Reactionless drive with conservation of momentum

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This paper describes an electromagnetic drive that, while conserving momentum and energy, does not obey the action/reaction principle and results in translation through space with no external forces. This drive consists of light moving from vacuum into, and then out of, a dielectric, returning to the starting point through vacuum. While all that is required for the drive to work is for the photon momentum in a dielectric to differ from its vacuum value, the system is analyzed in terms of a recently published momentum model (the MP model) that resolves the Abraham-Minkowski controversy. This model shows the velocity of the center of energy changes from zero, when photon in vacuum, to a non-zero value when the photon propagates through the dielectric, then back to zero when the photon reenters vacuum. It is commonly assumed that such a velocity change is forbidden by conservation of momentum (uniform velocity of center of energy). This belief is incorrect, as this paper shows the such a velocity change is required when reaction mass is temporarily reduced, as required by the MP model.

Keywords: Electrodynamics; Abraham-Minkowski controversy, reactionless drive, conservation of momentum

Introduction

The idea of a reactionless drive for an isolated system, which violates conservation of momentum, is treated by most physicists on par with perpetual motion. Recently, experimental results, along with highly speculative theoretical justification, in support of such a drive have entered the literature[1]. Recent testing revealed that the thrust was an experimental artifact[2].

A reactionless drive need not violate momentum conservation however. Such a drive would be unique as conservation of momentum requires that the net force, and hence net acceleration, is 0. Given such a limitation, it would seem impossible for an object to translate arbitrary distances through space without ejection of mass or interaction with external fields. Such a translation however is shown to be required in order to conserve momentum. It is

tempting to rule out the possibility of such translation first as a violation of Newton's third law. Such a law is well known not to apply in the presence of fields. A more sophisticated argument against the possibility of a reactionless drive stems from what is often, but mistakenly, assumed to be a corollary to conservation of momentum; the Center of Mass theorem which states the velocity of the center of energy is uniform for an isolated system.

This paper will show by direct calculation that a non-zero displacement for an isolated system occurs when none would be expected. The velocity of the center of energy is then calculated, using a new model for light in a dielectric. This velocity changes from, for a system initially at rest, from 0 to a non-zero value and then back to 0. Finally, the Einstein covariance condition will be used to show explicitly that the center of energy velocity is not uniform and used to explain uniform speed of the center of mass does not hold in certain situations even though momentum is conserved.

The first example of translation without external forces was by Wisdom[3] who showed translation in curved space-time through cyclic shape changes, called "Swimming in Spacetime". He found that for a one-meter sized object in the space-time curvature near Earth's surface the displacement is of the order of 10^{-23}m for one cycle.

In this paper I introduce an example of a reactionless drive in flat space-time that takes advantage of the change in momentum a photon experiences on entering a dielectric.
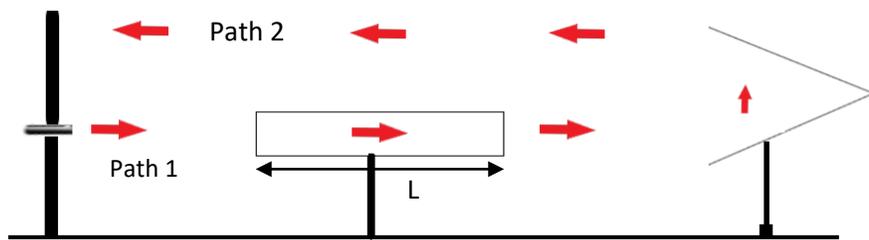

Figure 1. A photon travels from a laser, in vacuum, through a dielectric (black rectangle) of length L, back into vacuum, then into a retroreflector which returns the photon to the left where it is absorbed. The dielectric, the laser, and reflector as well as all mounts are mounted to the isolated platform of length D.

Figure 1 shows a platform of length D with a laser at the left end and a retroreflector at the other end that reflects the photon back to the left where it is absorbed. A dielectric material (the rectangular block which I refer to as glass) partially fills the space as shown. The dielectric, of length L, is non-reflecting with an anti-reflection coating (not shown), non-absorbing, non-dispersive with a real refractive index n and mass M. The platform, which is isolated from the rest of the universe, has total mass M_P which is the mass of all the components including the laser, dielectric, retroreflector and mounts. This platform mass does not include the mass-energy of light.

The closed path (the photon could be reflected downward and absorbed arbitrarily close to the source) the light takes can be considered as consisting of two paths. In path 1 (photon traveling left to right), the photon travels a distance in vacuum, enters and travels through a dielectric, before exiting the dielectric and reaching the retroreflector. In path 2 the photon travels only through vacuum back to the left side where it is absorbed.

