]$ SUSCETTANZA

resist. $G \neq 0$

cond. $G = 0$

indutt. $G = 0$

$i_1(t) + i_2(t) + \dots + i_N(t) = 0$

$\mathrm{Re}[I_1 e^{j\omega t}] + \mathrm{Re}[I_2 e^{j\omega t}]$

$\mathrm{Re}[(I_1 + I_2 + \dots + I_N) e^{j\omega t}] = 0$

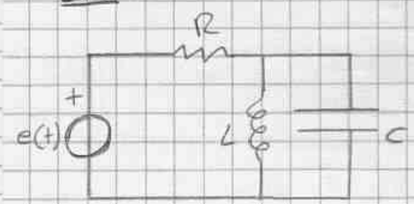
$\Downarrow$

$I_1 + I_2 + \dots + I_N = 0$ LKC

$V_1 + V_2 + \dots + V_N = 0$ LKT

mantengono la stessa struttura

ES



$e(t) = 20\cos(4t - 15°)$, V $\quad \omega = 4$ rad/s

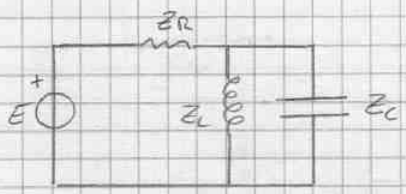
$R = 60\ \Omega$

$C = 10\,mF = 10 \cdot 10^{-3} F$

$L = 5H$

$v_o(t)$ a regime

Dominio Fasori



$E = 20\, e^{-j15°}$

$Z_R = 60\ \Omega$

$Z_L = j\omega L = j20\ [\Omega]$

$Z_C = -j\frac{1}{\omega C} = -j\frac{1}{4 \cdot 10 \cdot 10^{-3}} = -j\,25\ [\Omega]$

$V_o = \frac{Z_L \parallel Z_C}{Z_L \parallel Z_C + Z_R} \cdot E =$

$Z_L \parallel Z_C = \frac{Z_L \cdot Z_C}{Z_L + Z_C} = \frac{500}{-j5} = j100\ [\Omega]$

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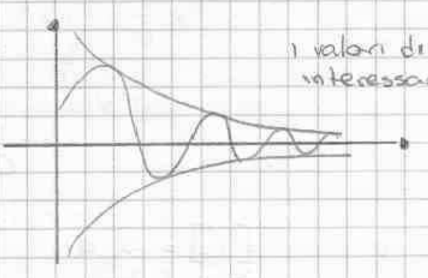
$e^{\sigma t}(A\cos\omega t + B\sin\omega t)$

Se A e B sono 2 valori elevati alla fine arrivo sempre a zero



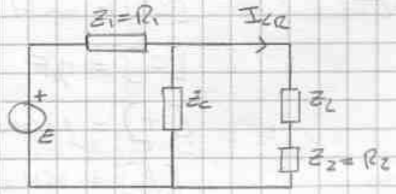
I valori di A e B non mi interessano perché l'esponenziale me la porta a $\phi$

↓

con un tempo relativamente lungo arrivo a regime ovvero a $\phi$

Per calcolare il termine di regime

$\omega' = 2\ \frac{rad}{sec}$




$E = -5j \qquad Z_1 = 2\,\Omega$

$Z_2 = 1\,\Omega$

$Z_C = jX_C \qquad X_C = -\frac{1}{\omega' C} = -1\,\Omega$

$Z_L = jX_L \qquad X_L = \omega' L = 2\,\Omega$

$$I_{LR} = \frac{E}{Z_1 + Z_C \parallel (Z_L + Z_2)} \cdot \frac{Z_C}{Z_C + Z_L + Z_2} =$$

$$= \frac{E}{R_1 + jX_C \parallel (jX_L + R_2)} \cdot \frac{jX_C}{jX_C + jX_L + R_2} =$$

$$= \frac{-5j}{2 + (-j) \parallel (2j+1)} \cdot \frac{(-j)}{-j + 2j + 1} = \frac{-5j}{2 + \frac{(-j)(2j+1)}{-j+2j+1}} \cdot \frac{-j}{-j + 2j + 1} =$$

$$= \frac{-5j}{2 + \frac{2-j}{j+1}} \cdot \frac{-j}{j+1} = \frac{-5}{2j + 2 + 2 - j} = \frac{-5}{4+j}$$

$$= \frac{-5(4+j)}{16-1} \quad \frac{-20}{15} - \frac{5}{15}j = \quad = -\frac{4}{3} - \frac{1}{3}j$$

$|I_{LR}| = \frac{5}{\sqrt{17}}$

$\angle I_{LR} = \pi - \arctan\frac{1}{4}$

$$i_{LR}(t) = \frac{5}{\sqrt{17}}\cos\left(2t + \pi - \operatorname{atan}\frac{1}{4}\right) =$$

$$= -\frac{5}{\sqrt{17}}\cos\left(2t - \operatorname{atan}\frac{1}{4}\right)$$

$I_{LR} = -\frac{20}{17} + \frac{5}{17}j$

$$i_{LR}(t) = \mathcal{F}^{-1}\left(-\frac{20}{17} + \frac{5}{17}j\right) = \mathcal{F}^{-1}\left(-\frac{20}{17}\right) + \mathcal{F}^{-1}\left(\frac{5}{17}j\right) =$$

$$= -\frac{20}{17}\cos(2t) - \frac{5}{17}\sin(2t)$$

$$i_{LR}(t) = -\frac{5}{\sqrt{17}}\cos\left(2t - \operatorname{atan}\frac{1}{4}\right) = -\frac{5}{\sqrt{17}}\left[\cos(2t)\cos\left(\operatorname{atan}\frac{1}{4}\right) + \sin(2t)\sin\left(\operatorname{atan}\frac{1}{4}\right)\right]$$

$Q = \frac{V_m I_m}{2} \sin(\varphi_V - \varphi_I)$ potenza REATTIVA [VAR] (no significato fisico)

complesso coniugato

$\frac{1}{2} V I^* = \frac{1}{2} \left(V_m e^{j\varphi_V}\right)\left(I_m e^{j\varphi_I}\right)^* = \frac{1}{2} V_m e^{j\varphi_V} I_m e^{-j\varphi_I} =$

viene fuori dalle formule trigonometriche

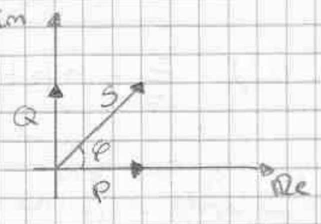
$= \frac{1}{2} V_m I_m e^{j(\varphi_V - \varphi_I)} = S \in \mathbb{C}$ potenza COMPLESSA

$$S = \frac{1}{2} V_m I_m \left[\cos(\varphi_V - \varphi_I) + j \sin(\varphi_V - \varphi_I)\right] =$$

$$= \underbrace{\frac{1}{2} V_m I_m \cos(\varphi_V - \varphi_I)}_{P} + j \underbrace{\frac{1}{2} V_m I_m \sin(\varphi_V - \varphi_I)}_{Q}$$

Quindi la parte reale di $S$ è la potenza attiva.