The photon momentum changes when it enters the dielectric from the vacuum, details of how the momentum changes are discussed in subsequent sections. Momentum conservation requires the dielectric momentum change as well, as $\vec{p}_{dielectric} = \vec{p}_{\gamma_{vacuum}} - \vec{p}_{\gamma_{dielectric}}$. The dielectric moves as a reaction to the momentum change which occurs as the photon travels through the vacuum-dielectric interface. As the photon continues to travel through the dielectric, with no change in energy or momentum, the dielectric continues to have this constant momentum value. When the photon exits the dielectric, the photon returns to its initial momentum state and the dielectric returns to its initial state of zero momentum. The net force is 0, forces at the interfaces are equal and directed in opposite directions, but the two forces do occur at *different times.* It is during this difference in time that the dielectric translates through space. A retroreflector (or an absorber and another laser) returns the photon to the left-hand side, except the return trip is through vacuum only.

Clearly, if the round trip took place only in a vacuum, there would be no net translation in space. The platform initially moves to the left, due to emission of the photon, then stops when the photon is reflected or absorbed. Launching an identical photon from the right to the left causes the platform to move the same distance to the right, returning the platform to its initial position.

While the net displacement of the platform for a photon making a round trip will be calculated in detail in Appendix A, it is illustrative to look at the calculation to first order to see the plausibility of the argument. I suppress vector signs and define motion to the left as negative and motion to the right as positive. The laser, emitting light, gives momentum, $-p_1$, to the platform for a duration, $t \approx \frac{(D-L)}{c}$ while the light in the glass gives the platform an additional momentum, p_{glass}, which acts for a time $t \approx \frac{nL}{c}$. The lights return trip gives the platform momentum p_1 for time, $t \approx \frac{D}{c}$.

The total displacement, due to round trip of the light, is

$$d_{net} \approx \frac{L}{M_P c}\left(n\left(p_{glass} - p_1\right) + p_1\right). \tag{1}$$

If p_γ is described with a "Minkowski momentum", $p_{\gamma_M} = \frac{n\hbar\omega}{c}$, then the net displacement is non-zero, in apparent contradiction to the center of mass theorem (C.M. theorem). If p_γ is described with a "Abraham momentum", $p_{\gamma_A} = \frac{\hbar\omega}{nc}$,the displacement is 0, but *only to first order*. More detailed displacement calculations show a non-zero displacement for even the Abraham momentum model[4].

I briefly review the issue of momentum of light in glass before discussing a recently published model of light entering a dielectric which provides a consistent explanation of the non-zero displacement that is consistent with momentum conservation

A correct expression for the momentum of the photon in the dielectric is not obvious. This situation is commonly called the "Abraham-Minkowski Controversy". There is extensive literature on this issue over the last 100 years, summarized in a review article[5] by Pfeifer et al. Abraham[6] and Minkowski[7,8] provide differing definitions for the electromagnetic momentum-

energy tensor of light in a dielectric. These differing tensors result in different predictions for the momentum density;

$$\vec{g}_{\text{Abraham}} = \vec{E} \times \frac{\vec{H}}{c^2}, \vec{g}_{\text{Minkowski}} = \vec{D} \times \vec{B} . \tag{2}$$

These differing momentum densities then give contradictory predictions for the momentum of a photon in a dielectric of index n;

$$\left| \vec{p}_{\gamma \text{Abraham}} \right| = \frac{\hbar \omega}{nc}, \left| \vec{p}_{\gamma \text{Minkowski}} \right| = \frac{n \hbar \omega}{c} . \tag{3}$$

While both momenta equations agree in a vacuum, they predict very different behavior in a material. Both momenta expressions have compelling theoretical and experimental support which is summarized in the aforementioned review article. Either of these models, or any reasonable model for photon momentum in a dielectric which differs from the vacuum value, predicts the platform describe previously will have a non-zero, reactionless, displacement.

While these models were the leading candidates, a new description, the Mass Polariton quasiparticle model, (MP model)[9], was recently published. Neither the Abraham nor Minkowski model, or any other description of momentum inside the dielectric, satisfies the covariance condition of special relativity which requires the photon inside the dielectric to have a mass.

The fact that photon momenta in a dielectric that differs from the vacuum value is sufficient to create a reactionless drive. However, as both the Abraham and Minkowski models fail to satisfy this fundamental relationship, I do not analyze the drive with these models in this paper. It is only the MP model which provides an explanation for how the Center of Mass theorem is violated.