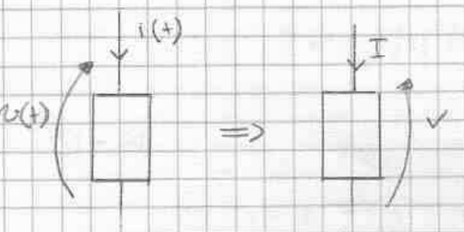
$Re[S] = P \quad Im[S] = Q$




$S = P + jQ \qquad \sqrt{P^2 + Q^2} = |S| = A = \frac{1}{2} V_m I_m$ pot. APPARENTE

$\frac{Q}{P} = \tan(\angle S) = \tan(\varphi_V - \varphi_I) = \tan\varphi$

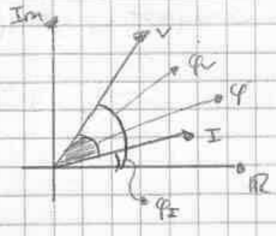
$\varphi = \varphi_V - \varphi_I$




$v(t) = V_m \cos(\omega t + \varphi_V) \Rightarrow V = V_m e^{j\varphi_V}$

$Z = \frac{V}{I} = \frac{V_m}{I_m} \frac{e^{j\varphi_V}}{e^{j\varphi_I}}$

$Z = \frac{V_m}{I_m} e^{j(\varphi_V - \varphi_I)} \qquad \varphi = \varphi_V + \varphi_I = \angle Z$

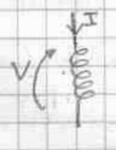



<u>resistore</u> $(Z = R)$

$V = RI \qquad \varphi_V - \varphi_I = 0$

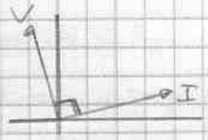

$P_R = \frac{1}{2} V_m I_m \qquad Q_R = 0$

<u>induttore</u>




$V = j\omega L I$

$Z = j\omega L$




$\varphi_V - \varphi_I = \frac{\pi}{2} \qquad P_L = 0$

$Q_L = \frac{1}{2} V_m I_m > 0$

l'induttore non assorbe energia → assorbe una

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$$= \frac{1}{2} R_2 \, |I_L|^2 = \frac{1}{2} (6,32)^2 \cdot 5 = 100 \text{ W}$$

$$P_{R_1} = \frac{1}{2} R_1 |I_L|^2 = 80 \text{ W} \qquad Q_L = \frac{1}{2} x_{L_2} |I_L|^2 = \frac{1}{2} \omega L_2 |I_L|^2 =$$

$$= \frac{1}{2} \, 9 \, (6,32)^2 = 180 \text{ VAR}$$

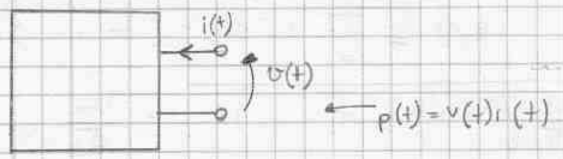
$$Q_C = \frac{1}{2} x_C |I_L|^2 = \frac{1}{2} \left( - \frac{1}{\omega C} \right) (I_L)^2 =$$

$$= \frac{1}{2} (6,32)^2 \cdot (-2) = -40 \text{ VAR}$$

$$S_L = \frac{1}{2} \underbrace{V_L}_{(R_2 + j\omega L_2 - j\frac{1}{\omega C}) I_L} I_L^* = P_{R_2} + j (Q_{L_2} + Q_C) = 100 + j\,140 \ [\text{VA}]$$

$$P_E = \mathrm{Re} \left[ \frac{1}{2} E I_L^* \right] = 180 \text{ W} = P_{R_1} + P_{R_2}$$

17-12




$$v(t) = V_M \cos(\omega t + \varphi_V)$$

$$i(t) = I_M \cos(\omega t + \varphi_I)$$

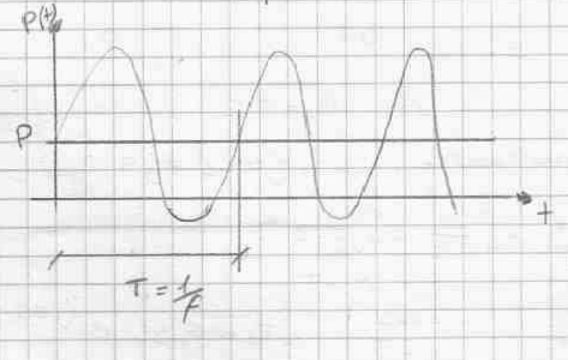
$$V = V_M e^{j\varphi_V} \qquad |V| = V_M$$

$$I = I_M e^{j\varphi_I} \qquad |I| = I_M$$

$$p(t) = \frac{1}{2} \mathrm{Re} \left( V_M e^{j\varphi_V} I_M e^{-j\varphi_I} \right) + \frac{1}{2} \mathrm{Re} \left( V_M e^{j\varphi_V} I_M e^{j\varphi_I} e^{2j\omega t} \right) =$$

$$= \frac{1}{2} V_M I_M \cos(\varphi_V - \varphi_I) + \frac{1}{2} V_M I_M \cos(2\omega t + \varphi_V + \varphi_I)$$

$$p(t) = \underbrace{\frac{1}{2} |V| |I| \cos(\varphi_V - \varphi_I)}_{P} + \frac{1}{2} |V| |I| \cos(2\omega t + \varphi_V + \varphi_I)$$




$$p(t) = \frac{dW(t)}{dt}$$

$$W(T) = \int_0^T p(t)\,dt = P \cdot T + \emptyset$$

dal punto di vista energetico conta solo il valor medio

Nel <u>resistore</u>

$$\varphi_V = \varphi_I \qquad p(t) = \frac{1}{2} |V| |I| + \frac{1}{2} |V| |I| \cos(2\cos t + \varphi_V + \varphi_I)$$

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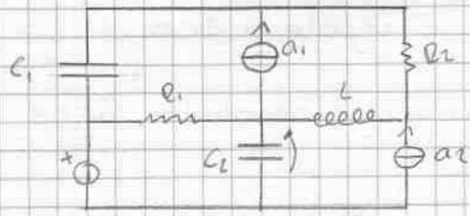
$E = -5j \qquad x_L = \omega L = 3\,\Omega$

$x_C = \dfrac{-1}{\omega C} = -\dfrac{1}{3}\,\Omega$

$$I = \frac{E}{R + jx_L /\!/ jx_C} = \frac{-5j}{1 + \dfrac{3j\left(-\frac{1}{3}j\right)}{3j - \frac{1}{3}j}} = \frac{-5j}{1 + \dfrac{1}{\frac{8}{3}j}} = \frac{-5\,\frac{8}{3}\,j}{\frac{8}{3} - j} = \frac{-40j}{8 - 3j}$$

$$|I|^2 = \frac{40^2}{64+9} = \frac{40^2}{73} \qquad P = \frac{1}{2} R\,|I|^2 = \frac{1}{2}\,\frac{40^2}{73} = \frac{800}{73}\ W$$

Esempio 10-12



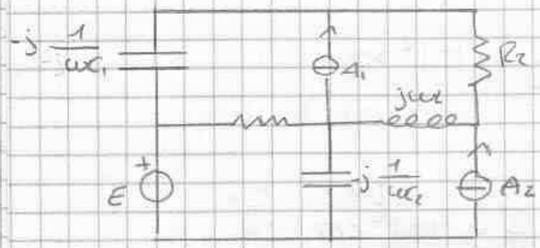

$e(t) = 10\cos\omega t \qquad R_1 = 8\,\Omega$

$a_1(t) = 4\cos\omega t \qquad R_2 = 6\,\Omega$

$a_2(t) = 3\cos\omega t \qquad L = 6H \qquad C_1 = \frac{1}{4}F$

$\omega = 1\ rad/s \qquad C_2 = \frac{1}{2}F$

Dominio fasori $(\omega = 1\ rad/s)$




$j\omega L = j5\,\Omega \qquad E = 10e^{j0} = 10V$

$-j\dfrac{1}{\omega C_1} = -j4\,\Omega \qquad A_1 = 4e^{j0} = 4A$

$-j\dfrac{1}{\omega C_2} = -j2\,\Omega \qquad A_2 = 3e^{j0} = 3A$

$V_0 = V_0|_E + V_0|_{A_2}$ (sovrapp.)