Section 1: MP Model

Both Abraham and Minkowski models violate the covariance principle,

$$E^2 - (pc)^2 = (mc^2)^2 . \tag{4}$$

Since photons have no mass, no model that only considers the field term can be correct. In addition to a field term, a term with mass, from the stress tensor of the material, must be

included. The "splitting" of momentum between the field and material terms is not arbitrary, as commonly thought. The MP model posits that the photon/dielectric forms a mass-polariton quasi-particle (MP) that satisfies the covariance principle. The authors point out that their definition of mass-polariton

We is distinct from the conventional usage of the term as it does not involve a close resonance with internal states of the system.

Henceforth, I adopt the terminology of the MP authors. I suppress vector signs on all momenta, with positive values meaning momentum to the right and negative values meaning momentum to the left, consistent with Figure 1. The vacuum photon, with energy $E = \hbar\omega$ and momentum $p_{\gamma_{vacuum}} = \dfrac{\hbar\omega}{c}$, on entering the dielectric creates an MP, which consists of a field component and a mass density wave (MDW), propagating together at speed $v = \dfrac{c}{n}$ inside the dielectric. In the lab frame, the momentum of the MP (field and matter wave) is

$$p_{MP} = \frac{n\hbar\omega}{c},$$ (5)

same as the Minkowski description, and the energy of the MP is

$$E_{MP} = n^2\hbar\omega = \gamma m_0 c^2,$$ (6)

Where m_0 is the effective mass of the MP. The momentum of the MP is greater than the momentum of the photon in vacuum, causing the dielectric to move in the opposite direction to propagation direction of the MP, conserving momentum; to the left in Figure 1. The MP also has n^2 more energy than the vacuum photon. This energy increase comes from a *mass loss* of the dielectric,

$$\partial m = \left(n^2 - 1\right)\frac{\hbar\omega}{c^2}.$$ (7)

It is recognition of this mass loss that allows consistent explanation of why a reactionless drive is possible while conserving momentum in my system. As pointed out by the MP authors, the reaction mass of the dielectric is reduced to

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$$M_R = (M - \partial m), \tag{8}$$

where M is the mass of the dielectric before the photon enters, or after it exits, the dielectric.

While the MP is the real physical entity, momentum of the field and MDW can be separately calculated. The field momentum is

$$p_{field_{MP}} = \frac{\hbar\omega}{nc} \tag{9}$$

and the MDW momentum is

$$p_{MDW_{MP}} = \left(n - \frac{1}{n}\right)\frac{\hbar\omega}{c} \tag{10}$$

in the Lab frame. The field momentum is given by the same expression as the Abraham momentum and the sum of the momenta is the Minkowski momentum.

The mass of the MDW, which is required in Section 2 but is not calculated in the MP paper, is calculated from the covariance condition, $(M_{MDW}c^2)^2 = E_{MDW}{}^2 - (p_{MDW}c)^2$, yielding

$$M_{MDW} = (n^2 - 1)\sqrt{\frac{n^2 - 1}{n^2}}\left(\frac{\hbar\omega}{c^2}\right). \tag{11}$$

Since the MP has the same momentum as described by Minkowski model one might assume that the identical result would be obtained as assuming a Minkowski momentum for the photon, but that is incorrect. The "recoil mass", the mass of the dielectric that moves as a result of the momentum change as the photon enters the dielectric, is smaller than the mass of the dielectric, M, before the photon enters, leaving $M_R = M - \partial m$ as the reaction mass of the dielectric that moves to the left (see Figure 3 of reference 9). In addition to this mass loss, there are two other things different between the Minkowski and the MP model. (1) The momentum of the field term *increases* when the MP reaches the end of the dielectric, i.e. when the photon reenters the vacuum. (2) The MDW remains inside the dielectric until it reaches the end of the dielectric and transfers all of its momentum to the dielectric.

As always, the momentum change when the photon enters the dielectric is equal and opposite to the momentum change when the photon leaves the dielectric. The dielectric comes to rest once the photon exits and the sum of the forces is 0.

Section 2 of this paper provides two major results. The first result is the velocity of the center of energy is calculated directly from the MP model to show this velocity is not uniform when the photon enters the dielectric, in contradiction to the C.M. theorem. The second result of this section uses the change of mass into energy as predicted by the MP model, to extend the C.M. theorem, showing the isolated system does have a change in momentum.