$$V_0|_E = \frac{-j\frac{1}{\omega C_2}}{-j\frac{1}{\omega C_2} + \left[R_1 \,||\, (j\omega L + R_2 - j\frac{1}{\omega C_1})\right]} \cdot E$$

$$V_0|_{A_1} = -\left[R_1 \,||\, \left(-j\frac{1}{\omega C_2}\right)\right] \cdot \frac{(R_2 + j\omega L)\cdot A_1}{\underbrace{(R_2 + j\omega L) + \left[R_1 \,||\, \left(-j\frac{1}{\omega C_2}\right)\right] - j\frac{1}{\omega C_1}}}$$

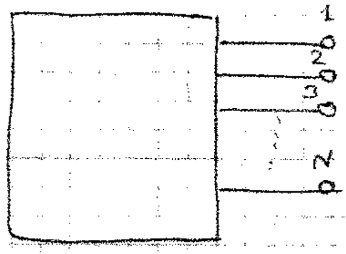
$$V_0|_{A_2} = \left[R_1 \,||\, \left(-j\frac{1}{\omega C_2}\right)\right] \cdot \frac{(R_2 - j\frac{1}{\omega C_1}) \overset{I^*}{\cdot} A_2}{R_2 - j\frac{1}{\omega C_1} + j\omega L + \left[R_1 \,||\, \left(-j\frac{1}{\omega C_2}\right)\right]}$$

$V_0 = 9{,}756\, e^{j222{,}32}\ [V]$

$v_0(t) = 9{,}756\,\cos(\omega t + 222{,}32°)$

# TEORIA DEI CIRCUITI A PARAMETRI CONCENTRATI

## OGGETTO ELETTRICO



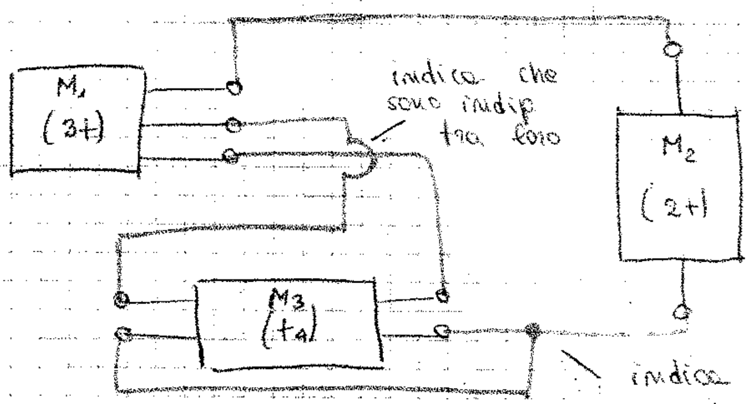

TERMINALI ELETTRICI (o POLI)

un oggetto caratterizzato + poli è il MULTIPOLI ↙ termine tecnico indicare l'oggetto

|| CIRCUITO (rete) ELETTRICO è una struttura costituita dalle connessione arbitrarie di multipoli ||

CONNESSIONI

es.




indica che sono indip tra loro

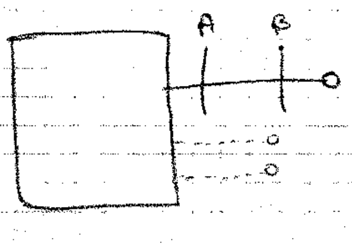
indica che sono connessi tra loro (prende nome di NODO)

- Come si studia un circuito?

Sono necessarie delle grandezze fisiche:

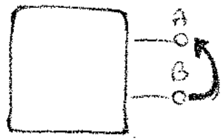
- INTENSITÀ DI CORRENTE ELETTRICA

Dato un terminale (di un multipolo) e definite 2 A e B sul terminale, l'intensità di corrente elettrica si indica con $i_{AB}(t)$ è la QUANTITÀ DI CARICA ELETTRICA che nell'unità di tempo transita nel te da A verso B.



La tensione si misura in VOLT e, data una c[...] di terminali, bisogna stabilire quello da cui si parte. Esistono più notazioni utilizzate:

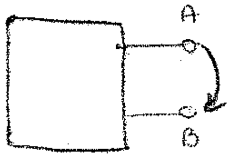
① NOTAZIONE ITALIANA




$v(t) = v_{AB}(t)$

la freccia indica il terminale di partenza

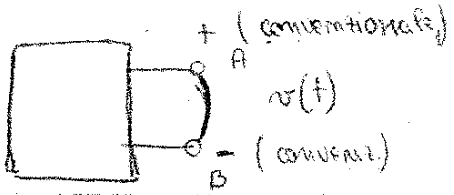
② NOTAZIONE TEDESCA




$v(t)$

la freccia indica il terminale di arrivo

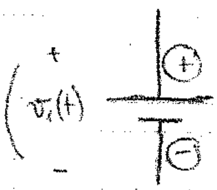
③ NOTAZIONE USA




si mette un archetto e per indicare i 2 terminali con segni + e − che risu[...] convenzionali

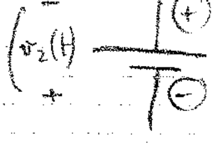
P.S. Noi useremo quella italiana e quella USA!

es. prendiamo una batteria da 12 V



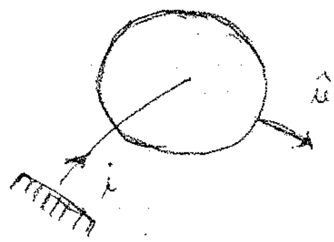

FISICAMENTE POSITIVO (e la tensione è costante)

Se faccio coincidere il termine convenzionalmente positivo quello fisicamente negativo, allora $v_1(t) = 12$ V; ma non è detto che i 2 coincidano e dunque viene $v_2(t) = -12$ V.



es 2 (5)

Se abbiamo un' antenna




$-i(t) = \phi$

$i(t) \neq \phi$

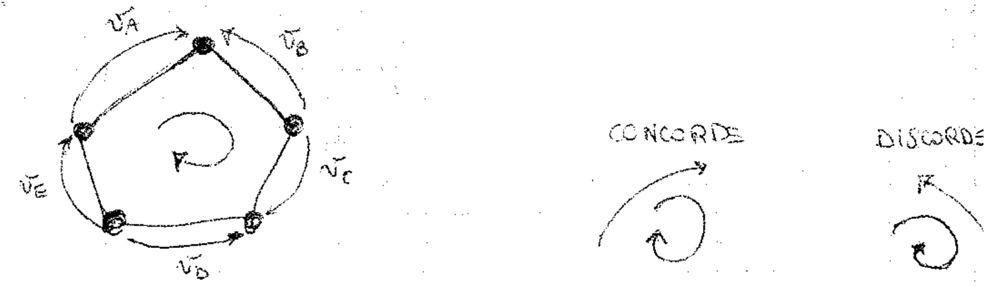
in questo caso (rispetto al primo) il sistema non è confinato da un punto di vista spaziale e quindi non si può applicare la legge di Kirchhoff

LEGGE DI KIRCHHOFF DELLE TENSIONI

Sia dato un insieme di terminali (di multipoli di una rete elettrica) ed il poligono chiuso che si ottiene connettendo le estremità dei terminali e si definisca un verso di percorrenza del poligono.