Section 3 considers another surprising consequence of the MP model, that the MP transiting the dielectric *must* lose energy, in contradiction to the well-known belief that the energy of a photon in a dielectric is identical to its vacuum energy. In Appendix A, I follow the photon on its round-trip path and use momentum conservation to determine the momentum of all the entities in this problem along with the net displacement of the platform using the MP model extending the calculation that lead to the non-zero displacement in Equation 1.   Appendix B uses the mass loss predicted by the MP model to provide a qualitative picture of how the center of mass moves in the lab frame.

Section 2: Velocity of Center of Energy

The velocity of the center of energy is given by

$$V_{CE} = \frac{\sum_i E_i v_i}{\sum_i E_i} .$$ (12)

The numerator is proportional to the total momentum of the system while the denominator is its total energy.

Working in the lab frame with the platform initially at rest, when the laser first emits the photon, the platform recoils to the left as the photon travels to the right. The velocity of the center of energy is

$$V_{CE} = \frac{\sum_i E_i v_i}{\sum_i E_i} = \frac{\hbar \omega c + M_P c^2 V_{R1}}{\hbar \omega + M_P c^2}. \tag{13}$$

The recoil velocity of the platform is $V_{R1} = \frac{-\hbar\omega}{M_P c}$. Thus, the center of energy velocity is 0, as expected.

    The situation changes dramatically when the photon enters the dielectric and creates an MP;

$$V_{CE_{dielectric}} = \frac{\sum_i E_i v_i}{\sum_i E_i} = \frac{\gamma m_0 c^2 \left(\dfrac{\dfrac{c}{n} + V_{R1} + V_{R2}}{1 + \dfrac{(V_{R1} + V_{R2})}{nc}} \right) + (M_P - \partial m) c^2 (V_{R1} + V_{R2})}{\gamma m_0 c^2 + (M_P - \partial m) c^2}. \tag{14}$$

In the rest frame of the dielectric the MP travels at c/n. This speed is reduced in the Lab frame and given by the relativistic velocity addition law, recall V_{R1} and V_{R2} are negative. To first order, the velocity of the MP in the dielectric in the Lab frame is

$$V_{MP} = \frac{c}{n} + (V_{R1} + V_{R2}) \left(1 - \frac{1}{n^2} \right). \tag{15}$$

Before the photon enters the dielectric, the platform, and hence the dielectric, has velocity V_{R1}. When the photon creates the MP, momentum is conserved and the dielectric velocity becomes $(V_{R1} + V_{R2})$. As previously shown, both these velocities are in the same direction, to the left. The second velocity change is $V_{R2} = (1-n) \dfrac{\hbar\omega}{(M_P - \partial m) c}$. The MP parameters are

$$M_{MDW} = (n^2 - 1) \sqrt{\frac{n^2 - 1}{n^2}} \left(\frac{\hbar\omega}{c^2} \right), \partial m = (n^2 - 1) \frac{\hbar\omega}{c^2}. \tag{16}$$

Making these substitutions, the center of energy velocity in the lab frame when the photon enters the dielectric is

$$V_{CE_{dielectric}} = \frac{\gamma m_0 c^2 \left[\dfrac{c}{n} - \left(\dfrac{\hbar\omega}{M_P c} + \dfrac{(n-1)\hbar\omega}{(M_P - \partial m)c} \right)\left(1 - \dfrac{1}{n^2} \right) \right] + (M_P - \partial m)c^2 \left(\dfrac{(1-n)\hbar\omega}{(M_P - \partial m)c} - \dfrac{\hbar\omega}{M_P c} \right)}{\gamma m_0 c^2 + (M_P - \partial m)c^2}.$$

(17)

Cancelations in the numerator occur, leaving a non-zero result,

$$V_{CE_{dielectric}} = \frac{-\dfrac{(\hbar\omega)^2}{c}\left[\dfrac{(n^2 - 1)(n-1)}{(M_P - \partial m)} \right]}{\gamma m_0 c^2 + (M_P - \partial m)c^2}.$$

(18)

This velocity is negative, hence to the left. The velocity of the center of energy changes from 0, before photon enters the dielectric and returns to 0 when the photon exits the dielectric, in contradiction to the C.M. theorem. A non-zero velocity is also found when the calculation is done adding all velocities relativistically.

This change in velocity of the center of energy from 0 to a non-zero value is surprising. It is widely, but incorrectly, held that the constancy of the center of energy velocity is a direct consequence of conservation of momentum. This work, which has conserved momentum throughout, shows this change in the center of energy velocity directly.