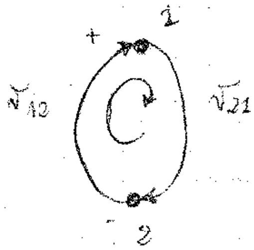
CONNESSIONE GEOMETR

La somma delle tensioni definite sui lati del poligono (assumi con segno "più" quelle concordi e con "−" quelle discordi) uguale a 0.




$$+v_A(t) - v_B(t) + v_C(t) - v_D(t) + v_E(t) = \phi$$

es 1



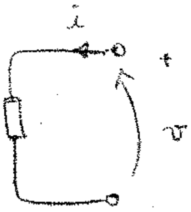

$v_{12} + v_{21} = \phi$

$v_{21} = -v_{12}$

posso scegliere arbitrariamente il terminale positivo e viceversa

9

BIPOLO CON CONV. UTILIZZATORE




$P_e(t) = v(t)\,i(t)$ → POTENZA ENTRANTE

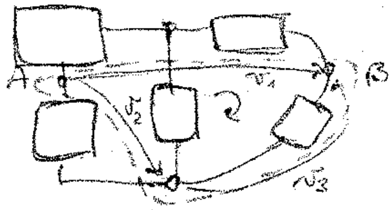
$P_u(t) = -v(t)\,i(t)$ → POTENZA USCENTE

## ESERCITAZIONE 2 26/9/2009

LEGGI DI KIRCHHOFF

(1) Somma correnti entranti = 0
" " uscenti = 0 } correnti entranti = uscen

(2)



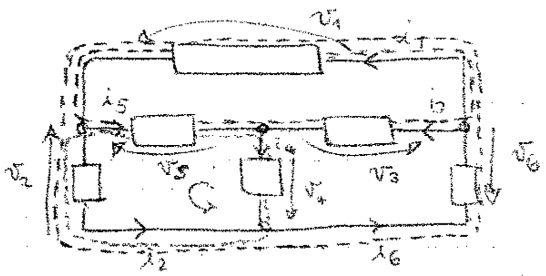

$v_1 - v_2 - v_3 = 0$ (senso orario)

se avessi scelto il senso o

$v_2 + v_3 - v_1 = 0$

Somma tensioni che agiscono in un verso è uguale a somma delle tensioni che agiscono in senso opposto

ES 1




$v_4 = 7\,V$
$v_5 = 9\,V$
$v_6 = 8\,V$

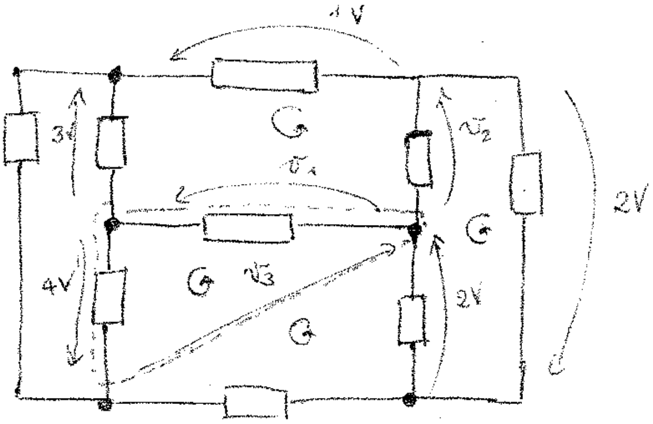
$v_1, v_2, v_3, i_1, i_2$

$$-v_2 - v_4 + v_5 = 0$$
$$-v_2 - 7V + 9V = 0 \qquad \boxed{v_2 = 2V}$$

$$v_1 - v_2 - v_6 = 0$$
$$v_1 - 2 - 8 = 0 \qquad \boxed{v_1 = 10\,V}$$

$$v_1 - v_5 + v_3 = 0$$
$$10 - 9 + v_3 = 0 \qquad \boxed{v_3 = 1\,V}$$

ES 4



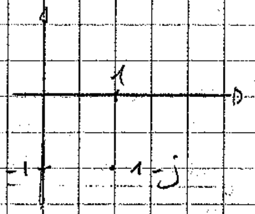

$v_1$ ?
$v_2$ ?
$v_3$ ?

$1 - 3 - v_1 + v_2 = 0$ $\boxed{v_1 = -6\,V}$

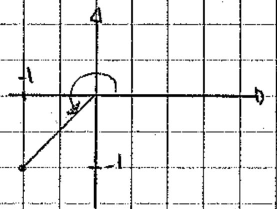
$-2 - v_2 - 2 = 0 \quad -v_2 = 4$ $\boxed{v_2 = -4\,V}$

$v_1 + 4 + v_3 = 0 \quad -6 + 4 + v_3 = 0$ $\boxed{v_3 = 2\,V}$

$z = 1 - j$ $\quad |z| = \sqrt{2} \quad \theta = \operatorname{arctg}\left(\frac{-1}{1}\right) = -\frac{\pi}{4}$




$z = -1 - j$ $\quad |z| = \sqrt{2} \quad \theta = \operatorname{arctg}\left(\frac{-1}{-1}\right) = \frac{\pi}{4}$ sbagliato!




$\theta = \operatorname{arctg}\left(\frac{-1}{-1}\right) + \pi = \frac{5}{4}\pi$ per i num complessi che sono nel II° e III° quadrante

$$\begin{cases} \theta = \operatorname{arctg}\left(\frac{y}{x}\right) & \text{se } x > 0 \\ \theta = \operatorname{arctg}\left(\frac{y}{x}\right) + \pi & \text{se } x < 0 \end{cases}$$

$z = -1 + j$

$|z| = \sqrt{2} \quad \theta = \operatorname{arctg}\left(\frac{1}{-1}\right) + \pi = \frac{3}{4}\pi$

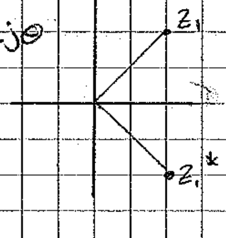
$z = x + jy = |z|\left(\cos\theta + j\sin\theta\right) = |z| e^{j\theta}$

Formula di Eulero $\quad e^{j\theta} = \cos\theta + j\sin\theta$

OPERAZIONI NUM. COMPLESSI

$z_1 = x_1 + jy_1 = |z_1| e^{j\theta_1}$

$z_2 = x_2 + jy_2 = |z_2| e^{j\theta_2}$ $\qquad z_1^* = x_1 - jy_1 = |z_1| e^{-j\theta}$




$z_1 \pm z_2 = (x_1 \pm x_2) + j(y_1 \pm y_2)$

$z_1 z_2 = |z_1||z_2| e^{j(\theta_1 + \theta_2)} \implies z_1 z_1^* = |z_1|^2$

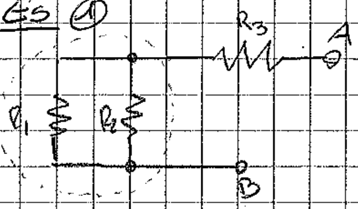
$\left[(x_1 + jy_1)(x_2 + jy_2) = (x_1x_2 - y_1y_2) + j(x_1y_2 + y_1x_2)\right]$

$\frac{z_1}{z_2} = \frac{|z_1|}{|z_2|} e^{j(\theta_1 - \theta_2)}$ $\qquad \left[\frac{(x_1 + jy_1)}{(x_2 + jy_2)} = \frac{(x_1 + jy_1)(x_2 - jy_2)}{x_2^2 + y_2^2} = \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} + j\,\frac{y_1x_2 - x_1y_2}{x_2^2 + y_2^2}\right]$

$\Downarrow$

$\frac{1}{z_2} = \frac{1}{|z|} e^{-j\theta}$

## Es 1



$R_1 = 4\,\Omega$
$R_2 = 4\,\Omega$
$R_3 = 5\,\Omega$

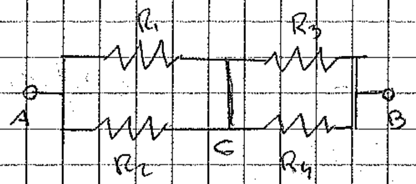
$R_{AB} = ?$

parallelo con resistori uguali

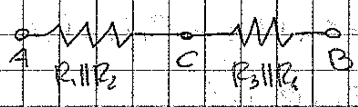
$R \| R \| R = \left(\frac{RR}{2R}\right) \| R = \frac{R^2/2}{\frac{3}{2}R} = \frac{R}{3}$

$R_{AB} = R_3 + (R_1 \| R_2)$

## Es 2

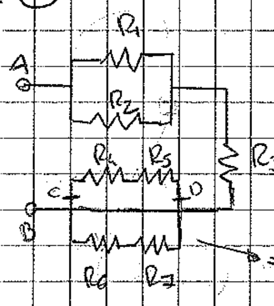


$R_{AB} = ?$ la $v$ applicata in A da $R_1$ è uguale a $v$ in A da $R_2$



$R_{AB} = (R_1 \| R_2) + (R_3 \| R_4)$

## Es 3

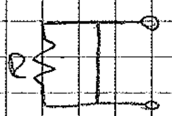


$R_{AB} = (R_1 \| R_2) + R_3 + 0$

$= \phi$ in un cortocircuito $v = \phi$
Resistenza nulla

(CD) $0 = (R_1 \| R_2)\, i \Rightarrow i = \phi$

resistore in parallelo con c.c.



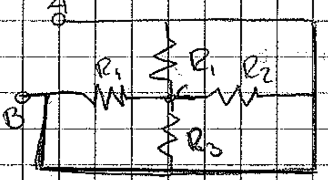
resistore in parallelo con circuito aperto

$i = \phi \quad \forall v \Rightarrow G = \phi \Rightarrow R = \infty$

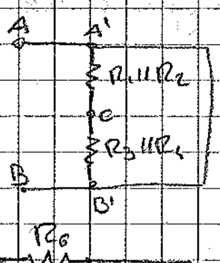
$R' \| R'' = \frac{R'R''}{R' + R''} = R'$

## Es 4

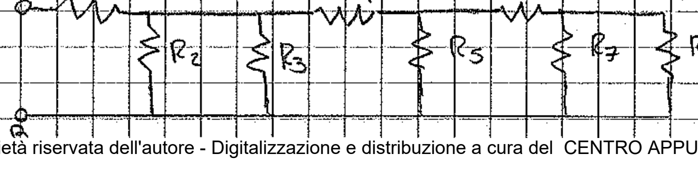
$R_{AB} = (R_1 \| R_2) \| (R_2 \| R_3) \| (R_3 \| R_4)$ sbagliato



c.c. $R_{AB} = \phi$

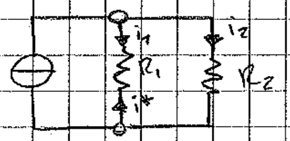


## Es 5



$R_{AB} = R_2 \| R_3 + R_7 \| R_8$

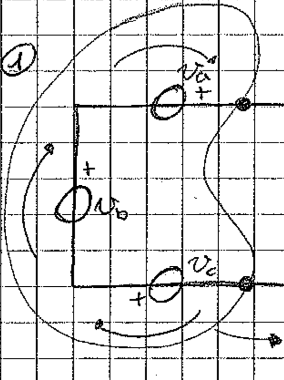
# PARTITORE di CORRENTE




$$i_1 = \frac{R_2}{R_1+R_2}\, a \qquad i_2 = \frac{R_1}{R_1+R_2}\, a$$

$$i_1 + i_2 = a \qquad i^* = -\frac{R_2}{R_1+R_2}$$

(1)



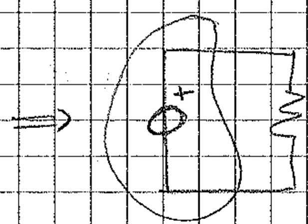

$i = ?$ $\quad Ri = v_R$ legge di Ohm

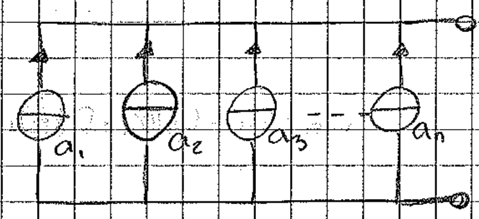
$v_a + v_b + v_c = v_R$ LKT

$\Downarrow$

bipolo composto da generatori di tensione in serie

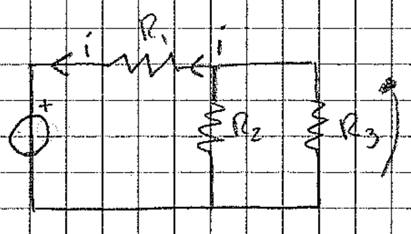
$$i = \frac{v_a + v_b + v_c}{R}$$






bipolo costituito da generatori di corrente in parallelo

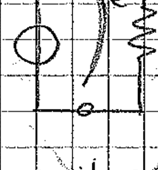
(2)




$v$? $i$?

per calcolare $i$

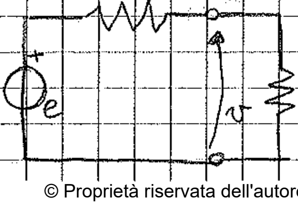
in qst caso basta sfruttare la legge di ohm




$(R_2 // R_3) + R_1$

$$i = -\frac{e}{(R_2 // R_3) + R_1}$$

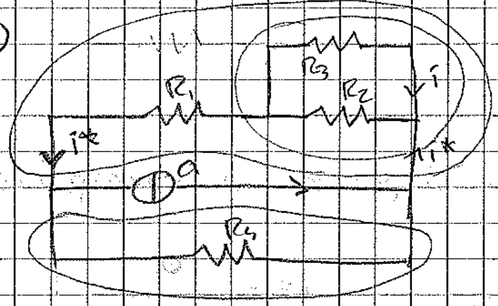
per calcolare $v$ (utilizziamo il partitore di tensione)




$R_2 // R_3$

$$v = \frac{R_2 // R_3}{R_2 // R_3 + R_1}\, e$$

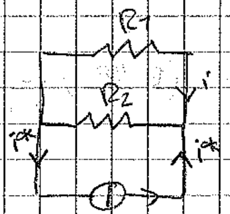
6



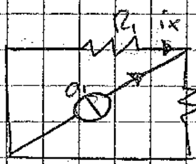

$i$?

$$i^* = \frac{R_4}{R_4 + R_1 + (R_2 \| R_3)} a$$

$$i = \frac{R_2}{R_2 + R_3}(-i^*) = -\frac{R_2}{R_2 + R_3} \cdot \frac{R_4}{R_4 + R + (R_2 \| R_3)} a$$