We have what appears to be a serious contradiction; conservation of momentum, as demonstrated in this paper, requires a change in velocity while conservation of momentum is commonly used to show that the velocity cannot change. The model by Partanen et al provides a clear explanation of how the change in velocity comes about and why the usual derivation of this velocity from momentum conservation is incomplete.

Consider the system with the photon in vacuum as described by Eq. (12). Working in the center of energy frame of the system, the system has zero momentum and is described by $E^2 = M_P^2 c^4$. When the photon enters the dielectric, the mass of the dielectric, and hence of the

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platform is reduced, $(M_P - \partial m)$. Working in the *same frame of reference,* energy is conserved, giving

$$E^2 = M_P{}^2 c^4 = (pc)^2 + (M_P - \partial m + M_{MDW})^2 c^4. \tag{19}$$

Substituting in the masses from Eq (16) into Eq (19) yields a non-zero momentum for the center of mass frame;

$$p^2 = \hbar\omega(n^2 - 1)\left(1 + \sqrt{1 - \frac{1}{n^2}}\right)\left[2M_P - \frac{\hbar\omega}{c^2}(n^2 - 1)\left(1 + \sqrt{1 - \frac{1}{n^2}}\right)\right]. \tag{20}$$

It is the conversion of mass into energy that requires that the momentum of the system changes, thus giving rise to the reactionless drive. The conversion of mass into energy is ignored in the usual derivations of the "constancy" of the velocity of the center of energy.

Section 3: Consequence of energy loss.

As shown, conservation of momentum and fundamental electrodynamics allows creation of a reactionless drive. The assumptions used are that standard ones; energy of a photon is constant throughout and the energy and momentum of the photon that exits the dielectric is identical to the momentum and energy of the photon that entered. What is the consequence if this assumption is not true, not due to imperfections of the material, but due to a fundamental energy loss?

The MP model allows analysis of the situation in terms of electrodynamics of continuous media, which not only gives the same results as the MP model, it allows computer simulations of the MP pulse and the reaction of the material as the MP travels through it.

The MP authors simulated a diamond crystal cube (100mm on a side) subjected to a titanium-sapphire laser pulse ($\lambda_0 = 800nm; \hbar\omega = 1.55eV$ and total energy 5mJ) traveling as a Gaussian wave packet. As the MP propagates through the crystal, the MDW displaces atoms. In

the bulk the displacement is of the order of 10^{-17}m while the displacement at the interfaces is approximately 100 times larger although the authors warn that calculations near the interface should be considered only approximate. These distortions relax elastically and thus dissipate energy in the process with the dominant energy loss resulting from distortion at the interface. The energy loss is very small, depending on the length scale chosen for the interface (interface length of 250μm, energy loss 10^{-14}eV; interface length 333nm, the wavelength in diamond, energy loss 10^{-11}eV).

Through the interface, and much lesser extent through the bulk of the material, the energy of the MDW decreases by an amount ΔE with a commensurate decrease in momentum of $\Delta p = \dfrac{\Delta E}{c}$. As this is a decrease in momentum of the MP, the momentum of the diamond must increase the same amount, in the direction of propagation of the MP and opposite the much larger momentum change of the diamond due to the photon entering the diamond. The energy loss mechanism thus decreases the momentum of the diamond to the left, but by a negligible amount.

When the lower energy MP reaches the interface and a vacuum photon created, the momentum change will be slightly smaller than that shown in cell labelled (2) in the rightmost column of Table 2. This small change cancels out the small change in momentum at the entering interface due to energy loss and once more the material comes to rest when the photon returns to vacuum. Even with a fundamental energy loss mechanism due to the creation of the MDW, the reactionless effect remains.

Conclusion:

This paper shows that a reactionless drive is predicted in flat space-time using classical electrodynamics. The recent resolution to the Abraham-Minkowski controversy provides a microscopic model to analyze the momenta transfers and allows calculation of the velocity of the center of energy which changes from 0 to a non-zero value and back to 0 as the photon "transits" the dielectric. The well-known "rule" of the constancy of the velocity of the center of energy as a result of momentum conservation is shown to not be true when some of the mass of the object is converted into energy. In this case the momentum is required to change. This change in mass is

distinct from the well understood variable mass system such as a conventional rocket where reaction mass is expended.

Appendix A: Direct calculation of displacement.

In the description that follows the dielectric has length L, real index n, and mass M. The platform is of length D, such that D>L, and total mass M_P. The platform, except for the dielectric, is in vacuum. Table 1 summarizes the momenta of the various quantities in three regimes; (1) vacuum, before entering the dielectric, (2) inside the dielectric, and (3) in vacuum after exiting the dielectric. X's represent the item not existing. During the transition from dielectric to vacuum the MDW ceases to exist, transferring all of its momentum to the dielectric. When the photon reenters the vacuum the momentum of the dielectric returns to 0. This calculation is shown in Table 2 which examines the momentum transfer from dielectric to vacuum in more detail.

	1 (vacuum)	2(dielectric)	3 (vacuum)
p_γ	$\dfrac{\hbar\omega}{c}$	X	$\dfrac{\hbar\omega}{c}$
$p_{field_{MP}}$	X	$\dfrac{\hbar\omega}{nc}$	X
$p_{MDW_{MP}}$	X	$\left(n-\dfrac{1}{n}\right)\dfrac{\hbar\omega}{c}$	X
$p_{MP} = p_{field_{MP}} + p_{MDW_{MP}}$	X	$\dfrac{n\hbar\omega}{c}$	X
$p_{dielectric}$	0	$-(n-1)\dfrac{\hbar\omega}{c}$	0

Table 1. Momenta of the various quantities as the photon travels from vacuum (1) into dielectric (2) and back into vacuum (3). An X indicates that the quantity does not exist during this state. Positive quantities represent momenta to the right while negative values represent momenta to the left. The dielectric is given a momentum to the left when the photon enters it, returning to 0 when the photon exits.

The rightmost column in Table 2 gives the momentum transfer to the dielectric as the photon reenters the vacuum from the dielectric. The first merged cell in this column shows that the field momentum increases from its Abraham value to vacuum value, giving a momentum to the dielectric to the left (negative). The second cell gives the momentum transfer to the dielectric when the MDW interacts with the dielectric/vacuum interface. The sum of these momentum transfers is given in the third cell. This value is equal and opposite the momentum given to the dielectric when the photon first enters the dielectric. The final momentum of the dielectric is 0.

	2(dielectric)	3 (vacuum)	MP transition to vacuum
p_γ	X	$\dfrac{\hbar\omega}{c}$	$\Delta p_{dielectric_{\text{field}\to\text{vacvuum}}} = -\left(1-\dfrac{1}{n}\right)\dfrac{\hbar\omega}{c}$ (1)
$p_{field_{MP}}$	$\dfrac{\hbar\omega}{nc}$	X	
$p_{MDW_{MP}}$	$\left(n-\dfrac{1}{n}\right)\dfrac{\hbar\omega}{c}$	X	$\Delta p_{dielectric_{MDW\to 0}} = \left(n-\dfrac{1}{n}\right)\dfrac{\hbar\omega}{c}$ (2)
			$\Delta p_{\text{dielectric}_{MP\to vacuum}} = \dfrac{\hbar\omega}{c}(n-1)$ (3)
$p_{dielectric}$	$-(n-1)\dfrac{\hbar\omega}{c}$	0	$p_{\text{dielectric}_{vacuum\to dielectric}} + p_{\text{dielectric}_{dielctric\to vacuum}} = 0$ (4)

Table 2. Extension of Table 1 focusing on photon transition from dielectric to vacuum. The cell marked (1) in the last column gives the momentum transfer to the dielectric as the field term transitions to a vacuum photon. The cell marked (2) gives the momentum transferred to the dielectric when the MDW reaches the interface and vanishes. The cell marked (3) is the sum of the momentum transfer to the dielectric from the previous two cells. The final cell, marked (4), gives the total momentum of the dielectric after the photon reenters the vacuum, which is 0.

The momentum applied to the dielectric, and hence the platform to which it is mounted, causes the platform to have a net displacement. I calculate the total displacement of the platform for when the photon makes a round trip (path 1 + path 2).

Table 3 shows the momenta and displacements for the platform. For clarity I shall do the calculation for a single photon. The mass of the platform, M_P, is the mass of all components of the platform, the laser, retroreflector, the dielectric and all supports, but not energy of the photon.