$\boxed{III}$ effetto $(a_1)$



sono in //

$$i''_x = -a_1 \frac{R_2}{R_1 + R_2}$$ partitore di corrente

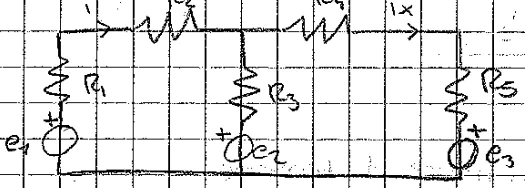
$\boxed{IV}$

le resistenze sono in // con un cc

$$i^{IV}_x = \emptyset$$

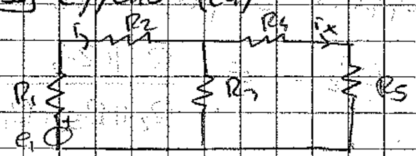
$$i_x = i'_x + i''_x + i'''_x + i^{IV}_x = \frac{e_1 - e_2 - a_1 R_2}{R_1 + R_2}$$

③



$i_x = ?$

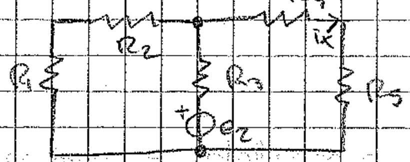
$\boxed{I}$ effetto $(e_1)$

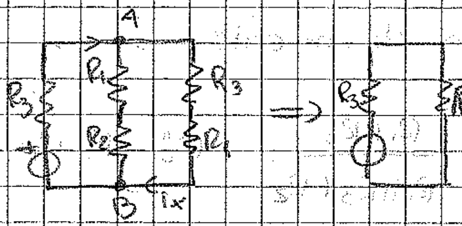


$$i = \frac{e_1}{(R_4+R_5) \parallel R_3 + R_1 + R_2}$$

$$i'_x = i \frac{R_3}{R_3 + R_4 + R_5}$$ p. corrente

$\boxed{II}$ effetto $(e_2)$



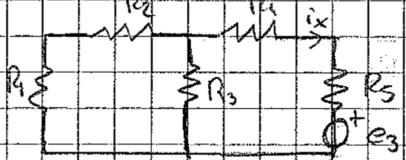


$$R_{eq} = (R_1 + R_2) \parallel (R_4 + R_5)$$

$$v_{ab} = e_2 \frac{(R_1+R_2) \parallel (R_4+R_5)}{(R_1+R_2) \parallel (R_4+R_5) + R_3}$$

$$i''_x = \frac{v_{ab}}{R_4 + R_5} = \frac{R_1 + R_2}{(R_1+R_2) \parallel (R_4+R_5) + R_3}$$

$\boxed{III}$ effetto $(e_3)$

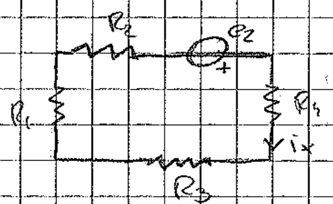


$$i'''_x = -\frac{e_3}{(R_1+R_2) \parallel R_3 + R_4 + R_5}$$

la corrente è nello stesso ramo del generatore e applico la legge di Ohm

$$i_x = i'_x + i''_x + i'''_x$$

$\boxed{II}$ effetto $(e_2)$

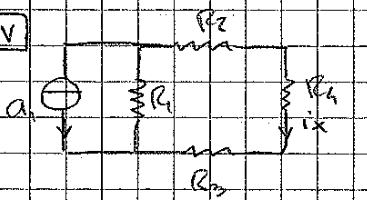



$$i''_x = \frac{e_2}{R_4+R_3+R_2+R_1}$$

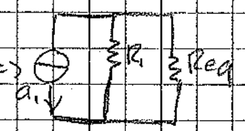
$\boxed{III}$ effetto $(e_3)$

$$i'''_x = \frac{e_3}{R_4+R_3+R_2+R_1}$$

$\boxed{IV}$

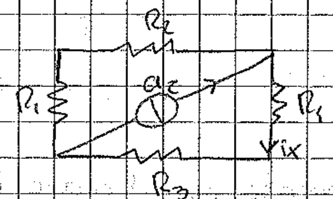



$\Rightarrow$




$$i^{IV}_x = -a_1 \frac{R_1}{R_1+R_2+R_3+R_4}$$

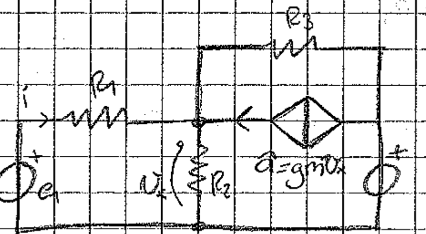
$\boxed{V}$ effetto $(a_2)$




$$i^{V}_x = a_2 \frac{\overbrace{R_1+R_2}^{\text{l'altro ramo}}}{\underbrace{R_3+R_4}_{\text{ramo che mi interessa}}+R_1+R_2}$$

$$i_x = i'_x + i''_x + i'''_x + i^{IV}_x + i^{V}_x$$

Es 22-10

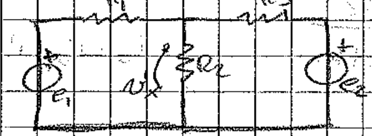



$i = ?$ $e_1 = 6V$ $e_2 = 8V$

$R_1 = R_2 = 2\,\Omega$ $R_3 = 4\,\Omega$ $gm = \frac{1}{4}\,\Omega^{-1}$

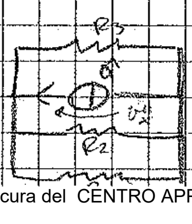
① assumo che gen. pilota sia generatore indip.

② Calcolare il pilota $\rightarrow$ calcolo $v_x$ $\rightarrow$ $\rightarrow$ effetto di $e_1$ e $e_2$ $(v'_x)$

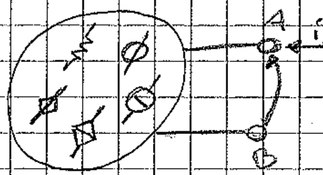


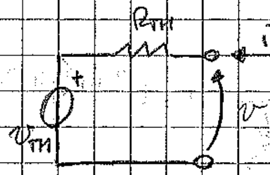

applico Millman $\rightarrow$ $$v'_x = \frac{\frac{e_1}{R_1}+\frac{e_2}{R_2}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$$