	1 & 3 (vacuum)	2 (dielectric)
p_{Platform}	Path 1: $\dfrac{-\hbar\omega}{c}$ Path 2: $\dfrac{\hbar\omega}{c}$	$-(n-1)\dfrac{\hbar\omega}{c}$
V_{Platform}	Path 1: $V_{1_1} = V_{3_1} = \dfrac{-\hbar\omega}{M_P c}$ Path 2: $V_{1_2} = \dfrac{\hbar\omega}{M_P c}$	$V_{2_1} = V_{1_1} - \dfrac{(n-1)\hbar\omega}{(Mp-\partial m)c}$ $= \dfrac{-\hbar\omega}{c}\left(\dfrac{1}{M_P} + \dfrac{(n-1)}{(Mp-\partial m)}\right)$
Duration	Path 1: $\left(\dfrac{D-L}{c-V_{1_1}}\right)$ Path 2: $\left(\dfrac{D}{c+V_{1_2}}\right)$	$\dfrac{L}{\dfrac{c}{n}+(V_{1_1}+V_{2_2})}\Big/{1+\dfrac{(V_{1_1}+V_{2_2})}{nc}} \approx$ $\dfrac{Ln}{c}\left(1 - \dfrac{n(V_{1_1}+V_{2_2})}{c}\left(1-\dfrac{1}{n^2}\right)\right)$
Displacement	Path 1: $-\dfrac{\hbar\omega}{M_P c^2}(D-L)\left(1-\dfrac{\hbar\omega}{M_P c^2}\right)$ Path 2: $\dfrac{\hbar\omega}{M_P c^2}D\left(1-\dfrac{\hbar\omega}{M_P c^2}\right)$	$\left(\dfrac{-\hbar\omega n L}{c^2} + L\left(\dfrac{\hbar\omega}{c^2}\right)^2(n^2-1)\right)\left(\dfrac{1}{M_P}+\dfrac{(n-1)}{(Mp-\partial m)}\right)$
Total Displacement	$-\dfrac{\hbar\omega(n-1)L}{c^2}\left(\dfrac{1}{Mp}+\dfrac{n}{(Mp-\partial m)}\right) -$ $\left(\dfrac{\hbar\omega}{c^2}\right)^2 L\left[\dfrac{1}{M_P{}^2}+(n^2-1)\left(\dfrac{1}{Mp}+\dfrac{n}{(Mp-\partial m)}\right)^2\right]$ Path 1 and 2	

Table 3. The momentum of platform, the velocity of the platform, the duration of the momentum transfer, and the displacement of the platform is given for Path 1 and Path 2 (columns 1 & 2, where Column 1 is for the vacuum path of the photon and Column 2 for the path through the dielectric). Note that the mass of the dielectric is reduced when the MDW is formed. The bottom row of the table gives the total displacement for Path 1 + Path 2 (round trip).

Table 3 summarizes the calculations that give the velocity and displacement of the platform. As the photon travels Path 1 (left to right) the platform moves to the left. When the photon exits the laser, the platform initially moves at velocity v_{1_1}. When the photon enters the dielectric, the velocity is changed by v_{2_1}, giving the platform a velocity $v_1 = v_{1_1} + v_{2_1}$. These velocities are in the same direction, towards the left. The sub-subscript represents the path.

Since the platform cannot be infinitely rigid the platform will not move as a single unit with a single velocity. I use the velocities as an "effective" velocity that allows calculation of the displacement after the platform has "settled down" on a time scale set by $\dfrac{D}{v_{sound}}$;

The duration of the momentum transfer is corrected for the small velocity of the platform while the photon is traveling in vacuum. In each case the photon moves at speed c and the velocity in the denominator is the closing velocity. The velocity of the MP in the dielectric, which is c/n in the dielectric frame, is calculated in the Lab frame use the relativistic velocity addition formula. As v/c <<1, the velocity of the MP is calculated to first order, giving the Fresnel formula, $v \approx \dfrac{c}{n} + \left(v_{1_1} + v_{2_1}\right)\left(1 - \dfrac{1}{n^2}\right)$. Recall that $v_1 \& v_2 \approx \dfrac{\hbar\omega}{M_P c} << 1$. In the row labelled displacement, the product of the velocity and duration is given to 2nd order in $\left(\dfrac{\hbar\omega}{c^2}\right)$. As the photon travels right to left along Path 2, making a complete round trip, the platform is displaced to the right. This displacement is *smaller* than the displacement that occurs during Path1, giving rise to a *net displacement to the left* with no reaction mass. The total displacement is independent of the length of the platform and linear with the length of the dielectric, it is given in the last cell in Table 3, to 2nd order in $\left(\dfrac{\hbar\omega}{c^2}\right)$. The displacements were also calculated numerically with no approximations[10] and a non-zero displacement was found.

In simplest terms, this reactionless drive occurs because the momentum change when the photon enters the dielectric produces an "extra" velocity. This extra velocity, $-\dfrac{(n-1)\hbar\omega}{(Mp - \partial m)c}$,

occurs over a longer time, $\left(\dfrac{nL}{c}\right)$, than an equivalent path in vacuum, giving an extra-large

displacement, $-\dfrac{(n-1)\hbar\omega}{(Mp-\partial m)c}\left(\dfrac{nL}{c}\right)$. The return path, without this extra velocity and a shorter

time, $\left(\dfrac{L}{c}\right)$, cannot cancel out the extra displacement due to the dielectric. This picture is

supported by analysis of the velocity of the center of energy in the next section.

Appendix B: Qualitative explanation

The movement of the C.M. of this isolated system with momentum conservation can be understood qualitatively as follows.

The MP model has the glass losing mass, $\partial m = \left(n^2 - 1\right)\dfrac{\hbar\omega}{c^2}$, when the photon enters the glass.

This "mass loss bubble" travels through the glass at $\dfrac{c}{n}$. When reaching the rightmost end of the

glass, energy of the MP is converted back to mass, restoring full mass to the glass, and the energy to the photon which travels on to the right.

The "mass loss bubble" provides an easy way to understand how the center of mass moves during a photon transit.

The change in the location of the center of mass, of the platform in the platform frame, calculated from the left edge of the platform, is

$$\Delta CM = \frac{-\partial m}{\left(M_P - \partial m\right)} x, \tag{1.1}$$

where $\partial m = 0$ when light is outside of the glass, and $\partial m = \left(n^2 - 1\right)\dfrac{\hbar\omega}{c^2}$ when inside the glass and

x is measured from the left edge of the platform. The effect of the moving "mass loss bubble" on the shift of the C.M. is shown schematically in Figure 2.
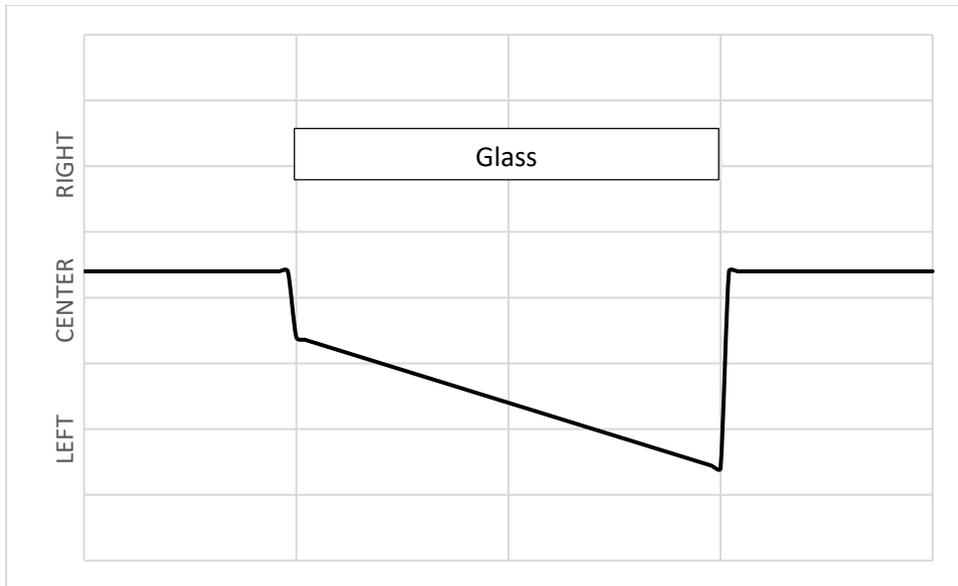
Figure 2. Schematic representation of change in center of mass of platform (arbitrary units) as light traverses the glass, as measured in the Platform frame. The C.M. changes only when the light is inside the glass due to the "mass loss bubble" moving left to right. The shift at the boundary depends on details of how the light transitions from vacuum to the bulk glass.

While the C.M. of the platform, in the platform frame returns to its initial position once the photon exits the glass, the situation is different as viewed from the lab frame.

In the lab frame, the C.M. initially moves to the left due to the relative velocity of the two frames. The C.M. then jumps an additional distance to the left when light enters the glass. The velocity of the platform increases and, in addition, the C.M. continues moving to the left as the mass loss bubble travels to the right, until the light exits the glass and the C.M. just moves at the speed of the platform. The net result is the C.M., as measured in the lab frame, has moved an additional distance to the left, beyond what is expected just due to the relative velocity between the platform and lab frame.

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