effetto di $\bar{a}$ $(v''_x)$




legge di Ohm $v''_x = (R_1 \| R_2 \| R_3)\,\bar{a}$



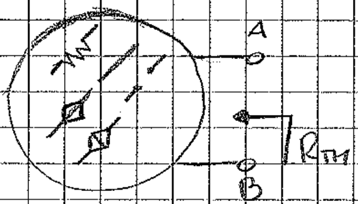



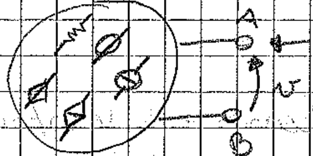

$v = v_{TH} + R_{TH}\, i$

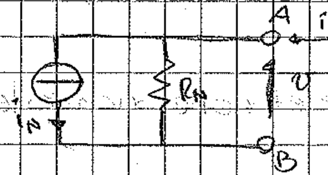
$v_{TH} = v|_{i=\emptyset}$

$R_{TH} = \frac{v}{i}\Big|_{v_{TH}=\emptyset}$

gen. INDIP. SPENTI



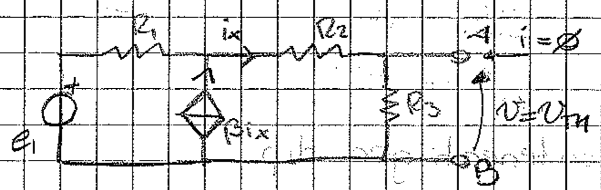

$i = i_N + G_N v$


$i_N = i|_{v=\emptyset}$

$G_N = \frac{i}{v}\Big|_{i_N=\emptyset}$

gen indip SPENTI

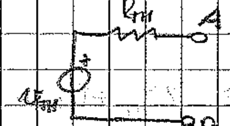
## Esempio




con gen vuoto $R_2$ serie $R_3$

$e = 6\,V \quad R_1 = 5\,\Omega \quad R_2 = 3\,\Omega \quad R_3 = 4\,\Omega$

$\beta = \frac{3}{2}$




$v_{TH} = v|_{i=\emptyset} \qquad R_{TH} = \frac{v}{i}\Big|_{\text{gen. IND.}}$

- Calcolo $v_{TH}$

Metodo del pilota $i=\emptyset$ e $R_2$ serie $R_3$

$$i_x = \underbrace{\frac{e}{R_1+R_2+R_3}}_{\text{spengo g.c.}} + \underbrace{\frac{R_1}{R_1+R_2+R_3}}_{\text{metodo partitore di corrente}} \cdot \overset{\beta i_x}{a} = \frac{e}{R_1(1-\beta)+R_2+R_3}$$

$$v_{TH} = R_3\, i_x = \frac{R_3}{R_1(1-\beta)+R_2+R_3} \cdot e$$

**b-c** • Calcolo $v_{TH}$ $(i=\phi)$


configurazione di Millman

$$v_{TH}=\frac{\frac{e}{R_1+R_2+R_3}+a}{\frac{1}{R_1+R_2+R_3}+\frac{1}{R_4}}$$

$$v_{TH}=\frac{R_4}{R_1+R_2+R_3+R_4}\,e+\frac{R_1+R_2+R_3}{R_4+(R_1+R_2+R_3)}\,a\cdot R_4$$

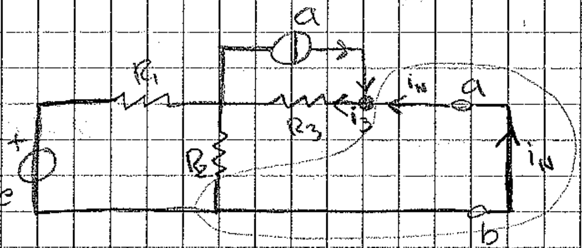
$i^*$

$[R_4 \,//\, (R_1+R_2+R_3)]\ \Omega$

• Calcolo $R_{TH}$

$R_{TH}=(R_1+R_2+R_3)\,//\,R_4$

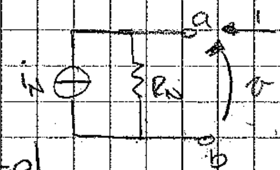
(2)




Norton a-b

$a$ — $i=0$

$v_{AB}$



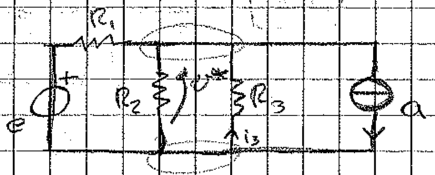

$i=i_N+G_N v$

$i_N=i|_{v=0}$

$G_N=\frac{i}{v}\Big|$ spengo gen. ind.

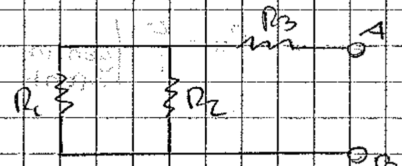
• Calcolo $i_N$ $(v=0)$

$i_N=i_3-a_3$




$$i_3=\frac{1}{R_3}\cdot\underbrace{\frac{\frac{e}{R_1}-a}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}}_{v^*}$$

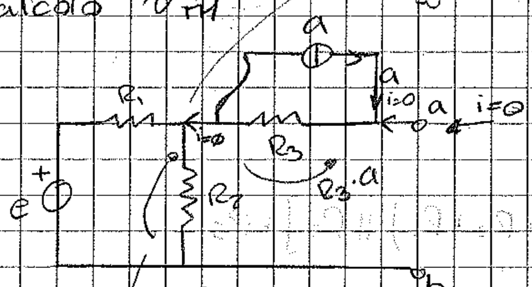
• Calcolo $R_N$




$R_N=(R_1//R_2)+R_3$

$$\boxed{v_{TH}=R_N\, i_N}$$

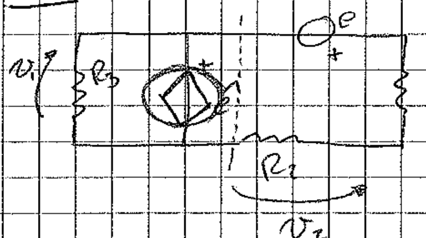
• Calcolo $v_{TH}$ — per la legge di K., i che entra è uguale a i che esce




$\frac{R_2}{R_2+R_1}\,e$

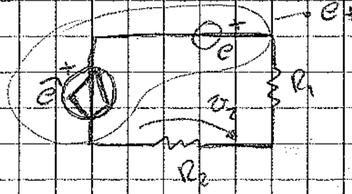
$$v_{TH}=\frac{R_2}{R_2+R_1}\,e+R_3 a$$

**Es**




$\hat{e} = \alpha v_2 \qquad v_1 = ?$

Metodo del pilota

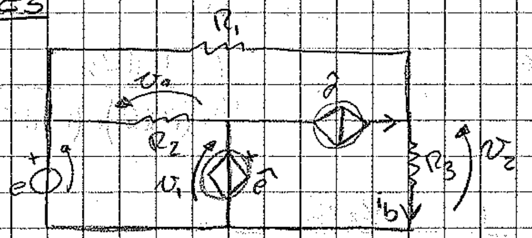



2. $v_2 = \dfrac{R_2}{R_1+R_2}(e+\hat{e})$

3. $v_2 = \dfrac{R_2}{R_1+R_2(1-\alpha)} \cdot e$

4. $v_1 = \hat{e} = \alpha v_2 = \dfrac{\alpha R_2}{R_1+R_2(1-\alpha)}\, e$

**Es**




$\hat{e} = r_m i_b \qquad \hat{a} = g_m v_0 \qquad v_1, v_2\ ?$

2. calcolo $v_0$ e $i_b$

$v_0 = e - \hat{e} = e - r_m i_b$

il contributo di $\hat{e} = \phi$ per sovr. degli effetti

$$i_b = \underbrace{\frac{e}{R_1+R_3}}_{e} + \underbrace{0}_{\hat{e}} + \underbrace{\frac{R_1}{R_1+R_3}\hat{a}}_{\hat{a}} \qquad \hat{a} \to g_m v_0$$

3.
$$\begin{cases} v_0 + r_m i_b = e \\ -\dfrac{R_1}{R_1+R_3} g_m v_0 + i_b = \dfrac{e}{R_1+R_3} \end{cases}$$

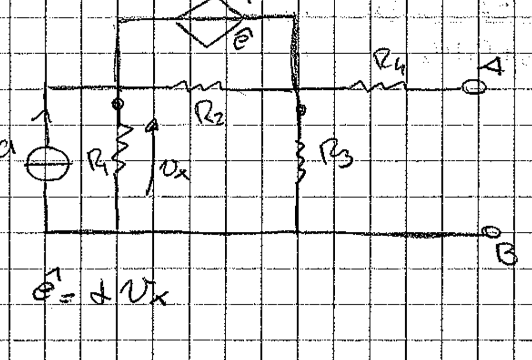
le incognite sono $v_0$ e $i_b$

$$\begin{pmatrix} 1 & r_m \\ -\dfrac{R_1 g_m}{R_1+R_3} & 1 \end{pmatrix} \begin{pmatrix} v_0 \\ i_b \end{pmatrix} = \begin{pmatrix} e \\ \dfrac{e}{R_1+R_3} \end{pmatrix} \Rightarrow$$ risolvo e ottengo $v_0, i_b$

4.
$v_1 = \hat{e} = r_m i_b$

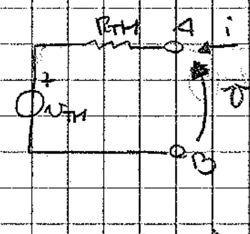
$v_2 = R_3 \cdot i_b$ legge di Ohm

**Es**




$\hat{e} = \alpha v_x$

Thev ?

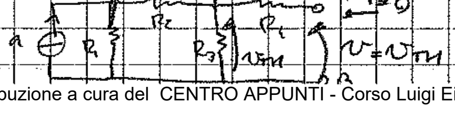



$v = v_{TH} + R_{TH} \cdot i$

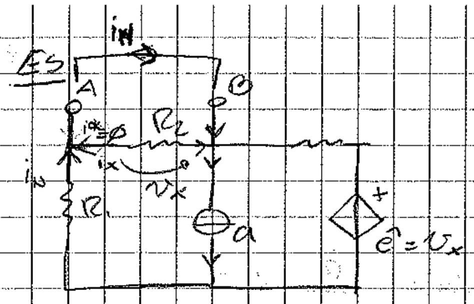
$v_{TH} = v|_{i=\phi}$

$R_{TH} = \dfrac{v}{i}\Big|_{v_{TH}=\phi}$

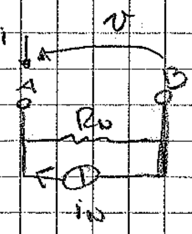
Calcolo di $v_{TH}$ ($i=0$)



Es




Norton




$i = i_N + G_N v$

$i_N = i \big|_{v=\varnothing}$

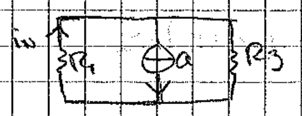
$G_N = \dfrac{i}{v}\Big|_{\text{spengo gen. indip.}}$

° Calcolo $i_N$ $(v = \varnothing)$

Metodo del pilota

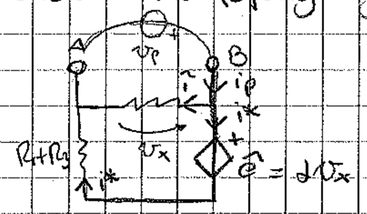
2. $v_x = \varnothing$

4. calcolo $i_N$




$i_N = \dfrac{R_3}{R_3 + R_1}\, a$

Calcolo $R_N$ (spengo gen. indip.)




$R_N = \dfrac{v_p}{i_p}$

metodo del pilota

2. $v_x = v_p$

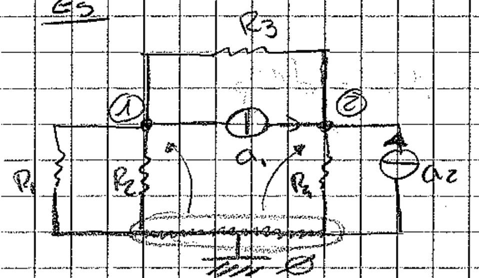
4. calcolo $i_p$

$i_p = i^* + \hat{i} \qquad \hat{i} = \dfrac{v_x}{R_2} = \dfrac{v_p}{R_2} \qquad i^* = \dfrac{v^*}{R_1+R_3} = \dfrac{(1-\alpha)}{R_1+R_3}\, v_p$

$i_p = \left[\dfrac{1}{R_2} + \dfrac{(1-\alpha)}{R_1+R_3}\right] v_p$

$R_N = \dfrac{v_p}{i_p} = \dfrac{1}{\dfrac{1}{R_2} + \dfrac{1-\alpha}{R_1+R_3}}$

Es



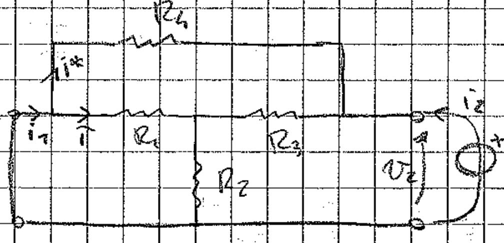

$\underline{\underline{G}} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \underline{A}$

$\begin{pmatrix} G_1+G_2+G_3 & -G_3 \\ -G_3 & G_3+G_4 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} -a_1 \\ a_1 + a_2 \end{pmatrix}$

$$G_{11} = \frac{i_1}{v_1}\Big|_{v_2=0} = \frac{1}{R_4 \parallel [R_1 + (R_2 \parallel R_3)]}$$

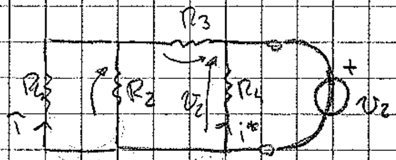
$$G_{22} = \frac{i_2}{v_2}\Big|_{v_1=0} = \frac{1}{R_4 \parallel [R_3 + (R_1 \parallel R_2)]}$$

$$G_{12} = \frac{i_1}{v_2}\Big|_{v_1=0}$$




$$i_1 = i^* + \tilde{i}$$

$$i^* = -\frac{v_2}{R_4}$$



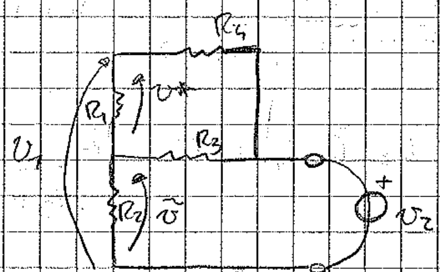

$$\tilde{i} = -\frac{1}{R_1} \cdot \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_3} \cdot v_2$$

$$G_{12} = -\left(\frac{1}{R_4} + \frac{1}{(R_1 \parallel R_2) + R_3} \cdot \frac{R_2}{R_1 + R_2}\right)$$

$$H_{11} = \frac{v_1}{i_1}\Big|_{v_2=0} = \frac{1}{G_{11}} = R_4 \parallel [R_1 + (R_2 \parallel R_3)]$$

$$H_{22} = \frac{i_2}{v_2}\Big|_{i_1=0} = \frac{1}{[(R_1+R_4) \parallel R_3] + R_2} = \frac{1}{R_{22}}$$

$$H_{12} = \frac{v_1}{v_2}\Big|_{i_1=0}$$




$$v_1 = \tilde{v} + v^*$$

$$\tilde{v} = \frac{R_2 \cdot v_2}{R_2 + [R_3 \parallel (R_1 + R_4)]}$$

$$v^* = \frac{R_3 \parallel (R_1 + R_4)}{[R_3 \parallel (R_1 + R_4)] + R_2} \cdot \frac{R_1}{R_1 + R_4}\, v_2$$

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